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On massive higher spin interactions in Fradkin-Vasiliev approach

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Here we discuss Fradkin-Vasiliev approach for investigation higher spin fields interactions. Initially this approach was developed for investigation of massless fields interactions, but using frame-like gauge invariant formalism for massive higher spin fields it can be straightforwardly applied to any combination of massive and/or massless fields. After brief description of such approach we consider the simplest possible examples — self-interaction and gravitational interaction for partially massless spin 2 field.

Keywords: *higher spins, gauge invariance, interactions.*

1 Fradkin-Vasilev approach for massive fields

First of all, let us briefly recall what frame-like formalism for higher spins is [1–3]. Really it is just a natural and straightforward generalization of well-known frame-like formalism for gravity

$$e_\mu{}^a, \omega_\mu{}^{ab} \quad \Longrightarrow \quad \Phi_\mu{}^{a_1\dots a_{s-1},b_1\dots b_k}, \quad 0 \le k \le s-1$$

where now instead of frame $e_\mu{}^a$ and Lorentz connection $\omega_\mu{}^{ab}$ one introduces a whole bunch of fields $\Phi_\mu{}^{a_1\dots a_{s-1},b_1\dots b_k}$, $0 \le k \le s-1$. It is very important that all these fields are gauge ones so that each field has its own gauge transformation:

$$\delta\Phi_\mu{}^{a_1\dots a_{s-1},b_1\dots b_k} \sim D_\mu \xi^{a_1\dots a_{s-1},b_1\dots b_k} + \dots$$

where dots stand for the terms without derivatives. Moreover, each one has its own gauge invariant field strength (which we generally will call curvatures):

$$\mathcal{R}_{\mu\nu}{}^{a_1\dots a_{s-1},b_1\dots b_k} \sim D_{[\mu}\Phi_{\nu]}{}^{a_1\dots a_{s-1},b_1\dots b_k} + \dots$$

where again dots stand for the terms without derivatives. Remarkably that the free Lagrangian can be rewritten in terms of these gauge invariant curvatures as

$$\mathcal{L}_0 = \sum \mathcal{R}\mathcal{R}$$

very similar to the usual Yang-Mills theories.

Now let us turn to the so-called constructive approach to investigation of possible interactions. Schematically it can be described as follows.

- Construct cubic vertex such that its free variations vanish on-shell:

$$\mathcal{L}_1 \sim \Phi\Phi\Phi \quad \Leftrightarrow \quad \delta_0\mathcal{L}_1 \approx 0$$

- Find corresponding corrections to gauge transformations such that now all variations in the linear approximation vanish off-shell:

$$\delta_1\Phi \sim \Phi\xi \quad \Leftrightarrow \quad \delta_0\mathcal{L}_1 + \delta_1\mathcal{L}_0 = 0$$

Note that these two steps are completely general and common to any constructive approach based on metric-like, frame-like or any other formalism one uses. But in a frame-like formalism there are two more steps.

- There exist quadratic deformations for all curvatures such that deformed curvatures transform covariantly:

$$\Delta\mathcal{R} \sim \Phi\Phi \quad \Longrightarrow \quad \delta\hat{\mathcal{R}} \sim \mathcal{R}\xi$$

- Moreover, interacting Lagrangian can be written in terms of these deformed curvatures as

$$\mathcal{L} \sim \sum \hat{\mathcal{R}}\hat{\mathcal{R}}$$

Such approach is straightforward and do allows one investigate possible interactions. But its first two (and the most hard) steps do not take into account the existence of gauge invariant curvatures though it is clear that in any gauge invariant theory the most simple and elegant formulation is the one in terms of gauge invariant objects. Roughly speaking, the Fradkin-Vasiliev approach [4,5] modifies the order of calculations to take advantages of these curvatures existence. Schematically, it can be described as follows.

- Construct deformations for curvatures and corresponding corrections to gauge transformations

$$\Delta\mathcal{R} \sim \Phi\Phi \quad \oplus \quad \delta_1\Phi \sim \Phi\xi \quad \Longrightarrow \quad \delta\hat{\mathcal{R}} \sim \mathcal{R}\xi$$

  so that they transform covariantly. In this, there is still some ambiguity because this guarantees that the equations are gauge invariant, but not necessarily Lagrangean.

- Put them into the Lagrangian and require it to be gauge invariant
$$\mathcal{L} \sim \sum \hat{\mathcal{R}}\hat{\mathcal{R}} \quad \Leftrightarrow \quad \delta\mathcal{L} = 0$$
thus fixing all remaining ambiguities.

As we have seen the main ingredients of such approach are frame-like formalism and gauge invariance. But gauge invariant frame-like formalism exists for massive higher spin particles as well [6, 7]. Thus such approach can be used for investigations of interactions for any combination of massless and/or massless fields. In the next section as an illustration we consider the most simple example — self-interaction and gravitational interaction for so-called partially massless spin 2 field.

2 Example: partially massless spin-2

First of all let us remind what is partially massless spin 2 [8–10]. It is an exotic representation that exists in de Sitter space only and has four physical degrees of freedom — helicities (± 2, ± 1). In a frame-like gauge invariant formalism it can be described by the following free Lagrangian:

$$\begin{aligned} \mathcal{L}_0 &= \frac{1}{2}\left\{ {}^{\mu\nu}_{ab} \right\} \Omega_\mu{}^{ac}\Omega_\nu{}^{bc} - \frac{1}{2}\left\{ {}^{\mu\nu\alpha}_{abc} \right\} \Omega_\mu{}^{ab} D_\nu f_\alpha{}^c + \\ &+ \frac{1}{4} B_{ab}{}^2 - \frac{1}{2}\left\{ {}^{\mu\nu}_{ab} \right\} B^{ab} D_\mu B_\nu + \\ &+ m[\left\{ {}^{\mu\nu}_{ab} \right\} \Omega_\mu{}^{ab} B_\nu + e^\mu{}_a B^{ab} f_\mu{}^b] \end{aligned}$$

which is invariant under the following gauge transformations:

$$\begin{aligned} \delta_0 f_\mu{}^a &= D_\mu \xi^a + \eta_\mu{}^a + m e_\mu{}^a \xi, \quad \delta_0 \Omega_\mu{}^{ab} = D_\mu \eta^{ab} \\ \delta_0 B_\mu &= D_\mu \xi + m \xi_\mu, \qquad \delta_0 B^{ab} = -2m\eta^{ab} \end{aligned}$$

For all four fields there exist gauge invariant curvatures:

$$\begin{aligned} \mathcal{F}_{\mu\nu}{}^{ab} &= D_{[\mu}\Omega_{\nu]}{}^{ab} - \frac{m}{2} e_{[\mu}{}^{[a} B_{\nu]}{}^{b]} \\ \mathcal{T}_{\mu\nu}{}^a &= D_{[\mu} f_{\nu]}{}^a - \Omega_{[\mu,\nu]}{}^a + m e_{[\mu}{}^a B_{\nu]} \\ \mathcal{B}_\mu{}^{ab} &= D_\mu B^{ab} + 2m\Omega_\mu{}^{ab} \\ \mathcal{B}_{\mu\nu} &= D_{[\mu} B_{\nu]} - B_{\mu\nu} - m f_{[\mu,\nu]} \end{aligned}$$

In this, free Lagrangian can be written in terms of these curvatures as follows:

$$\begin{aligned} \mathcal{L}_0 &\sim \frac{1}{4}\left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} \mathcal{F}_{\mu\nu}{}^{ab}\mathcal{F}_{\alpha\beta}{}^{cd} + 2\left\{ {}^{\mu\nu}_{ab} \right\} \mathcal{B}_\mu{}^{ac}\mathcal{B}_\nu{}^{bc} + \\ &+ m\left\{ {}^{\mu\nu\alpha}_{abc} \right\} \mathcal{T}_{\mu\nu}{}^a \mathcal{B}_\alpha{}^{bc} \end{aligned}$$

Self-interaction. By straightforward calculations it is not hard to find quadratic deformations for curvatures (note that we have already fixed all ambiguities here taking into account the invariance of the Lagrangian):

$$\begin{aligned} \Delta\mathcal{F}_{\mu\nu}{}^{ab} &= \frac{1}{4}[B_{[\mu}{}^{[a} B_{\nu]}{}^{b]} - e_{[\mu}{}^{[a} B^{b]c} B_{\nu]}{}^c] + \\ &+ \Omega_{[\mu}{}^{c[a}\Omega_{\nu]}{}^{b]c} \\ \Delta\mathcal{T}_{\mu\nu}{}^a &= 2\Omega_{[\mu}{}^{ab} f_{\nu]}{}^b - B_{[\mu}{}^a B_{\nu]} \\ \Delta\mathcal{B}_\mu{}^{ab} &= \Omega_\mu{}^{c[a} B^{b]c}, \qquad \Delta\mathcal{B}_{\mu\nu} = -B_{[\mu}{}^a f_{\nu]}{}^a \end{aligned}$$

In this, corresponding corrections to gauge transformations look like:

$$\begin{aligned} \delta_1 \Omega_\mu{}^{ab} &= -2\eta^{c[a}\Omega_\mu{}^{b]c} \\ \delta_1 f_\mu{}^a &= -2\eta^{ab} f_\mu{}^b + 2\Omega_\mu{}^{ab}\xi^b - B_\mu{}^a \xi \\ \delta_1 B^{ab} &= -\eta^{c[a} B^{b]c}, \qquad \delta_1 B_\mu = -B_\mu{}^a \xi^a \end{aligned}$$

It is not hard to check that under such corrected gauge transformations all curvatures transform covariantly:

$$\begin{aligned} \delta\hat{\mathcal{F}}_{\mu\nu}{}^{ab} &= 2\eta^{c[a}\mathcal{F}_{\mu\nu}{}^{b]c} \\ \delta\hat{\mathcal{T}}_{\mu\nu}{}^a &= 2\eta^{ab}\mathcal{T}_{\mu\nu}{}^b + 2\mathcal{F}_{\mu\nu}{}^{ab}\xi^b - \mathcal{B}_{[\mu,\nu]}{}^a \xi \\ \delta\hat{\mathcal{B}}_\mu{}^{ab} &= \eta^{c[a}\mathcal{B}_\mu{}^{b]c}, \qquad \delta\hat{\mathcal{B}}_{\mu\nu} = -\mathcal{B}_{[\mu,\nu]}{}^a \xi^a \end{aligned}$$

Gravitational interaction It turns out that in this case deformations for partially massless curvatures are minimal, i.e. correspond to standard minimal gravitational interaction:

$$\begin{aligned} \Delta\mathcal{F}_{\mu\nu}{}^{ab} &= \frac{m}{2}[h_{[\mu}{}^{[a} B_{\nu]}{}^{b]} - e_{[\mu}{}^{[a} B^{b]c} h_{\nu]}{}^c] - \\ &- \omega_{[\mu}{}^{c[a}\Omega_{\nu]}{}^{b]c} \\ \Delta\mathcal{T}_{\mu\nu}{}^a &= -\omega_{[\mu}{}^{ab} f_{\nu]}{}^b - \Omega_{[\mu}{}^{ab} h_{\nu]}{}^b - m h_{[\mu}{}^a B_{\nu]} \\ \Delta\mathcal{B}_\mu{}^{ab} &= -\omega_\mu{}^{c[a} B^{b]c} \\ \Delta\mathcal{B}_{\mu\nu} &= B_{[\mu}{}^a h_{\nu]}{}^a + m f_{[\mu}{}^a h_{\nu]}{}^a \end{aligned}$$

where $h_\mu{}^a$ and $\omega_\mu{}^a$ are usual gravitational frame field and Lorentz connection. But to find deformation for Riemann tensor and torsion requires much more work with the result:

$$\begin{aligned} \Delta R_{\mu\nu}{}^{ab} &= -\frac{1}{2}\Omega_{[\mu}{}^{c[a}\Omega_{\nu]}{}^{b]c} - \frac{1}{4} B_{[\mu}{}^{[a} B_{\nu]}{}^{b]} + \\ &+ \frac{1}{4} e_{[\mu}{}^{[a} B^{b]c} B_{\nu]}{}^c - \frac{m}{2} B_{[\mu}{}^{[a} f_{\nu]}{}^{b]} + \\ &+ \frac{m}{2} e_{[\mu}{}^{[a} B^{b]c} f_{\nu]}{}^c - \frac{m^2}{2} f_{[\mu}{}^{[a} f_{\nu]}{}^{b]} \\ \Delta T_{\mu\nu}{}^a &= -\Omega_{[\mu}{}^{ab} f_{\nu]}{}^b + B_{[\mu}{}^a B_{\nu]} + m f_{[\mu}{}^a B_{\nu]} \end{aligned}$$

Here we again take into account invariance of the complete Lagrangian i.e. the sum of Lagrangian for massless gravitational spin 2 and partially massless spin 2 fields. Similarly to the self-interaction case, form the formulas given it straightforward to find appropriate corrections to gauge transformations and check that all deformed curvatures do transform covariantly.

Conclusion

Thus Fradkin-Vasiliev approach provides effective framework for investigation of cubic vertices for massless and massive fields. It also allows investigate possibilities to go beyond linear approximation, though it requires more work.

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О ВЗАИМОДЕЙСТВИИ МАССИВНЫХ ВЫСШИХ СПИНОВ В ПОДХОДЕ ФРАДКИНА-ВАСИЛЬЕВА

Мы обсуждаем подход Фрадкина-Васильва к исследованию взаимодействий полей с высшими спинами. Изначально этот подход был развит для исследования взаимодействий безмассовых полей. Однако, используя реперный калибровочно инвариантный формализм для масивных полей, его можно применять для любой комбинации массивных и/или безмассовых полей. После краткого описания такого подхода мы рассмтриваем простейший возможный пример — самодействие и гравитационное взаимодействие для частично безмассового спина 2.

Ключевые слова: *высшие спины, калибровочная инвариантность, взаимодействия.*

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