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PALATINI FORMULATION OF $L(R)$ GRAVITY

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I. Introduction

It now seems well-established that the expansion of our universe is currently in an accelerating phase. The most direct evidence for this is from the measurements of type Ia supernova [1]. Other indirect evidences such as the observations of CMB by the WMAP satellite [2],

large-scale galaxy surveys by 2dF and SDSS [3] also seem supporting this.

But now the mechanisms responsible for this acceleration are not very clear. Many authors introduce a mysterious cosmic fluid called dark energy to explain this (see Ref. [4] for a review). On the other hand, some authors suggest that maybe there does not exist

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such mysterious dark energy, but the observed cosmic acceleration is a signal of our first real lack of understanding of gravitational physics [5]. An example is the braneworld theory of Dvali et al. [7]. Recently, there are active discussions in this direction by modifying the action for gravity [7-17]. Specifically, a $1/R$ term is suggested to be added to the action [7]: the so called $1/R$ gravity. It is interesting that such term may be predicted by string/M-theory [8]. In Ref. [12], Vollick used Palatini variational principle to derive the field equations for $1/R$ gravity. In Ref. [11], Dolgov *et al.* argued that the fourth order field equations following from the metric variation suffer serious instability problem. If this is indeed the case, the Palatini formulation appears quite appealing, because the second order field equations following from Palatini variation are free of this sort of instability [13]. However, it is also interesting to note that quantum effects may resolve the instabilities, see Ref. [14]. In this paper, we will review deriving the full Modified Friedmann equation for Palatini formulation of the $L(R)$ gravity. Then we will discuss the application of this derivation to $1/R$, R^2 , $1/R + R^2$ and $\ln R$ gravity. Recently, Palatini formulation of $L(R)$ gravity was also considered in Ref. [15].

II The Modified Friedmann Equation

In general, when handled in Palatini formulation, one considers the action to be a functional of the metric $\bar{g}_{\mu\nu}$ and a connection $\hat{\nabla}_\mu$ which is another independent variable besides the metric. The resulting modified gravity action can be written as

$$S[\bar{g}_{\mu\nu}, \hat{\nabla}_\mu] = \int d^4x \sqrt{-\bar{g}} \frac{1}{2\kappa^2} L(\hat{R}) + S_m \quad (2)$$

where we use the metric signature $\{-, +, +, +\}$, $\kappa^2 = 8\pi G$, $\hat{R}_{\mu\nu}$ is the Ricci tensor of the connection $\hat{\nabla}_\mu$, $\hat{R} = \bar{g}^{\mu\nu} \hat{R}_{\mu\nu}$, and S_m is the matter action.

Varying the action (1) with respect to $\bar{g}_{\mu\nu}$ gives

$$L'(\hat{R}) \hat{R}_{\mu\nu} - \frac{1}{2} L(\hat{R}) \bar{g}_{\mu\nu} = \kappa^2 T_{\mu\nu} \quad (2)$$

where a prime denotes differentiation with respect to \hat{R} and $T_{\mu\nu}$ is the energy-momentum tensor given by

$$T_{\mu\nu} = -\frac{2}{\sqrt{-\bar{g}}} \frac{\delta S_M}{\delta \bar{g}^{\mu\nu}} \quad (3)$$

where S_M is the matter action. For a universe filled with perfect fluid, $T_{\mu\nu} = \{-\rho, p, p, p\}$. Note that the local conservation of energy momentum $\bar{\nabla}_\mu T^{\mu\nu} = 0$ is a result of the covariance of the action (1) and Noether theorem, thus it is independent of the gravitational field

equations. Then the energy conservation equation $\dot{\rho} + 3H(\rho + p) = 0$ is unchanged.

In the Palatini formulation, the connection is associated with $\hat{g}_{\mu\nu} \equiv L'(\hat{R}) \bar{g}_{\mu\nu}$, which is known from varying the action with respect to $\hat{\Gamma}_{\mu\nu}^\lambda$. Thus the Christoffel symbol with respect to $\hat{g}_{\mu\nu}$ is given in terms of the Christoffel symbol with respect to $\bar{g}_{\mu\nu}$ by

$$\hat{\Gamma}_{\mu\nu}^\lambda = \bar{\Gamma}_{\mu\nu}^\lambda + \frac{1}{2L'} [2\delta_{(\mu}^\lambda \partial_{\nu)} L' - \bar{g}_{\mu\nu} \bar{g}^{\lambda\sigma} \partial_\sigma L'] \quad (4)$$

Thus the Ricci curvature tensor is given by

$$\hat{R}_{\mu\nu} = \bar{R}_{\mu\nu} + \frac{3}{2} (L')^{-2} \bar{\nabla}_\mu L' \bar{\nabla}_\nu L' - (L')^{-1} \bar{\nabla}_\mu \bar{\nabla}_\nu L' - \frac{1}{2} (L')^{-1} \bar{g}_{\mu\nu} \bar{\nabla}_\sigma \bar{\nabla}^\sigma L' \quad (5)$$

and

$$\hat{R} = \bar{R} - 3(L')^{-1} \bar{\nabla}_\mu \bar{\nabla}^\mu L' + \frac{3}{2} (L')^{-2} \bar{\nabla}_\mu L' \bar{\nabla}^\mu L' \quad (6)$$

where $\bar{R}_{\mu\nu}$ is the Ricci tensor with respect to $\bar{g}_{\mu\nu}$ and

$\hat{R} = \bar{g}^{\mu\nu} \hat{R}_{\mu\nu}$. Note by contracting Eq.(2), we get:

$$L'(\hat{R}) \hat{R} - 2L(\hat{R}) = \kappa^2 T \quad (7)$$

Assume we can solve \hat{R} as a function of T from Eq. (7). Thus Eqs. (6), do define the Ricci tensor with respect to $h_{\mu\nu}$.

We will consider the general Robertson-Walker metric (note that this is an ansatz for $\bar{g}_{\mu\nu}$ and is the result of the assumed homogenization and isotropy of the universe, thus its form is independent of the gravity theory):

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right) \quad (8)$$

where k is the spatial curvature and $k = -1, 0, 1$ correspond to open, flat and closed universe respectively. The $a(t)$ is called the scale factor of the universe.

From equations (8), and (6), we can get the non-vanishing components of the Ricci tensor:

$$\hat{R}_{00} = -3\frac{\ddot{a}}{a} + \frac{3}{2} (L')^{-2} (\partial_0 L')^2 - \frac{3}{2} (L')^{-1} \bar{\nabla}_0 \bar{\nabla}_0 L' \quad (9)$$

$$\hat{R}_{ij} = [a\ddot{a} + 2\dot{a}^2 + 2k + (L')^{-1} \bar{\Gamma}_{ij}^0 \partial_0 L' + \frac{a^2}{2} (L')^{-1} \bar{\nabla}_0 \bar{\nabla}_0 L'] \delta_{ij} \quad (10)$$

where a dot denotes differentiation with respect to t .

Substituting equations (9) and (10) into the field equations (2), we can get

$$6H^2 + 3H(L')^{-1} \partial_0 L' + \frac{3}{2} (L')^{-2} (\partial_0 L')^2 + 6\frac{k}{a^2} = \frac{\kappa^2(\rho + 3p) + L}{L'} \quad (11)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter, ρ and p are the total energy density and total pressure respectively.

Using the energy conservation equation $\dot{\rho} + 3H(\rho + p) = 0$, we have

$$\partial_0 L' = \frac{2\kappa^2}{\beta}(1 - 3c_s^2)(\rho + p) \quad (12)$$

where $c_s^2 = dp/d\rho$ is the sound velocity.

Substituting Eq. (12) into Eq. (13) we can get the Modified Friedmann (MF) equation of $L(R)$ gravity in Palatini formulation.

III. Applications

In this section, we review the application of the general Modified Friedmann equation to cosmology.

A. $1/R$ gravity

The Lagrangian is given by $L(\hat{R}) = \hat{R} - \mu^4/\hat{R}$ [7]. It is interesting to note that, in Ref. [8], Nojiri and Odintsov have shown that this action can be derived from string/M theory.

The MF equation follows from eq. (13) [13]

$$H^2 = \frac{\kappa\rho - \alpha \left(G\left(\frac{\kappa\rho}{\alpha}\right) - \frac{1}{3G\left(\frac{\kappa\rho}{\alpha}\right)} \right)}{\left(1 + \frac{1}{3G\left(\frac{\kappa\rho}{\alpha}\right)^2} \right) \left(6 + 3F\left(\frac{\kappa\rho}{\alpha}\right) \left[1 + \frac{1}{2}F\left(\frac{\kappa\rho}{\alpha}\right) \right] \right)} \quad (13)$$

where the two functions G and F are defined as

$$G(x) = -\left(\frac{1}{2}x + \sqrt{1 + \frac{1}{4}x^2} \right) \quad (14)$$

$$F(x) = \frac{x}{\left(G(x)^2 + \frac{1}{3} \right) \sqrt{1 + \frac{1}{4}x^2}} \quad (15)$$

In Ref. [13], we have shown that the above MF equation can fit the current SN Ia data at an acceptable level. However, the effective equation of state it gives shows some pathological behaviors.

B. R^2 gravity

It is a well-known result that a R^2 term in the Lagrangian can drive an early universe inflation without inflaton [24] (See also Ref. [25] for a comprehensive discussion of R^2 gravity).

In the Palatini formulation, the MF equation is [19]

$$H^2 = \frac{2\kappa^2(\rho_m + \rho_r) + \frac{(\kappa^2\rho_m)^2}{3\beta}}{\left(1 + \frac{2\kappa^2\rho_m}{3\beta} \right) \left(6 + 3F_0\left(\frac{\kappa^2\rho_m}{\beta}\right) \left(1 + \frac{1}{2}F_0\left(\frac{\kappa^2\rho_m}{\beta}\right) \right) \right)} \quad (16)$$

where the function F_0 is given by

$$F_0(x) = -\frac{2x}{1 + \frac{2}{3}x} \quad (17)$$

It is interesting to see from Eq. (16) that all the effects of the R^2 term are determined by ρ_m . If $\rho_m = 0$, Eq. (16) simply reduces to the standard Friedmann equation.

At late cosmological times when $\kappa^2\rho_m/\beta \ll 1$, $F \sim 0$, the MF equation (16) reduces to the standard Friedmann equation:

$$H^2 = \frac{\kappa^2}{3}(\rho_m + \rho_r) \quad (18)$$

Thus from the BBN constraints on the Friedmann equation [26], β should be sufficiently large so that the condition $\kappa^2\rho_m/\beta \ll 1$ is satisfied in the era of BBN. In typical model of R^2 inflation, β is often taken to be the order of the Plank scale [24].

Then we will see that whatever we assume on the value of ρ_m during inflation, a R^2 driven inflation can not happen.

Firstly, since inflation happens in very early universe, where the temperature is typical of the 10^{15} Gev order, if we assume that almost all the matter in the universe at that time is relativistic so that $\kappa^2\rho_m/\beta \ll 1$, then as we have shown above, the MF equation reduces to the standard Friedmann equation and thus no inflation happens. Note that at the inflation energy scale, all the standard model particles will be relativistic.

Secondly, if there are enough exotic objects other than the standard model particles that will be non-relativistic during the inflation era so that $\kappa^2\rho_m/\beta \gg 1$. Those objects may be primordial black holes, various topological defects which we will not specify here. In this case, from Eq. (17), the MF equation (16) will reduce to

$$H^2 = \frac{\kappa^2\rho_m}{\beta} + \frac{2\beta\rho_r}{7\rho_m} + \frac{2\beta}{7} \quad (19)$$

Then we can see that if the β term dominates over the other two terms, it will drive an inflation. But this equation is derived under the assumption that $\beta \ll \kappa^2\rho_m$. Thus no inflation will appear, too.

Thus, the difference between Palatini and metric formulation of the same higher derivative gravity theory is quite obvious here.

C. $1/R + R^2$ gravity

In Ref. [17], Nojiri and Odintsov showed that a combination of the $1/R$ and R^2 terms can drive both the current acceleration and inflation. The Palatini form of this theory is studied in Ref. [19].

The MF equation reads,

$$H^2 = \frac{\kappa^2 \rho_m + 2\kappa^2 \rho_r + \alpha \left[G(x) - \frac{1}{3G(x)} + \frac{\alpha}{3\beta} G(x)^2 \right]}{\left[1 + \frac{1}{3G(x)^2} + \frac{2\alpha}{3\beta} G(x) \right] \left[6 + 3F(x) \left(1 + \frac{1}{2} F(x) \right) \right]} \quad (20)$$

where $x \equiv \frac{\kappa^2 \rho_m}{\alpha}$ and the two functions G and F are given by

$$G(x) = \frac{1}{2} \left[x + 2\sqrt{1 + \frac{1}{4}x^2} \right] \quad (21)$$

$$F(x) = \frac{\left(1 - \frac{\alpha}{\beta} G(x)^3 \right) x}{\left(G(x)^2 + \frac{2\alpha}{3\beta} G(x)^3 + \frac{1}{3} \right) \sqrt{1 + \frac{1}{4}x^2}} \quad (22)$$

In order to be consistent with observations, we should have $\alpha \ll \beta$. We can see this in two different ways.

Firstly, when $\kappa^2 \rho_m \gg \alpha$, from Eq. (21), $G \sim \kappa^2 \rho_m / \alpha$. From the BBN constraints, we know the MF equation should reduce to the standard one in the BBN era [26]. This can be achieved only when $F \sim 0$ and from Eq. (22), this can be achieved only when $\alpha \ll \beta$ and $1 \ll \kappa^2 \rho_m / \alpha \ll (\beta/\alpha)^{1/3}$.

Secondly, when $\kappa^2 \rho_m \ll \alpha$, we can expand the RHS of Eq. (20) to first order in $\kappa^2 \rho_m / \alpha$:

$$H^2 = \frac{\frac{11-\alpha/\beta}{8-4\alpha/\beta} \kappa^2 \rho_m + \frac{3}{2-\alpha/\beta} \kappa^2 \rho_r + \frac{1}{2} \alpha}{6 + \frac{9}{4-2\alpha/\beta} (1+\alpha/\beta) \frac{\kappa^2 \rho_m}{\alpha}} \quad (23)$$

When $\alpha \ll \beta$, this will reduce exactly to the first order MF equation in the $1/R$ theory [13]. Since we have shown there that the MF equation in $1/R$ theory can fit the SN Ia data at an acceptable level, the above MF equation can not deviate from it too large, thus the condition $\alpha \ll \beta$ should be satisfied.

D. $\ln R$ gravity

In Ref. [18] Nojiri and Odintsov proposed using a single $\ln R$ term to explain both the current acceleration and inflation. The Palatini formulation of $\ln R$ gravity is studied in Ref. [27].

The Modified Friedmann equation reads,

$$H^2 = \frac{\kappa \rho_m + 2\kappa \rho_r - \beta \left(\frac{R}{\beta} - \ln \frac{R}{-\alpha} \right)}{\left(1 - \frac{\beta}{R} \right) \left(6 + 3F(x) \left(1 + \frac{1}{2} F(x) \right) \right)} \quad (24)$$

where $x \equiv \frac{\kappa \rho_m}{\beta}$ and F is defined as

$$F(x) = \frac{3x}{(R(x)/\beta)^2 - 2R(x)/\beta} \quad (25)$$

It can be seen from equations (24) and (25) that when $\beta \rightarrow 0$, the MF equation will reduce continuously to the standard Friedmann equation. Thus, the $\ln R$ modification is a smooth and continuous modification.

Let us first discuss the cosmological evolution without matter and radiation. Define the parameter n as $R_0 = -\alpha e^{-n}$. Substitute this to the vacuum field equation $L(R) = 0$, we can get $\alpha = e^n (2n+1)\beta$ and $R_0 = -(2n+1)\beta$. Substitute those to the vacuum MF equation and set $t = 0$, we have

$$H_0^2 = \frac{\beta(n+1)}{6 \left(1 + \frac{1}{2n+1} \right)} \quad (26)$$

Thus when $\alpha \sim H_0^2 \sim (10^{-33} eV)^2$ and $n > -1/2$, the $\ln R$ modified gravity can indeed drive a current exponential acceleration compatible with the observation. The role of the parameter β is similar to a cosmological constant or the coefficient of the $1/R$ term in the $1/R$ gravity [13].

When the energy density of dust can not be neglected, i.e. $\kappa \rho_m / \beta \gg 1$, $F(x) \sim 0$ and if α satisfies $|\ln(\kappa \rho_m / \alpha)| \ll \kappa \rho_m / \beta$, i.e. $\exp(-\kappa \rho_m / \beta) \ll \alpha / \beta \ll \exp(\kappa \rho_m / \beta)$, then $R \sim -\kappa \rho_m$. Then the MF equation (24) reduces to the standard Friedmann equation

$$H^2 = \frac{\kappa}{3} (\rho_m + \rho_r) \quad (27)$$

Thus if $\exp(-\kappa \rho_{m,BBN} / \beta) \ll \alpha / \beta \ll \exp(\kappa \rho_{m,BBN} / \beta)$ where $\rho_{m,BBN}$ is the energy density of dust in the epoch of BBN, the $\ln R$ gravity can be consistent with the BBN constraints on the form of Friedmann equation [26]. One possible choice is $\alpha = \beta$, for which the vacuum solution can be solved exactly $R_0 = -\alpha$. Since

$\beta \sim H_0^2$, the condition $\kappa_p / \beta \gg 1$ breaks down only in recent cosmological time. Thus the universe evolves in the standard way until recently, when $\ln R$ term begins to dominate and drives the observed cosmic acceleration.

IV. Conclusions and Discussions

In this paper we reviewed the Palatini formulation of $L(R)$ gravity and its applications to cosmology. The nature of dark energy is so mysterious that it is promising to seek further the idea that there does not exist such mysterious dark energy, but it is General Relativity that fails at large scale.

The current "standard theory" of gravitation, Einstein's General Relativity (GR) has passed many tests within Solar system. To reconcile the successful

GR predictions within the solar system, the extended gravity theories may be required to be scale sensitive. It could be challenging and profound to locate the additional curvature terms in our above discussions what form of scale dependence.

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