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INFINITE-DIMENSIONAL SYMMETRIES OF THE TWISTOR STRING MODELS

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It is shown that similarly to massless superparticle models, space-time symmetry of the classical action of the Berkovits twistor string is infinite-dimensional. Its superalgebra contains finite-dimensional subalgebra that includes the generators of $psl(4|4,\mathbb{R})$ superalgebra. In quantum theory this infinite-dimensional symmetry breaks down to $SL(4|4,\mathbb{R})$ one.

Keywords: *supertwistor, twistor string, infinite-dimensional symmetry, anomaly.*

1 Introduction

Twistor string theory [1, 2] inspired remarkable progress in understanding spinor and twistor structures underlying scattering amplitudes in gauge theories and gravity. To gain further insights into the properties of twistor strings it is helpful to identify their symmetries both classical and quantum. It was shown in [1,3] that except for an obvious $PSL(4|4,\mathbb{R})$ global symmetry twistor strings are also invariant under its Yangian extension that is closely related to infinite-dimensional symmetry of integrable $N=4$ superYang-Mills theory.

In this note we exhibit infinite-dimensional global symmetry of the world-sheet action of the Berkovits twistor string and its generalization with ungauged $GL(1,\mathbb{R})$ symmetry. In the quantum theory infinite-dimensional symmetries break down to $SL(4|4,\mathbb{R})$ one, whose consistency was proved in [3]. Let us mention that infinite-dimensional nature of massless superparticles' symmetries was revealed already in [4].

2 Twistor string models and their symmetries

For Lorentzian signature world sheet the simplest twistor string action can be presented as

$$\begin{aligned} S &= \int d\tau d\sigma(\mathscr{L}_L+\mathscr{L}_R): \\ \mathscr{L}_L &= -2(Y_\alpha\partial_- Z^\alpha+\eta_i\partial_-\xi^i)+\mathscr{L}_{\mathrm{L-mat}}, \\ \mathscr{L}_R &= -2(\bar{Y}_\alpha\partial_+\bar{Z}^\alpha+\bar{\eta}_i\partial_+\bar{\xi}^i)+\mathscr{L}_{\mathrm{R-mat}}, \end{aligned} \tag{1}$$

where $\partial_\pm=\frac{1}{2}(\partial_\tau\pm\partial_\sigma)$, $\sigma^\pm=\tau\pm\sigma$, $Y_{+\alpha}\equiv Y_\alpha$, $\bar{Y}_{-\alpha}\equiv\bar{Y}_\alpha$, $\eta_{+i}\equiv\eta_i$, $\bar{\eta}_{-i}\equiv\bar{\eta}_i$ and $\mathscr{L}_{\mathrm{L(R)-mat}}$ are Lagrangians for left- and right-moving non-twistor matter variables, whose contribution to the world-sheet conformal anomaly equals $c=\bar{c}=26$ to cancel that of $(b,c)-$ghosts. Such variables may contain a current algebra for some Lie group (see, e.g., [5]). In Berkovits twistor-string model [2] global scale symmetry for both left- and right-movers

$$\begin{aligned} &\delta Z^\alpha=\Lambda Z^\alpha,\ \delta Y_\alpha=-\Lambda Y_\alpha,\ \delta\xi^i=\Lambda\xi^i,\ \delta\eta_i=-\Lambda\eta_i; \\ &\delta\bar{Z}^\alpha=\bar{\Lambda}\bar{Z}^\alpha,\ \delta\bar{Y}_\alpha=-\bar{\Lambda}\bar{Y}_\alpha,\ \delta\bar{\xi}^i=\bar{\Lambda}\bar{\xi}^i,\ \delta\bar{\eta}_i=-\bar{\Lambda}\bar{\eta}_i \end{aligned} \tag{2}$$

is gauged by adding to the action (1) appropriate constraints $T=Y_\alpha Z^\alpha+\eta_i\xi^i\approx 0$ and $\bar{T}=\bar{Y}_\alpha\bar{Z}^\alpha+\bar{\eta}_i\bar{\xi}^i\approx 0$ with the Lagrange multipliers

$$S_{GL(1,\mathbb{R})}=\int d\tau d\sigma(\lambda T+\bar{\lambda}\bar{T}). \tag{3}$$

This necessitates add two units to the central charges of the matter variables to compensate that of (b,c)-ghosts and ghosts for the gauged $GL(1,\mathbb{R})$ symmetry.

Definition of the open string sector, that to date is the only one well-understood, is based on the conditions $\mathcal{Z}^A=\bar{\mathcal{Z}}^A$, $\mathcal{Y}_B=\bar{\mathcal{Y}}_B$ imposed on the world-sheet boundary on the supertwistors $\mathcal{Z}^A=(Z^\alpha,\xi^i)$, $\bar{\mathcal{Z}}^A=(\bar{Z}^\alpha,\bar{\xi}^i)$ and their duals $\mathcal{Y}_B=(Y_\beta,\eta_j)$, $\bar{\mathcal{Y}}_B=(\bar{Y}_\beta,\bar{\eta}_j)$. So taking into account reality condition of the Lagrangian one is led to consider left(right)-moving supertwistors $\mathcal{Z}^A$ ($\bar{\mathcal{Z}}^A$) and dual supertwistors $\mathcal{Y}_B$ ($\bar{\mathcal{Y}}_B$) as independent variables with real components. Such supertwistors are adapted for the description of fields on $D=4$ $N=4$ superspace for the space-time of signature $(++--)$[1]. Conformal group of Minkowski space-time of such a signature is $SO(3,3)\sim SL(4,\mathbb{R})$ and its minimal $N=4$ supersymmetric extension is $PSL(4|4,\mathbb{R})$ with the bosonic subgroup $SL(4,\mathbb{R})\times SL(4,\mathbb{R})$ implying that bosonic and odd components of $\mathcal{Z}^A$ belong to the fundamental representation of $SL(4,\mathbb{R})_L\times SL(4,\mathbb{R})_L$, whereas bosonic and odd components of $\mathcal{Y}_A$ belong to the antifundamental representation. Correspondingly components of $\bar{\mathcal{Z}}^A$ and $\bar{\mathcal{Y}}_A$ transform according to the (anti)fundamental representation of $SL(4,\mathbb{R})_R\times SL(4,\mathbb{R})_R$.

[1]Detailed discussion of the reality conditions of the twistor string Lagrangian for both Lorentzian and Euclidean world sheets, and different real structures in the complex supertwistor space associated with $D=4$ space-times of various signatures can be found, e.g., in [6].

Focusing on the sector of left-movers of the model (1) and applying the Dirac approach yields equal-time D.B. relations

$$\begin{aligned}\{Z^\alpha(\sigma), Y_\beta(\sigma')\}_{D.B.} &= \delta^\alpha_\beta\delta(\sigma-\sigma'),\\ \{\xi^i(\sigma), \eta_j(\sigma')\}_{D.B.} &= \delta^i_j\delta(\sigma-\sigma')\end{aligned} \quad (4)$$

that in terms of the $PSL(4|4,\mathbb{R})$ supertwistors can be written as

$$\{\mathcal{Z}^A(\sigma), \mathcal{Y}_B(\sigma')\}_{D.B.} = \delta^A_B\delta(\sigma-\sigma'). \quad (5)$$

Similar relations hold for the right-movers.

2.1 Classical symmetries of twistor strings

Global symmetry of the left-moving part of the action (1) is generated on D.B. by the function

$$\begin{aligned}&G = \int d\sigma \sum_{L\geq 0} G_{(L)}(\sigma),\\ &G_{(L)}(\sigma) = \mathcal{Y}_B(\sigma)\Lambda^B{}_{A_L\ldots A_1}\mathcal{Z}^{A_1}(\sigma)\cdots\mathcal{Z}^{A_L}(\sigma).\end{aligned} \quad (6)$$

For arbitrary value of the order L transformation rules for the supertwistors read

$$\begin{aligned}&\delta\mathcal{Z}^A(\sigma) = \Lambda^A{}_{B(L)}\mathcal{Z}^{B(L)}(\sigma),\\ &\delta\mathcal{Y}_A(\sigma) = -L\mathcal{Y}_C(\sigma)\Lambda^C{}_{AB_{L-1}\ldots B_1}\mathcal{Z}^{B(L-1)}(\sigma),\end{aligned} \quad (7)$$

where convenient notation to be used below is $\mathcal{Z}^{A(L)} = \mathcal{Z}^{A_1}\cdots\mathcal{Z}^{A_L}$ ($\mathcal{Z}^{A(0)} = 1$, $\mathcal{Z}^{A(1)} = \mathcal{Z}^A$) and $\bar{\mathcal{Z}}_{A(L)} = \bar{\mathcal{Z}}_{A_1}\cdots\bar{\mathcal{Z}}_{A_L}$ ($\bar{\mathcal{Z}}_{A(0)} = 1$, $\bar{\mathcal{Z}}_{A(1)} = \bar{\mathcal{Z}}_A$)[2]. Associated Noether current densities up to irrelevant numerical factor are given by the monomials

$$T^{(L)}{}_B{}^{A(L)}(\sigma) = \mathcal{Y}_B\mathcal{Z}^{A(L)}(\sigma), \quad L \geq 0 \quad (8)$$

that enter generating functions $G_{(L)}$. On D.B. they generate the twistor string algebra (TSA)[3]

$$\begin{aligned}&\{T^{(L)}{}_B{}^{A(L)}(\sigma), T^{(M)}{}_D{}^{C(M)}(\sigma')\}_{D.B.}\\ &= (\delta^{A(1)}_D T^{(L+M-1)}{}_B{}^{A(L-1)C(M)}\\ &-\delta^{C(1)}_B T^{(L+M-1)}{}_D{}^{A(L)C(M-1)})(\sigma)\delta(\sigma-\sigma').\end{aligned} \quad (9)$$

The finite-dimensional subalgebra of TSA is spanned, apart from the order 0 generator $\mathcal{Y}_A(\sigma)$ that is responsible for constant shift of the supertwistor components, by quadratic monomial

$$T_A{}^B(\sigma) = \mathcal{Y}_A\mathcal{Z}^B(\sigma) \quad (10)$$

generating $gl(4|4,\mathbb{R})$ superalgebra

$$\begin{aligned}&\{T_A{}^B(\sigma), T_C{}^D(\sigma')\}_{D.B.}\\ &= (\delta^B_C T_A{}^D - (-)^{\varepsilon^b_a\varepsilon^d_c}\delta^D_A T_C{}^B)(\sigma)\delta(\sigma-\sigma'),\end{aligned} \quad (11)$$

where $\varepsilon^b_a = (-)^{a+b}$.

Irreducible components of $gl(4|4,\mathbb{R})$ current densities (10) are

$$\begin{aligned}&T_A{}^B(\sigma) = \{\widetilde{T}_\alpha{}^\beta,\ \widetilde{T}_i{}^j;\ Q_\alpha{}^j,\ Q_i{}^\beta;\ T,\ U\}:\\ &\widetilde{T}_\alpha{}^\beta = Y_\alpha Z^\beta - \tfrac{1}{4}\delta^\beta_\alpha(YZ),\ \widetilde{T}_i{}^j = \eta_i\xi^j - \tfrac{1}{4}\delta^j_i(\eta\xi);\\ &Q_\alpha{}^j = Y_\alpha\xi^j,\ Q_i{}^\beta = \eta_i Z^\beta;\\ &T = Y_\alpha Z^\alpha + \eta_i\xi^i,\ U = Y_\alpha Z^\alpha - \eta_i\xi^i.\end{aligned} \quad (12)$$

On D.B. they generate infinitesimal $SL(4,\mathbb{R})\times SL(4,\mathbb{R})$ rotations of the supertwistor components

$$\begin{aligned}&\delta Z^\alpha(\sigma) = \Lambda^\alpha{}_\beta Z^\beta(\sigma),\\ &\delta Y_\alpha(\sigma) = -Y_\beta(\sigma)\Lambda^\beta{}_\alpha,\ \Lambda^\alpha{}_\alpha = 0;\\ &\delta\xi^i(\sigma) = \Lambda^i{}_j\xi^j(\sigma),\ \delta\eta_i(\sigma) = -\eta_j(\sigma)\Lambda^j{}_i,\ \Lambda^i{}_i = 0\end{aligned} \quad (13)$$

and supersymmetry transformations

$$\begin{aligned}&\delta Z^\alpha(\sigma) = \varepsilon^\alpha{}_i\xi^i(\sigma),\ \delta\eta_i(\sigma) = -Y_\alpha(\sigma)\varepsilon^\alpha{}_i;\\ &\delta Y_\alpha(\sigma) = -\eta_i(\sigma)\epsilon^i{}_\alpha,\ \delta\xi^i(\sigma) = \epsilon^i{}_\alpha Z^\alpha(\sigma),\end{aligned} \quad (14)$$

where $\varepsilon^\alpha{}_i$ and $\epsilon^i{}_\alpha$ are independent odd parameters with 16 real components each. $T(\sigma)$ generates $GL(1,\mathbb{R})$ transformations (2) and $U(\sigma)$ – 'twisted' $GL_t(1,\mathbb{R})$ transformations

$$\begin{aligned}&\delta Z^\alpha(\sigma) = \Lambda_t Z^\alpha(\sigma),\ \delta Y_\alpha(\sigma) = -\Lambda_t Y_\alpha(\sigma),\\ &\delta\xi^i(\sigma) = -\Lambda_t\xi^i(\sigma),\ \delta\eta_i(\sigma) = \Lambda_t\eta_i(\sigma).\end{aligned} \quad (15)$$

2.2 Quantum symmetries of twistor strings

It was shown in [3] that $SL(4|4,\mathbb{R})$ symmetry is preserved at the quantum level, whereas the generator U of the 'twisted' $GL_t(1,\mathbb{R})$ symmetry has anomalous OPE with the world-sheet stress-energy tensor implying that corresponding symmetry is broken in twistor string theory. Thus possible type of infinite-dimensional symmetry that could survive in the quantum theory is restricted to that based on $sl(4|4,\mathbb{R})$ as finite-dimensional subalgebra. Since $sl(4|4,\mathbb{R})$ superalgebra belongs to the family of $sl(M|M,\mathbb{R})$ superalgebras, whose properties differ from those of $sl(M|N,\mathbb{R})$ superalgebras with $M \neq N$, one is forced to take components of supertwistors as building blocks of their generators.

[2]Composite objects like $\mathcal{Z}^{A(L)}$ and $\bar{\mathcal{Z}}_{A(L)}$ are graded symmetric in their indices. In general it is assumed graded symmetry in supertwistor indices denoted by the same letters. Similarly one defines the products of supertwistor bosonic and fermionic components as $Z^{\alpha(m)} = Z^{\alpha_1}\cdots Z^{\alpha_m}$, $\bar{Z}_{\alpha(m)} = \bar{Z}_{\alpha_1}\cdots\bar{Z}_{\alpha_m}$ and $\xi^{i[n]} = \xi^{i_1}\cdots\xi^{i_n}$, $\bar{\xi}_{i[n]} = \bar{\xi}_{i_1}\cdots\bar{\xi}_{i_n}$ ($n \leq N = 4$) that are (anti)symmetric. Antisymmetry in a set of n indices is indicated by placing n in square brackets. Both symmetrization and antisymmetrization are performed with unit weight.

[3]To be more precise one has to introduce TSA as an infinite-dimensional Lie superalgebra and then consider its loop version pertinent to twistor-string global symmetry. Let us also note that the subscript L in the notation of symmetry groups and algebras will be omitted as the discussion is concentrated on the sector of left-movers only. On the boundary left- and right-moving variables are identified and thus also no subscripts are needed.

2.2.1 Superalgebraic perspective on quantum higher-spin symmetries

In the bosonic limit TSA reduces to $TSA_{\mathfrak{b}}$ – an infinite-dimensional Lie algebra, whose generators are obtained from (8) by setting to zero fermionic components of the supertwistors. Order 0 and 1 generators are given by the dual bosonic twistor Y_α and $gl(4,\mathbb{R})$ generators $Y_\alpha Z^\beta$. The latter divide into $sl(4,\mathbb{R})$ $\widetilde{T}_\alpha{}^\beta$ and $gl(1,\mathbb{R})$ $T_0 = Y_\alpha Z^\alpha$ ones. Higher-order generators $Y_\alpha Z^{\beta(L)}$ divide into

$$\widetilde{T}_\alpha{}^{\beta(L)} = Y_\alpha Z^{\beta(L)} - \frac{1}{L+3}(YZ)\delta_\alpha^{\beta(1)} Z^{\beta(L-1)} \qquad (16)$$

and $T_0 Z^{\beta(L-1)}$. Expression (16) is an obvious generalization of $\widetilde{T}_\alpha{}^\beta$ from (12) to the case $L>1$.

Proceeding to TSA superalgebra, from (9) one infers that the D.B. relations of order L and M generators close on order $L+M-1$ generators. So that order 1 generators, i.e. $gl(4|4,\mathbb{R})$ ones (12), play a special role: D.B. relations of the generators of an arbitrary order L with those of order 1 yield again order L generators. This feature can be used to characterize irreducible higher-order generators.

Thus the form of irreducible order 2 generators can be found by D.B.-commuting corresponding bosonic generator (16) with order 1 supersymmetry generators $Q_i{}^\beta$ and $Q_\alpha{}^j$, dividing generators that appear on the r.h.s. into irreducible $SL(4,\mathbb{R})\times SL(4,\mathbb{R})$ tensors, then D.B.-commuting them with $Q_i{}^\beta$ and $Q_\alpha{}^j$ and so on. In such a way we obtain

$$\begin{aligned}&\{Q_i{}^\beta(\sigma), \widetilde{T}_\gamma{}^{\delta(2)}(\sigma')\}_{D.B.}\\ &= \left(\delta_\gamma^\beta Q_i{}^{\delta(2)} - \tfrac{1}{5}\delta_\gamma^{(\delta_1} Q_i{}^{\delta_2)\beta}\right)(\sigma)\delta(\sigma-\sigma'),\end{aligned} \qquad (17)$$

where

$$Q_i{}^{\delta(2)} = \eta_i Z^{\delta(2)} \qquad (18)$$

D.B.-commutes with $Q_i{}^\beta$. Analogously calculation of D.B. relations of $\widetilde{T}_\gamma{}^{\delta(2)}$ and $Q_\alpha{}^j$ yields

$$\begin{aligned}&\{Q_\alpha{}^j(\sigma), \widetilde{T}_\gamma{}^{\delta(2)}(\sigma')\}_{D.B.}\\ &= -\left(\delta_\alpha^{(\delta_1}\widetilde{Q}_\gamma{}^{\delta_2)j} - \tfrac{1}{5}\delta_\gamma^{(\delta_1}\widetilde{Q}_\alpha{}^{\delta_2)j}\right)(\sigma)\delta(\sigma-\sigma'),\end{aligned} \qquad (19)$$

where another order 2 supersymmetry generator

$$\widetilde{Q}_\gamma{}^{\delta i} = \widetilde{T}_\gamma{}^\delta \xi^i \qquad (20)$$

D.B.-commutes with $Q_\alpha{}^j$. Applying $Q_\alpha{}^j$ to $Q_k{}^{\delta(2)}$ gives

$$\begin{aligned}&\{Q_\alpha{}^j(\sigma), Q_k{}^{\delta(2)}(\sigma')\}_{D.B.}\\ &= \delta_k^j \widetilde{T}_\alpha{}^{\delta(2)}(\sigma)\delta(\sigma-\sigma') + \delta_\alpha^{(\delta_1}\Big[\widetilde{T}_k{}^{\delta_2)j}\\ &+\delta_k^j\left(\tfrac{9}{40}T - \tfrac{1}{40}U\right)Z^{\delta_2)}\Big](\sigma)\delta(\sigma-\sigma')\end{aligned} \qquad (21)$$

and similarly

$$\begin{aligned}&\{Q_i{}^\beta(\sigma), \widetilde{Q}_\gamma{}^{\delta l}(\sigma')\}_{D.B.}\\ &= \left(\delta_\gamma^\beta \widetilde{T}_i{}^{\delta l} - \tfrac{1}{4}\delta_\gamma^\delta \widetilde{T}_i{}^{\beta l}\right)(\sigma)\delta(\sigma-\sigma')\\ &+\delta_i^l\Big[\widetilde{T}_\gamma{}^{\beta\delta} + \left(\tfrac{9}{40}T - \tfrac{1}{40}U\right)\\ &\times\left(\delta_\gamma^\beta Z^\delta - \tfrac{1}{4}\delta_\gamma^\delta Z^\beta\right)\Big](\sigma)\delta(\sigma-\sigma'),\end{aligned} \qquad (22)$$

where

$$\widetilde{T}_i{}^{\beta j} = \widetilde{T}_i{}^j Z^\beta. \qquad (23)$$

Continuing further one recovers the set of irreducible order 2 generators

$$\begin{aligned}&\widetilde{T}_\alpha{}^{\beta(2)},\quad \widetilde{T}_i{}^{\alpha j},\quad T_\alpha{}^{j[2]} = Y_\alpha \xi^{j[2]};\\ &Q_i{}^{\alpha(2)},\ \widetilde{Q}_\alpha{}^{\beta j},\ \widetilde{Q}_i{}^{j[2]} = \eta_i\xi^{j[2]} - \tfrac{1}{3}(\eta\xi)\delta_i^{[j_1}\xi^{j_2]}\end{aligned} \qquad (24)$$

and

$$TZ^\alpha,\quad UZ^\alpha,\quad T\xi^i,\quad U\xi^i. \qquad (25)$$

The operators associated with the generators (25), as will be shown below, are not the primary fields in the world-sheet CFT and hence corresponding symmetries are broken at the quantum level. Since these generators appear on the r.h.s. of (21), (22) this implies breaking of the order 2 supersymmetries $Q_i{}^{\alpha(2)}$, $\widetilde{Q}_\alpha{}^{\beta j}$ and, in view of (17), (19) breaking of the bosonic symmetry generated by $\widetilde{T}_\gamma{}^{\delta(2)}$. So that classical odrer 2 symmetries break in the quantum theory.

For order $L>2$ calculation of D.B. relations of the corresponding bosonic generator (16) and order 1 supersymmetry generators gives

$$\begin{aligned}&\{Q_i{}^\beta(\sigma), \widetilde{T}_\gamma{}^{\delta(L)}(\sigma')\}_{D.B.}\\ &= \left(\delta_\gamma^\beta Q_i{}^{\delta(L)} - \tfrac{1}{L+3}\delta_\gamma^{\delta(1)} Q_i{}^{\delta(L-1)\beta}\right)(\sigma)\delta(\sigma-\sigma'),\end{aligned} \qquad (26)$$

and

$$\begin{aligned}&\{Q_\alpha{}^j(\sigma), \widetilde{T}_\gamma{}^{\delta(L)}(\sigma')\}_{D.B.}\\ &= -\Big(\delta_\alpha^{\delta(1)}\widetilde{Q}_\gamma{}^{\delta(L-1)j}\\ &-\tfrac{1}{L+3}\delta_\gamma^{\delta(1)}\widetilde{Q}_\alpha{}^{\delta(L-1)j}\Big)(\sigma)\delta(\sigma-\sigma'),\end{aligned} \qquad (27)$$

where order L supersymmetry generators are defined by the expressions

$$Q_i{}^{\delta(L)} = \eta_i Z^{\delta(L)},\quad \widetilde{Q}_\gamma{}^{\delta(L-1)j} = \widetilde{T}_\gamma{}^{\delta(L-1)}\xi^j. \qquad (28)$$

Their D.B. relations with order 1 supersymmetry generators read

$$\begin{aligned}&\{Q_\alpha{}^j(\sigma), Q_k{}^{\delta(L)}(\sigma')\}_{D.B.}\\ &= \delta_k^j\widetilde{T}_\alpha{}^{\delta(L)}(\sigma)\delta(\sigma-\sigma') + \delta_\alpha^{\delta(1)}\Big[\widetilde{T}_k{}^{\delta(L-1)j}\\ &+\delta_k^j\left(\tfrac{L+7}{8(L+3)}T - \tfrac{L-1}{8(L+3)}U\right)Z^{\delta(L-1)}\Big](\sigma)\delta(\sigma-\sigma')\end{aligned} \qquad (29)$$

and

$$\begin{aligned}
&\{Q_i{}^{\beta}(\sigma), \widetilde{Q}_{\gamma}{}^{\delta(L-1)l}(\sigma')\}_{D.B.} \\
&= \left(\delta^{\beta}_{\gamma}\widetilde{T}_i{}^{\delta(L-1)l} - \tfrac{1}{L+2}\delta^{\delta(1)}_{\gamma}\widetilde{T}_i{}^{\beta\delta(L-2)l}\right)(\sigma)\delta(\sigma-\sigma') \\
&+\delta^l_i\left[\widetilde{T}_{\gamma}{}^{\beta\delta(L-1)} + \left(\tfrac{L+7}{8(L+3)}T - \tfrac{L-1}{8(L+3)}U\right)\right. \\
&\times\left.\left(\delta^{\beta}_{\gamma}Z^{\delta(L-1)} - \tfrac{1}{L+2}\delta^{\delta(1)}_{\gamma}Z^{\beta}Z^{\delta(L-2)}\right)\right](\sigma)\delta(\sigma-\sigma'),
\end{aligned} \tag{30}$$

where

$$\widetilde{T}_k{}^{\delta(L-1)j} = \widetilde{T}_k{}^{j}Z^{\delta(L-1)}. \tag{31}$$

Continuing further calculation of D.B. relations of $gl(4|4,\mathbb{R})$ supersymmetry generators and order L generators allows to find complete set of irreducible order L bosonic

$$\begin{aligned}
&\widetilde{T}_{\alpha}{}^{\beta(p)j[q]} = \widetilde{T}_{\alpha}{}^{\beta(p)}\xi^{j[q]},\ q=0,2,4,\ p+q=L; \\
&\widetilde{T}_i{}^{\beta(p)j[q]} = \widetilde{T}_i{}^{j[q]}Z^{\beta(p)},\ \ q=1,3,\ \ p+q=L
\end{aligned} \tag{32}$$

and fermionic generators

$$\begin{aligned}
&\widetilde{Q}_{\alpha}{}^{\beta(p)j[q]} = \widetilde{T}_{\alpha}{}^{\beta(p)}\xi^{j[q]},\ \ q=1,3,\ \ p+q=L; \\
&\widetilde{Q}_i{}^{\beta(p)j[q]} = \widetilde{Q}_i{}^{j[q]}Z^{\beta(p)},\ q=0,2,4,\ p+q=L.
\end{aligned} \tag{33}$$

Relevant (traceless) products of bosonic components of supertwistors are defined in (16) and the definition of (traceless) products of fermionic components is given in (12), (24) and by the expressions

$$\begin{aligned}
&\widetilde{T}_i{}^{j[3]} = \eta_i\xi^{j[3]} - \tfrac{1}{2}(\eta\xi)\delta_i^{[j_1}\xi^{j_2}\xi^{j_3]}, \\
&Q_i \equiv \eta_i, \quad Q_i{}^{j[4]} = \eta_i\xi^{j[4]}.
\end{aligned} \tag{34}$$

There are also generators of the form

$$TZ^{\alpha(p)}\xi^{i[q]},\ \ UZ^{\alpha(p)}\xi^{i[q]},\ 0\le q\le 4,\ p+q=L-1. \tag{35}$$

It is these generators that correspond to non-tensor operators in the world-sheet CFT. They are present on the r.h.s. of (29) and (30) implying breaking of order L symmetries in analogy with those of order 2.

In Berkovits twistor string theory $GL(1,\mathbb{R})$ symmetry is gauged so that generators carrying the factor of T are set to zero. However, $GL_t(1,\mathbb{R})$ symmetry, being anomalous, cannot be gauged thus the generators carrying the factor of U cannot be put to zero. So we conclude that for any order L it is not possible to find a set of generators with closed D.B. relations that would correspond to the primary fields. As a result the quantum symmetry of the twistor string reduces to the symmetry $SL(4|4,\mathbb{R})\times SL(4|4,\mathbb{R})$ for the sector of closed strings and its diagonal subgroup for the sector of open strings.

2.2.2 Higher-spin symmetries from the world-sheet CFT perspective

This subsection is devoted to the twistor part of the left-moving world-sheet CFT justifying the arguments above discussion relied on. To apply the $2d$ CFT technique to the model (1) it is helpful to carry out Wick rotation to Euclidian signature world-sheet

$$\begin{aligned}
&\tau\to i\sigma^2,\ \sigma\to\sigma^1\ \Rightarrow \\
&\sigma^+\to z=\sigma^1+i\sigma^2,\ \sigma^-\to -\bar{z}=-(\sigma^1-i\sigma^2).
\end{aligned} \tag{36}$$

The following changes of the world-sheet derivatives

$$\begin{aligned}
&\partial_+\to\partial_z=\tfrac{1}{2}(\partial_1-i\partial_2)\equiv\partial, \\
&\partial_-\to-\partial_{\bar{z}}=-\tfrac{1}{2}(\partial_1+i\partial_2)\equiv-\bar{\partial},
\end{aligned} \tag{37}$$

$2d$ volume element

$$d\tau d\sigma\to id\sigma^1 d\sigma^2=\frac{i}{2}d^2z, \tag{38}$$

and supertwistor components

$$\mathcal{Y}_A\to\mathcal{Y}_{A(z)},\quad \bar{\mathcal{Y}}_A\to-\bar{\mathcal{Y}}_{A(\bar{z})}, \tag{39}$$

result in the Euclidean action

$$S_E=\int d^2z(\mathcal{Y}_A\bar{\partial}\mathcal{Z}^A+\bar{\mathcal{Y}}_A\partial\bar{\mathcal{Z}}^A). \tag{40}$$

Non-trivial OPE for the supertwistors of the left-moving sector is

$$\mathcal{Z}^A(z)\mathcal{Y}_B(w)\sim\frac{\delta^A_B}{z-w}. \tag{41}$$

The supertwistor part of the left-moving stress-energy tensor equals[4]

$$L_{\rm tw}(z)=-\mathcal{Y}_A\partial\mathcal{Z}^A \tag{42}$$

so that $\mathcal{Y}_B$ and $\mathcal{Z}^A$ are primary fields of conformal weight 1 and 0 respectively.

From the world-sheet CFT perspective the necessary condition for the considered global symmetries to survive in the quantum theory is that their generators become primary fields, i.e. their OPE's with the stress-energy tensor are anomaly free. As we find the generators containing the factor of T or U fail to comply with this requirement.

Direct calculation using the OPE definition (41) shows that $sl(4|4,\mathbb{R})$ generators $\widetilde{T}_{\alpha}{}^{\beta}$, $\widetilde{T}_i{}^{j}$, $Q_{\alpha}{}^{j}$, $Q_i{}^{\beta}$ and T are primary fields of unit weight, while U is not [3]

$$L_{\rm tw}(z)U(w)\sim\frac{-8}{(z-w)^3}+\mathcal{O}((z-w)^{-2}). \tag{43}$$

[4]It is assumed that composite operators depending on a single argument are normal-ordered but normal ordering signs : : will be omitted.

Higher-order generators (32), (33) also become primary fields of unit weight. While OPE's of the generators (35) with the stress-energy tensor are anomalous

$$\begin{aligned} &L_{\rm tw}(z)TZ^{\alpha(p)}\xi^{i[q]}(w) \\ &\sim -\tfrac{p+q}{(z-w)^3}Z^{\alpha(p)}\xi^{i[q]}(w)+\mathcal{O}((z-w)^{-2})\,, \\ &L_{\rm tw}(z)UZ^{\alpha(p)}\xi^{i[q]}(w) \\ &\sim -\tfrac{8+p-q}{(z-w)^3}Z^{\alpha(p)}\xi^{i[q]}(w)+\mathcal{O}((z-w)^{-2}). \end{aligned} \quad (44)$$

In the case $p=q=0$ one recovers the OPE's of $gl(1,\mathbb{R})$ and $gl_t(1,\mathbb{R})$ generators with the stress-energy tensor. For $p\neq 0$, $q\neq 0$ anomalous terms do not vanish so that associated symmetries are broken. Since generators (32), (33) are linked with other order L generators by order 1 supersymmetries (cf. Eqs. (26), (27), (29), (30)) it appears that higher-spin symmetry is broken for arbitrary value of L except for $L=1$, for which quantum-mechanically consistent global symmetry is isomorphic to $SL(4|4,\mathbb{R})$.

3 Conclusion

The main result reported in this note is the identification of the inifinite-dimensional classical symmetry of the Berkovits twistor string model and its extension with ungauged $GL(1,\mathbb{R})$ symmetry. Associated Noether current densities have been constructed in terms of $PSL(4|4,\mathbb{R})$ supertwistors. For the generalized twistor string model the D.B. relations of the Noether current densities have been shown to form the TSA inifinite-dimensional Lie superalgebra with the finite-dimensional subalgebra spanned by the $gl(4|4,\mathbb{R})$ generators and the generator of constant shifts of the supertwistor components. The full classical symmetry of the twistor string action is generated by the direct sum of two copies of TSA superalgebra for the left- and right-movers. Classical symmetry of the Berkovits model is described by the subalgebra of TSA obtained by going on the constraint shell $Y_\alpha Z^\alpha+\eta_i\xi^i\approx 0$. Its finite-dimensional subalgebra is spanned by $psl(4|4,\mathbb{R})$, 'twisted' $gl_t(1,\mathbb{R})$ generators and that of shifts of the supertwistor components.

More detailed account of the material covered in the present note can be found in Ref. [7].

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БЕСКОНЕЧНОМЕРНЫЕ СИММЕТРИИ МОДЕЛЕЙ ТВИСТОРНЫХ СТРУН

Показано, что, подобно моделям безмассовой суперчастицы, пространственно-временная симметрия классического действия твисторной струны Берковица является бесконечномерной. Ее супералгебра содержит конечномерную подалгебру, которая включает генераторы $psl(4|4,\mathbb{R})$ супералгебры. В квантовой теории данная бесконечномерная симметрия нарушается до $SL(4|4,\mathbb{R})$ симметрии.

Ключевые слова: *супертвистор, твисторная струна, бесконечномерная симметрия, аномалия.*

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