References

1. Perlmutter S. et al. // Astrophys. J. 1999. V. 517. P. 565, astro-ph/9812133; Nature. 1998. V. 391. P. 51, astro-ph/9712212; Riess A.G. et al. // Astron. J. 1998. V. 116. P. 1009, astro-ph/9805201.
2. de Bernardis P. et al. // Nature. 2000. V. 404. P. 955, astro-ph/0004404.
3. Hanany S. et al. // Astrophys. J. 2000. V. 545. P. L5, astro-ph/0005123.
4. Caldwell R.R., Dave R., Steinhardt P.J. // Phys. Rev. Lett. 1998. V. 80. P. 1582, astro-ph/9708069.
5. Caldwell R.R. // Phys. Lett. 2002. V. B545. P. 23, astro-ph/9908168.
6. Cline J.M., Jeon S.Y., Moore G.D. // to appear in Phys. Rev. D, hep-ph/0311312.
7. Shafi Q, Wetterich C. // Phys. Lett. 1983. V. B129. P. 387; Berkin A.L., Maeda K.-I., Yokoyama J. // Phys. Rev. Lett. 1990. V. 65. P. 141.
8. Russo J.G. // hep-th/0403010.
9. Elizalde E., Nojiri S., Odintsov S.D. // to appear in Rhys. Rev. D, hep-th/0405034.
10. Brans C., Dicke R.H. // Phys. Rev. 1961. V. 124. P. 925.
11. Carroll S.M., Duvvuri V., Trodden M., Turner M.S. // astro-ph/0306438.
12. Allemandi G., Borowiec A., Francaviglia M. // hep-th/0403264.
13. Nojiri S., Odintsov S.D. // Mod. Phys. Lett. 2004. V. A19. P. 627, hep-th/0310045.
14. Chiba T. // Phys. Lett. 2003. V. B575. P. 1, astro-ph/0307338.
15. Nojiri S., Odintsov S.D. // hep-th/0308176, to appear in General Relativity and Gravitation.
16. Dolgov A.D., Kawasaki M. // Phys. Lett. 2003. V. B573. P.1, astro-ph/0307285.
17. Nojiri S., Odintsov S.D. // Phys. Rev. 2003. V. D68. P. 123512, hep-th/0307288.
18. Starobinsky A.A. // Phys. Lett. 1980. V. B91. P. 99.
19. Mijic M.B., Morris M.S., Suen W.-M. // Phys. Rev. 1986. V. D34. P. 2934.
20. Hwang J., Noh H. // Phys. Lett. 2001. V. B506. P 13.
21. Nojiri S., Odintsov S.D. // to appear in Phys. Lett. B, hep-th/0405078.
22. Nojiri S., Odintsov S.D. // Phys. Lett. 2003. V. B562. P. 147, hep-th/0303117.
23. Onemli V.K., Woodard R.P. // Class. Quant. Grav. 2002. V. 19. P. 4607, gr-qc/0204065; Onemli V.K., Woodard R.P. // gr-qc/0406098.
24. Nojiri S., Odintsov S.D. // Phys.Lett. 2003. V. B571. P. 1, hep-th/0306212.
25. Elizalde E., Lidsey J.E., Nojiri S., Odintsov S.D. // Phys. Lett. 2003. V. B574. P.1, hep-th/0307177.
26. Khoury J., Weltman A. // astro-ph/0309300; Phys. Rev. 2004. V. D69. P. 044026, astro-ph/0309411.
27. Nojiri S., Odintsov S.D. // Mod. Phys. Lett. 2004. V. A19. P. 1273, hep-th/0310296.
28. Nojiri S., Odintsov S.D. // astro-ph/0403622.
29. Zhu Z.-H., Fujimoto M.-K. // Astrophys. J. 2002. V. 581. P.1, astro-ph/0212192; Z.-H. Zhu Z.-H., Fujimoto M.-K. // Astrophys J. 2003. V. 58[illegible] P. 52, astro-ph/0303021.
30. Deffayet C., Dvali R.G., Gabadadze G. // Phys. Rev. 2002. V. D65. P. 044023, astro-ph/0105068

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THE ASYMPTOTIC CONFORMAL INVARIANCE IN CHERN-SIMONS THEORY WITH MATTER IN CURVED SPACE-TIME

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1. The study of renormalizable field theories in curved space [1,2,3,4] proved the existance of the phenomenon of asymptotic conformal invariance. In this short review we discuss the asymptotic behaviour of 3d CS theories in curved space. Let us consider renormalizable abelian CS theory with scalar and spinor in three dimensions [5]. The Lagrangian looks like:

$$L=\frac{1}{2}\varepsilon^{\mu\nu\lambda}A_{\mu}\partial_{\nu}A_{\lambda}+|D_{\mu}\Phi|^2+ \\ +i\bar{\Psi}\hat{D}\Psi+\alpha\bar{\Psi}\Psi\Phi^{*}\Phi-h(\Phi^{*}\Phi)^3. \quad (1)$$

Here $D_{\mu}=\partial_{\mu}-ieA_{\mu}$, Φ,Ψ -complex scalar and dirac spinor consequently, coupling constants e,h,α are dimensiionless. The theory with Lagrangian (1) is multiplicatively renormalizable.

The two-loop RG equation for coupling constants has the form [6,7]:

$$(8\pi)^2\frac{de(t)}{dt}=0$$

$$(8\pi)^2\frac{d\alpha(t)}{dt}=\frac{14}{3}\alpha^3(t)-34\alpha(t)e^4(t)-24e^6(t) \quad (2)$$

$$(8\pi)^2\frac{dh(t)}{dt}=168h^2(t)-84h(t)\alpha^2(t)+36e^8(t)+ \\ +8\alpha(t)e^6(t)+4\alpha^2(t)e^6(t)-4\alpha^4(t).$$

It has been shown in paper [6] that for the theory with Lagrangian (1) exists finite four cases in which the theory is finite at two - loop level:

$$
\begin{aligned}
&1.\ \alpha = 3e^2, \quad h = -\frac{19}{4}e^4 \\
&2.\ \alpha = 3e^2, \quad h = e^4; \\
&3.\ \alpha \approx -2.23e^2, \quad h \approx -0.59e^4; \\
&4.\ \alpha \approx -2.23e^2, \quad h \approx 0.62e^4.
\end{aligned} \tag{3}
$$

In regime of finiteness the effective coupling constants are:

$$e(t) = e, \quad \alpha(t) = \alpha, \quad h(t) = h, \tag{4}$$

where values α, h give one from four variants (3). Let us consider the limit $t \to \infty$ (infrared) for solutions of equations (2). In this limit independently of initial values

$$h(t) \to e^4, \quad \alpha(t) \to 3e^2. \tag{5}$$

Therefore the theory becomes finite and supersymmetric in the asymptotic [8]. Consequently supersymmetry is infrared stable as far as in four dimensions [8].Let us study now the theory in the limit $t \to +\infty$. In this case we fix the initial value for $\alpha: \alpha(0) = 3e^2$. Then $\alpha(t) = 3e^2$ and at $t \to +\infty, h(t) \to -\frac{19}{4}e^4$ independent of initial value $h(0)$. Consequently in ultraviolet asymptotic under fixed value $\alpha(0) = 3e^2$ the theory effectively becomes finite (asymptotic finitness [8]). In this case initial value $h(t)$ is arbitrary. Fix now $\alpha(0) \approx -2.23e^2$. Then at $t \to \infty$, $h(t) \to 0.62e^4$ independently of initial value $h(0)$. The theory again appears to be asymptotic finite. In ultraviolet limit two regims of asymptotic finitness exist.

2. Let us consider the behavior of effective charge $\xi(t)$ in abelian CS theory with matter in curved space - time, the beta-function on two-loop level has the form [10].

$$\beta_\xi^{(2)} = \left(\xi - \frac{1}{8}\right)\gamma_{m^2}^{(2)}, \tag{6}$$

where $\gamma_{m^2}^{(2)}$ -gamma-function for mass of the scalar field in two - loop approximation. Then with supposition (6), two-loop RG equation for $\xi(t)$ has the form:

$$\frac{d\xi(t)}{dt} = \left(\xi(t) - \frac{1}{8}\right)\gamma_{m^2}^{(2)}(t), \tag{7}$$

where $\gamma_{m^2}^{(2)}$ is given in paper [7]. Then the solution of equation (7) in regimes of finiteness (3) has the form:

$$\xi(t) = \frac{1}{8} + \left(\xi - \frac{1}{8}\right)e^{\gamma_{m^2}^{(2)} \cdot t}, \tag{8}$$

where $\gamma_{m^2}^{(2)} = 0$ (for $N = 2$ supersymmetric theory) or

$$\gamma_{m^2}^{(2)} \approx -\frac{e^4}{24\pi^2} \cdot 11.23 \cdot 6.77.$$

Therefore at $\alpha \approx -2.23e^2$ the theory appears to be asymptotic conformally invariant in $d = 3$ at $t \to \infty$, $\xi(t) \to \frac{1}{8}$ independent of initial value. At $t \to -\infty$, $|\xi(t)| \to \infty$. For $N = 2$ there are 18 regimes of finiteness for $\gamma_{m^2}^{(2)}$ can be positive, negative or zero. Then at $t \to \infty$ the following situations are possible:

1. $\xi(t) \to \frac{1}{8}$ (asymptotic supersymmetry);
2. $\xi(t) = \xi$ (the asymptotic supersymmetry);
3. $|\xi(t)| \to \infty$.

For $t \to \infty$ the behavior of $\xi(t)$ in 1 and 3 is exchanged. Thus, it is shown that in $d3$ gauge theories also can be asymptotic conformal invariante of exponential type, like in [9]. This short review is mainly based on ref. [10].

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References

1. Buchbinder I.L., Odintsov S.D., Shapiro I.L. // Rivista Nuovo Cim. 1989. V.12. P. 1; Odintsov S.D. // Fortschr. Phys. 1991. V. 39. P. 621.
2. Buchbinder I.L., Odintsov S.D. // Sov. Phys. J. 1985. V. {2116}1. P. 86; Yad. Phys. 1984. V. 40. P. 1338; Lett. Nuovo Cim. 1985. V. 42. P. 379.
3. Lichtzier I.M., Odintsov S.D. // Yad. Phys. 1986. V. 47. P. 1782; Europhys. Lett. 1988. V. 7. P. 95; Buchbinder I.L., Odintsov S.D., Fonarev O.A. // Sov. Phys.J. 1990. V{2116}9. P. 74; Mod. Phys. Lett. 1989 V. A4. P. 2713.
4. Buchbinder I.L. // Theor.Math.Phys. 1984. V. 61. P. 393; Fortschr. Phys.1986. V. 34. P. 605.
5. Deser S., Jackiw R., Templeton S. // Ann. Phys. 1982. V. 140. P. 372.
6. Grigoryev G.V., Kazakov D.I. // Phys. Lett. 1991. V. 253B. P. 372.
7. Avdeev L.V., Grigoryev G.V., Kazakov D.I. // Preprint CERN -T.H.6091/91, 1991.
8. Odintsov S.D., Shapiro I.L. // Lett. JETF. 1989. V. 49. P. 125; Mod. Phys. Lett. 1989. V. A4. P. 1479; Odintsov S.D., Shapiro I.L., Toms D.J. // Int. J. Mod. Phys. 1991. V. A6. P. 1829.

9. Buchbinder I.L., Odintsov S.D., Lichtzier I.L. // Theor. Math. Phys. 1989. V. 79. P. 314; Class. Grav. 1989. V. 6. P. 605; Zaripov R.Sh., Odintsov S.D. // Theor. Math. Phys. 1990. V. 83. P. 399; Mod. Phys. Lett. 1989. V. A4. P. 1955.
10. Odintsov S.D., Timoshkin A.V. // Sov. Phys. J. 1992. V. {2116}6. P. 45.

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DYNAMICAL ORIGIN OF DUALITY BETWEEN GAUGE THEORY AND GRAVITY

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1 Gravity as collective excitaion of dual gauge fields

We are very happy to submit this paper to the special issue of Gravitation and Cosmology on the occasion of 70 years anniversary of Faculty of Physics and Mathematics of Tomsk State Pedagogical University.

It is popular now that by the AdS/CFT correspondence developed by J. Maldacena [1], gauge theory and gravity become dual with each other. In other words, the strong coupling regime of the former theory corresponds to the weak coupling regime of the latter, and *vice versa*. In this corresponcence D-branes and extra dimension play essential roles. This AdS/CFT correspondece may be considered within the context of the 't Hooft-Mandelstam duality [2].

In the late 70's and the early 80's, similar correspondence was known in which the closed string theory (with the Kalb-Ramond field as a gauge field of string) is dual to the gauge theory (being massive with or without Higgs field), and the extension of membranes is also considered [3,4,5,6].

Malcacena's AdS/CFT correspondence is, of cource, the more sophisticated, and provides an extremely powerful tool in studying the realistic hadron physics in QCD. Examples of such study can be found in a review article [7] and in a recent study of pentaquark baryons using the AdS/CFT correspondence [8].

In this paper, we study an origin of the dualiy between gauge theory and gravity (AdS/CFT) within the local field theory using the old-fashioned duality.

The idea which our study is based on is as follows. Starting from a gauge theory and applying a dual transformation to it, we obtain a dual theory. (We may start from a manifestly self-dual theory also.) If the gauge coupling of the original theory is e, then the gauge coupling of the dual theory is $g = 2\pi / e$. When e is small, g is large, so that bound states (or collective excitations) are formed in the dual theory by the exchange of strongly coupled dual gauge bosons. Among them we have a graviton as the collective excitaion of dual gauge bosons. Then, the graivity theory becomes dual to the original gauge theory, since the former is the low energy manifestation of the dual gauge theory in the strong coupling regime.

This idea is quite consistent with the fact that in string theories, a gauge boson is represented by an open string's mode, while a graviton is by a closed string's mode, so that a closed string can be considered as a bound state of two open strings.

We examine this idea in the usual $U(1)$ gauge theory, and also in its manifestly self-dual formulation by Zwanziger [9]. More detailed description of this paper can be found in the master theis written by one of the authors (A.N.) [10].

2 Duality between U(1) gauge theory and gravity

We start from the $U(1)$ gauge theory with a coupling e. The parition function of this theory reads,

$$Z[J] \propto \int \mathcal{D}A_\mu \exp\left\{ i \int d^4x \left[-\frac{e^2}{2} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2} F_{\mu\nu}J^{\mu\nu} \right] \right\}, \quad (1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of the gauge field A_μ, and J^μ is an external source.

Using an auxilliary field $W_{\mu\nu}$, the partition function becomes [3,4],

$$Z[J] \propto \int \mathcal{D}A_\mu \int \mathcal{D}W_{\mu\nu} \exp\left\{ i \int d^4x \right.$$
$$\left. \times\left[-\frac{e^2}{4} W_{\mu\nu}W^{\mu\nu} + \frac{1}{2}\tilde{W}_{\mu\nu}F^{\mu\nu} + \frac{1}{2}F_{\mu\nu}J^{\mu\nu} \right] \right\}, \quad (2)$$

where

$$\tilde{W}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}W^{\lambda\rho}, \quad (\epsilon_{0123} = -\epsilon^{0123} = 1). \quad (3)$$

Path integration over A_μ leads (1) to

$$\int \mathcal{D}B_\mu \exp\left\{ i \int d^4x \left[-\frac{1}{4g^2} G_{\mu\nu}G^{\mu\nu} \right.\right.$$
$$\left.\left. + \frac{1}{2}\tilde{G}_{\mu\nu}J^{\mu\nu} + \frac{1}{2}J_{\mu\nu}J^{\mu\nu} \right] \right\}, \quad (4)$$

where $G_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$ is the field strenght of the dual gauge field $B_{\mu\nu}$, and $eg = 2\pi$ holds.