UDC 530.1; 539.1

GAUGE INVARIANT LAGRANGIAN FORMULATIONS FOR MIXED-ANTISYMMETRIC FERMIONIC HIGHER SPIN FIELDS

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A Lagrangian formulation of irreducible half-integer higher-spin representations of the Poincare algebra with a Young tableaux having two columns is presented based on the BRST approach. Starting from Casimir constraints written by oscillator representation of Poincare algebra, which is the necessary condition of the irreducibility, we find closed higher spin superalgebra. In order to convert all constraints to the first class we introduce four auxiliary oscillators with γ-matrix and use Verma module method. To get nilpotent BRST operators we further introduce ghosts. After using restrictions on spin number and ghost number we construct Lagrangian having gauge symmetry with finite stage of reducibility.

Keywords: *higher-spin fields, gauge theories, BRST method, Lagrangian formulation.*

1 Introduction

Higher spin theory has been studied in the hopes as an unified description of elementary particles [2]. For the progress in higher-spin field theory, see the reviews [3]. For our study, we will construct unconstraint Lagrangian by using BRST approach [4]. In this construction, we make a connection between Lagrangian and space-time symmetry, like Poincare symmetry. Considering constraints from suitable representation of Casimir operators in Poincare algebra, one may find constraint algebra, that is, higher spin algebra which leads nil-potent BRST operators [5]. Once BRST operator is found, gauge invariant Lagrangian can be constructed straightforwardly.

BRST approach has been applied for various cases of constructing free Lagrangian. In the metric-like approach for bosonic tensors $\Phi_{\mu_1,\cdots,\mu_s}(x)$ or fermionic tensors $\Psi_{\mu_1,\cdots,\mu_s}(x)$ with general type of indexes $\mu_1,\cdots,\mu_s$, there are two different approaches; symmetric base approach and anti-symmetric base approach. In the former approach one start from totally symmetric tensor [4,6] and then generalize it to mixed symmetric case [7]. One introduce bosonic oscillators

$$[a'_{i\mu}, {a'}^{+}_{j\nu}] = -\delta_{ij}\eta_{\mu\nu}, i,j = 1,2,\cdots \tag{1}$$

to construct Fock space and consider suitable vanishing constraints on that for each case. For example, for massless fields in flat space-time, it is as follows.

	Totally sym.	Mixed sym.
Bosonic	$\partial_\mu\partial^\mu$, $a'_\mu\partial^\mu$, $\frac{1}{2}a'_\mu a'^\mu$	$\partial_\mu\partial^\mu$, $a'_{i\mu}\partial^\mu$, $\frac{1}{2}a'_{i\mu}{a'}^\mu_j$, ${a'}^+_{1\mu}{a'}^\mu_2$
Fermionic	$\tilde{\gamma}_\mu\partial^\mu$, $\tilde{\gamma}_\mu a'^\mu$	$\tilde{\gamma}_\mu\partial^\mu$, $\tilde{\gamma}_\mu {a'}^\mu_i$, ${a'}^+_{1\mu}{a'}^\mu_2$

In other word, these approaches correspond row based classification of associated Young-tableau. While totally symmetric case corresponds one row Young-tableau, mixed symmetric case corresponds Young-tableau with more than one row.

On the other hand, in the later approach one start from totally anti-symmetric tensor [8] and then generalize it to mixed anti-symmetric case [1]. One introduce fermionic oscillators

$$\{a_{i\mu}, a^{+}_{j\nu}\} = -\delta_{ij}\eta_{\mu\nu}, i,j = 1,2,\cdots \tag{2}$$

to construct Fock space and consider suitable vanishing constraints on that for each case. It is as follows for massless fields in flat space-time.

	Totally anti-sym.	Mixed anti-sym.
Bosonic	$\partial_\mu\partial^\mu$, $a_\mu\partial^\mu$	$\partial_\mu\partial^\mu$, $a_{i\mu}\partial^\mu$, $\frac{1}{2}a_{1\mu}a^\mu_2$, $a^+_{1\mu}a^\mu_2$
Fermionic	$\tilde{\gamma}_\mu\partial^\mu$, $\tilde{\gamma}_\mu a^\mu$	$\tilde{\gamma}_\mu\partial^\mu$, $\tilde{\gamma}_\mu a^\mu_i$, $a^+_{1\mu}a^\mu_2$

These approaches correspond column based classification of associated Young-tableau. While totally anti-symmetric case corresponds one column Young-tableau, mixed anti-symmetric case corresponds Young-tableau with more than one column.

In fact, for the antisymmetric case, there was no clear explanation about the fact that above condition gives irreducible representation of Poincare algebra. Therefore in the next section we will explain, by considering Casimir operators, how irreducible representation of Poincare algebra and the vanishing constraints are related.

[1] This talk is based on the collaboration with J. L. Buchbinder and A. A. Reshetnyak [1]

2 Irreducibility condition

In this section, we consider the meaning of the supplementary condition in terms of the irreducibility. To consider general type of tensor indexes we can use symmetric base or anti-symmetric case. Unlike the symmetric base we did not know how irreducibility condition for the general type of tensor can be written in the anti-symmetric base. We study it for massive bosonic tensor in four space-time dimension as the simplest case. Consider bosonic tensor $\Phi_{\mu_1\cdots\mu_{s1},\nu_1\cdots\nu_{s2}}(x)$ with constraint on indexes as

$$\Phi_{\mu_1\cdots\mu_{s1},\nu_1\cdots\nu_{s2}}(x) = \Phi_{[\mu_1\cdots\mu_{s1}],[\nu_1\cdots\nu_{s2}]}(x), \tag{3}$$

where square bracket describe anti-symmetrization. We are going to explain that the following conditions:

$$\Phi_{\mu_1\cdots[\mu_{s1},\nu_1\cdots\nu_{s2}]}(x) = 0 \tag{4}$$
$$(\Box - m^2)\Phi_{\mu_1\cdots\mu_{s1},\nu_1\cdots\nu_{s2}}(x) = 0 \tag{5}$$
$$\partial^{\mu_1}\Phi_{\mu_1\cdots\mu_{s1},\nu_1\cdots\nu_{s2}}(x) = 0 \tag{6}$$
$$\partial^{\nu_1}\Phi_{\mu_1\cdots\mu_{s1},\nu_1\cdots\nu_{s2}}(x) = 0 \tag{7}$$
$$\eta^{\mu_1\nu_2}\Phi_{\mu_1\cdots\mu_{s1},\nu_1\cdots\nu_{s2}}(x) = 0. \tag{8}$$

can be understood as the condition that Casimir operator C_2 is a multiple of the identity operator. For Poincare algebra we adopt oscillator representation on space $|\Phi\rangle$:

$$|\Phi\rangle = a_1^{+\mu_1}\cdots a_1^{+\mu_{s1}} a_2^{+\nu_1}\cdots a_2^{+\nu_{s2}}|0\rangle\Phi_{\mu_1\cdots\mu_{s1},\nu_1\cdots\nu_{s2}}(x)$$

with two sets of anti-commuting oscillators

$$\{a_i^\mu, a_j^{+\nu}\} \;=\; -\eta^{\mu\nu}\delta_{ij}, i,j = 1,2. \tag{9}$$

It is found that the representations of generators of Poincare algebra can be written as [9].

$$\begin{array}{rcl} P_\nu &=& -i\partial_\mu, \\ M_{\mu\nu} &=& L_{\mu\nu} + \Sigma_{\mu\nu}, \\ L_{\mu\nu} &=& -i(x_\mu\partial_\nu - x_\nu\partial_\mu), \\ \Sigma_{\mu\nu} &=& \Sigma^1_{\mu\nu} + \Sigma^2_{\mu\nu}, \\ \Sigma^j_{\mu\nu} &=& i(a_\mu^{j+}a_\nu^j - a_\nu^{j+}a_\mu^j), j = 1,2. \end{array} \tag{10}$$

Supplementary conditions (4)-(8) are also rewritten as[2]

$$\begin{array}{l} g_{12}|\Phi\rangle = l_0|\Phi\rangle = l_i\;|\Phi\rangle = l_{12}|\Phi\rangle \\ = (g_{ii} + s_i - 2)|\Phi\rangle = 0 \end{array} \tag{11}$$

where we have used definitions ($p^\mu = -i\partial^\mu$)

$$\begin{array}{lll} l_0 = -p^2 + m^2, & l_i = a_i^\mu p_\mu, & l_{12} = \frac{1}{2}a_{1\mu}a_2^\mu, \\ g_{ij} = a^+_{i\mu}a^\mu_j + 2\delta_{ij}, & l_i^+ = a_i^{\mu+}p_\mu, & l_{12}^+ = \frac{1}{2}a_2^{\mu+}a^+_{1\mu}. \end{array}$$

One can easily see that if the following constraints

$$\begin{array}{l} t_0|\Psi\rangle = t_1|\Psi\rangle = t_2|\Psi\rangle = g_{12}|\Psi\rangle \\ = (g_{ii} + s_i - \frac{d}{2})|\Psi\rangle = 0, \end{array} \tag{12}$$

The second Casimir operator C_2 can be written by the Pauli-Lubanski vector W_μ [10]

$$C_2 = W_\mu W^\mu,\;\; W_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu\Sigma^{\rho\sigma}. \tag{13}$$

By defining

$$C_{ij} = W^i_\mu W^{j\mu},\;\; W_{i\mu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu\Sigma_i^{\rho\sigma}, \tag{14}$$

the second Casimir can be written as $C_2 = W_{1\mu}W_1^\mu + W_{2\mu}W_2^\mu + 2W_{1\mu}W_2^\mu$. In the following we will show that C_2 are multiples of the identity operator on the vector. Note that these C_{ij} can be directly checked to commute to the generators. By inputting expressions of eqs (10) we get the following expression

$$\begin{array}{l} C_{ij} = \Big\{ -(a^{+i}\cdot a^{+j})(a^i\cdot a^j) + (a^{j+}\cdot a^i)(a^{+i}\cdot a^j) + (a^{+j}\cdot a^j) \\ \quad +2\delta^{ij}(a^{+j}\cdot a^j)\Big\}\dfrac{\partial^2}{2} - (a^{+j}\cdot\partial)(a^{+i}\cdot a^j)(a^i\cdot\partial) \\ \quad +(a^{+i}\cdot a^{+j})(a^i\cdot\partial)(a^j\cdot\partial) - \delta^{ij}(a^{+i}\cdot\partial)(a^j\cdot\partial) + h.c. \end{array} \tag{15}$$

When using above conditions eq. (11), it lead to the final result[3]

$$C_2|\Phi\rangle = m^2\left\{s_1(s_1-3) + s_2(s_2-5)\right\}|\Phi\rangle. \tag{16}$$

Thus we have shown that the second Casimir operator C_2 is a multiple of the identity operator on the vector which satisfies supplementary conditions eq. (11), that is a necessary condition of irreducibility of the representation.

3 HS Symmetry Superalgebra for mixed-antisymmetric fermionic fields

In this section, we consider a massless half-integer spin irreducible representation of Poincare group in a Minkowski space which is described by a tensor field $\Psi_{[\mu_1\cdots\mu_{s_1}],[\nu_1\cdots\nu_{s_2}]}(x)$ to be corresponding to a Young tableaux with 2 columns of height s_1, s_2, respectively

$$\Psi_{[\mu_1\cdots\mu_{s_1}],[\nu_1\cdots\nu_{s_2}]} \longleftrightarrow \begin{array}{|c|c|} \hline \mu_1 & \nu_1 \\ \hline \cdot & \cdot \\ \hline \mu_{s_2} & \nu_{s_2} \\ \hline \cdot & \multicolumn{1}{c}{} \\ \cline{1-1} \mu_{s_1} & \multicolumn{1}{c}{} \\ \cline{1-1} \end{array}\;. \tag{17}$$

We call this field as spin $[s_1, s_2]$ field, that is antisymmetric with respect to the permutations of each type of Lorentz indices μ, ν. This is realized in the space

[2] The first condition (3) is automatically satisfied.

[3] General formula for arbitrary Young tableaux in anti-symmetric base and in symmetric base are similary given as $C_2|\Phi\rangle = m^2\sum_{j=1}^{\#\text{ of colums}} s_j(s_j - 2j - 1)|\Phi\rangle$ and $C_2|\Phi\rangle = -m^2\sum_{j=1}^{\#\text{ of rows}} s'_j(s'_j - 2j + 3)|\Phi\rangle$, respectively. It can be easily proved that these two formulas are equal to each other for given fixed Young tableaux, independently from the choice of the base as expected.

of fermionic tensor field $\Psi_{[\mu_1\cdots\mu_{s_1}],[\nu_1\cdots\nu_{s_2}]}(x)$ satisfying the following constraints

$$\begin{aligned} \gamma^\mu\partial_\mu\Psi_{[\mu_1\cdots\mu_{s_1}],[\nu_1\cdots\nu_{s_2}]}(x) &= 0 \\ \gamma^{\mu_1}\Psi_{[\mu_1\cdots\mu_{s_1}],[\nu_1\cdots\nu_{s_2}]}(x) &= 0 \\ \gamma^{\nu_1}\Psi_{[\mu_1\cdots\mu_{s_1}],[\nu_1\cdots\nu_{s_2}]}(x) &= 0 \\ \Psi_{\mu_1\mu_2\cdots[\mu_{s_1},\nu_1\nu_1\cdots\nu_{s_2}]}(x) &= 0, \end{aligned} \quad (18)$$

those can be naturally guessed from the bosonic version (4)–(8) in the previous section. Our purpose is to construct a Lagrangian which reproduces these constraints as the consequences of the equations of motion. It is convenient to introduce grassmann-odd creation and annihilation operators with space-time indices ($\mu,\nu = 0,1,\ldots,d-1$) and column indices ($i,j = 1,2$)

$$\{a_i^\mu, a_j^{+\nu}\} = -\eta^{\mu\nu}\delta_{ij}, \quad \eta^{\mu\nu} = diag(+,-,\cdots,-) \quad (19)$$

We follow [6] and introduce a set of $(d+1)$ Grassmann-odd gamma-matrix-like objects $\tilde{\gamma}^\mu$, $\tilde{\gamma}$, subject to the conditions

$$\begin{aligned} &\{\tilde{\gamma}^\mu, \tilde{\gamma}^\nu\} = 2\eta^{\mu\nu}, \{\tilde{\gamma}^\mu, \tilde{\gamma}\} = \{\tilde{\gamma^\mu}, a_i^{(+)\nu}\} = 0, \\ &\tilde{\gamma}^2 = -1. \end{aligned} \quad (20)$$

An arbitrary vector in this Fock space has the form

$$|\Psi\rangle = \sum_{\{n_1,n_2\}=\{0,0\}}^{\{\infty,n_1\}} a_1^{+\mu_1}\cdots a_1^{+\mu_{n_1}} a_2^{+\nu_1}\cdots a_2^{+\nu_{n_2}}|0\rangle\Psi_{\{\mu\}_{n_1},\{\nu\}_{n_2}}(x). \quad (21)$$

We call the vector (21) as basic vector. The fields $\Psi_{\{\mu\}_{n_1},\{\nu\}_{n_2}}(x)$ are the coefficient functions of the vector $|\Psi\rangle$, where the simplified notation $\{\mu\}_n$ is used because its symmetry properties are stipulated by the properties of the product of the creation operators.

Let us define a set of operators being quadratic in powers of all oscillators, $a_i^{\mu(+)}$, odd gamma-matrices, $\tilde{\gamma}^\mu$, momenta p_μ (with notation $p_\mu = -i\partial_\mu$) and requirement to have a_i^μ, $a_i^{\mu+}$, p_μ in the products

$$\begin{aligned} &t_0 = \tilde{\gamma}^\mu p_\mu, && l_0 = -p^\mu p_\mu, && g_{ij} = a_{i\mu}^+ a_j^\mu + \tfrac{d}{2}\delta_{ij}, \\ &t_i = \tilde{\gamma}_\mu a_i^\mu, && l_i = a_i^\mu p_\mu, && l_{12} = \tfrac{1}{2}a_{1\mu}a_2^\mu, \\ &t_i^+ = a_i^{\mu+}\tilde{\gamma}_\mu, && l_i^+ = a_i^{\mu+}p_\mu, && l_{12}^+ = \tfrac{1}{2}a_2^{\mu+}a_{1\mu}^+, \end{aligned} \quad (22)$$

One can easily see that if the following constraints

$$\begin{aligned} &t_0|\Psi\rangle = t_1|\Psi\rangle = t_2|\Psi\rangle = g_{12}|\Psi\rangle \\ &= (g_{ii} + s_i - \tfrac{d}{2})|\Psi\rangle = 0, \end{aligned} \quad (23)$$

are satisfied for $|\Psi\rangle$ then each component $\Psi_{\{\mu\}_{n_1},\{\nu\}_{n_2}}$ of (21) obeys the conditions (18)–(18) Let us turn to the algebra generated by operators t_0, t_i, g_{12} and $g_{ii}, (i = 1,2)$. To get a real Lagrangian we need a Hermitian BRST operator. Therefore, we should add operators $t_i^+ (i = 1,2)$ and g_{21}. By taking super-commutators we get operators l_0, $l_i^{(+)}(i = 1,2)$,$l_{12}^{(+)}$. As a result we need all of operators defined in (22). Our task is to find a closed algebra including these operators. After simple calculation, we find the following closed algebra using these operators.

$[\downarrow,\rightarrow\}$	t_0	l_k	l_k^+	l_0	t_k	t_k^+	l_{12}	l_{12}^+	g_{kl}
t_0	$-2l_0$	0	0	0	$2l_i$	$-2l_k^+$	0	0	0
l_i	0	0	$\delta_{ik}l_0$	0	0	$-\delta_{ik}t_0$	0	$-\delta_{i[2}l_{1]}^+$	$-\delta_{ki}l_l$
l_i^+	0	$\delta_{ik}l_0$	0	0	$\delta_{ik}t_0$	0	$\delta_{i[2}l_{1]}$	0	$\delta_{li}l_k^+$
l_0	0	0	0	0	0	0	0	0	0
t_i	$-2l_i$	0	$-\delta_{ik}t_0$	0	$-4l_{12}\epsilon_{ik}$	$-2g_{ki}$	0	$-\delta_{i[1}t_{2]}^+$	$-\delta_{ik}t_l$
t_i^+	$2l_i^+$	$\delta_{ik}t_0$	0	0	$2g_{ik}$	$4l_{12}^+\epsilon_{ik}$	$\delta_{i[1}t_{2]}$	0	$\delta_{il}t_k^+$
l_{12}	0	0	$-\delta_{k[2}l_{1]}$	0	0	$-\delta_{k[1}t_{2]}$	0	$\frac{1}{4}(g_{11}+g_{22})$	$-\delta_{kl}l_{12}$
l_{12}^+	0	$\delta_{k[2}l_{1]}^+$	0	0	$\delta_{k[1}t_{2]}^+$	0	$-\frac{1}{4}(g_{11}+g_{22})$	0	$\delta_{kl}l_{12}^+$
g_{ij}	0	$\delta_{ki}l_j$	$-\delta_{kj}l_i^+$	0	$\delta_{ki}t_j$	$-\delta_{kj}t_i^+$	$\delta_{ij}l_{12}$	$-\delta_{ij}l_{12}^+$	$\delta_{il}g_{kj} - \delta_{jk}g_{il}$

HS superalgebra for massless fermionic field corresponding to two columns Young-tableaux. ($\epsilon_{ik} = -\epsilon_{ki}$, $\epsilon_{12} = 1$.)

In this table, the first and the second arguments of the super-commutators are listed in the first column and the first row, respectively. The algebra corresponding to this table is a base for massless half-integer higher spin field Lagrangian construction corresponding Young tableaux with 2 columns in flat space. We should emphasize some points. First, there are four hermitian operators l_0,t_0,g_{ii} for $i = 1,2$. Second, t_0, l_i and l_i^+ are grassmann-odd operators and others are grassmann-even. Third, a straightforward use of BRST-BFV construction as if all the operators are the first class constraints doesn't lead to the proper equations (23) for any spin $[s_1,s_2]$ (see e.g. [4,6]). In fact, in the table, there are second-class constraints caused by super-commutators among eight operators: $t_i^{(+)}(i = 1,2)$, $l_{12}^{(+)}$, g_{12} and g_{21}. Thus we must somehow get rid of these second class constraints. Method of elaboration of the second class constraints consists in constructing new representations of the superalgebra so that the hermitian operators g_{11} and g_{22} will be modified with constant parameters to be controlling possible values of spin $[s_1,s_2]$.

4 Conversion of HS Symmetry Superalgebra

In this section, to solve the problem of modification of t_i, t_i^+,l_{12},l_{12}^+, g_{12} and g_{21} we describe the method of auxiliary representation construction for the superalgebra with second-class constraints, in terms of new creation and annihilation operators. It implies the enlarging of $o_I = \{t_0, l_i, l_i^+, l_0, t_i, t_i^+, l_{12}, l_{12}^+, g_{ij}\}$ to $O_I = o_I + o_I'$, where additional parts o_I' are given on a new Fock space with requirement,

$$[o_I, o_J'\} = 0, \quad [o_I',\ o_J'\} = f_{IJ}^K o_K', \quad [O_I,\ O_J\} = f_{IJ}^K O_K$$

with the same structure constants f_{IJ}^K of the superalgebra in the table. Because of only sub-algebra of left-bottom part of the table has the second-class constraints, it is enough to get new operator realization form them. We will solve this problem by means of special procedure known as Verma module construction [11] for the latter algebra.

Corresponding to four pair of bosonic operators t_1, t_2,l_{12} and g_{12} and their conjugations, we introduce the same number of pair of auxiliary bosonic oscillators $[b_i, b_j^+] = \delta_{ij}$, $(i, j = 1, \cdots, 4)$. Owing to this conversion two arbitrary constants h_1 and h_2 can be introduced in the system. They, in fact, control the size of spin $[s_1, s_2]$. Detailed calculation and explicit form are written in [1].

As explained in the beginning of this section, new representation of the algebra is

$$\begin{array}{l} T_i = t_i + t_i' \\ T_i^+ = t_i^+ + t_i^{+\prime} \end{array}, \quad \begin{array}{l} L_{12} = l_{12} + l_{12}' \\ L_{12}^+ = l_{12}^+ + l_{12}^{+\prime} \end{array}, \quad G_{ij} = g_{ij} + g_{ij}$$

It is obvious that these operators with capital letter form the same algebra to the table.

5 BRST operator and fixed spin theory

In order to construct BRST operator one may use prescription [5]. It is as follows. Corresponding to the each generators O_I's of the algebra having only the first class constraints, we introduce a pair of creation -annihilation operator $\mathcal{C}^I$ and $\mathcal{P}_I$ satisfying to the canonical commutation relations, $[\mathcal{C}^I, \mathcal{P}_J\} = \delta_J^I$, whose Grassmann parity is opposite to the one of O_I. We call $\mathcal{C}^I$ ghost coordinate and $\mathcal{P}_I$ ghost momentum . Then nilpotent BRST-BFV operator Q' are given as

$$Q' = \mathcal{C}^I O_I + \frac{1}{2}\mathcal{C}^I\mathcal{C}^J f_{JI}^K \mathcal{P}_K (-1)^{\varepsilon(O_K)+\varepsilon(O_I)}, \tag{24}$$

corresponding to the structure of given Higher spin algebra

$$[O_I,\ O_J\} = f_{IJ}^K O_K,\ f_{IJ}^K = -(-1)^{\varepsilon(O_I)\varepsilon(O_J)} f_{JI}^K, \tag{25}$$

where $\varepsilon(O)$ describe Grassmann parity of O. Following to this prescription, we introduce ghost coordinate and ghost momenta and define Q' corresponding our superalgebra of the table. Including oscillators b_i^+ and creation operators of ghosts, Hilbert space is now extended from $|\Psi\rangle$ of eq. (21) to that we will write as $|\chi\rangle$. As usual prescription of BRST approach, one need specially treat Hermitian ghosts parts, which originate from hermite operators l_0, t_0 and G_{ii}. The detail is in [1]. Generalized spin number operators $\sigma_i + h_i$ are defined in Q' as the coefficient of the ghost coordinate that corresponds to g_{ii}. As a result for spin $[s_1, s_2]$ model, we fix parameters h_i to be

$$h_i = s_i - \left(\frac{d}{2} + 1 + (-1)^i\right). \tag{26}$$

Thus, we now study spin fixed theory.

6 Unconstrained Gauge-invariant Lagrangian

Nilpotent BRST operator Q' gives us equations of motion and gauge transformations in the space $|\chi\rangle$. After partially fixing gauge and using equation of motion those do not give any new constraints, eq. of motion and gauge transformations in reduced subspace are given as

$$\delta|\chi^{(n-1),0}\rangle = \Delta Q|\chi^{(n),0}\rangle + \frac{1}{2}\{T_0', q_i^+ q_i\}|\chi^{(n),1}\rangle, \tag{27}$$

$$\delta|\chi^{(n-1),1}\rangle = \Delta Q|\chi^{(n),1}\rangle + T_0'|\chi^{(n),0}\rangle, \tag{28}$$

where $n = 0$ giving equations of motion (l.h.s$\equiv 0$) and $n = 1, 2, \cdots \sum_{i=1}^2 s_i + 3$ giving reducible gauge transformations. ΔQ is defined as terms that independent of any Hermitian ghosts in Q' of (24). T_0' is also defined there as coefficient of the Hermitian ghost coordinate that corresponds to T_0. q_i^+ and q_i are ghost coordinates corresponding to L_i and L_i^+ respectively. Two independent vectors $|\chi^{(n),n_0}\rangle, (n_0 = 0, 1)$ are supposed to be independent of any Hermitian ghosts and to have ghost number $-(n + n_0)$, where ghost number $+1$ is assigned to the ghost coordinates and -1 to the ghost momenta. We can easily find Lagrangian that derive above equations of motion as

$$\begin{aligned} \mathcal{L} \ = \ & \langle\chi^0|KT_0'|\chi^0\rangle + \frac{1}{2}\langle\chi^1|K\{T_0', q_i^+ q_i\}|\chi^1\rangle \\ & +\langle\chi^0|K\Delta Q|\chi^1\rangle + \langle\chi^1|K\Delta Q|\chi^0\rangle, \end{aligned} \tag{29}$$

where $|\chi^{n_0}\rangle \equiv |\chi^{(0),n_0}\rangle$ and operator K has been introduced in order to make Lagrangian real [1]. This (29) is invariant under gauge transformations (27) and (28) with finite stage of reducibility, and is an unconstraint Lagrangian of massless free half-integer HS fields corresponding Young tableaux having two column in any space-time dimension. These are our final results.

7 Conclusion

In the present work, we have constructed a gauge-invariant Lagrangian description of massless free half-integer HS fields belonging to an irreducible representation of the Poincare group with the corresponding Young tableaux having two column in the metric-like formulation in a Minkowski space of any space-time dimension..

By using oscillator representation of Poincare algebra in mixed anti-symmetric base, an explicit explanation has been given, through the Casimir constraint (16), about initial conditions of BRST construction for massive bosonic fields in four space-time dimension.

We have started from initial condition (23) or (18) supposed to give an irreducible Poincare-group representation for massless fermionic field with corresponding two column Young-tableaux (17).

These constraints generate a closed Lie superalgebra of HS symmetry as in the table, whose representation can be additively converted to have only first class constraints.

The nilpotent BRST operator Q' (24) are defined and in its reduced subspace equations of motion, gauge transformations with finite stage of reducibility (27)-(28) and unconstraint lagrangian (29) for massless free half-integer HS fields corresponding Young tableaux having two column in any space-time dimension are given.

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Received 24.11.2014

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КАЛИБРОВОЧНО-ИНВАРИАНТНАЯ ЛАГРАНЖЕВА ФОРМУЛИРОВКА ДЛЯ СМЕШАННЫХ АНТИСИММЕТРИЧНЫХ ФЕРМИОННЫХ ПОЛЕЙ С ВЫСШИМИ СПИНАМИ

Представлена лагранжева формулировка неприводимого представления алгебры Пуанкаре с полуцелым высшим спином, основанная на методе БРСТ. Исходя из связей Казимира в осцилляторном представлении алгебры Пуанкаре, мы находим замкнутую суперaлгебру высших спинов. С целью преобразования всех связей к первому классу, мы вводим четыре вспомогательных осциллятора и используем метод Верма. Чтобы получить нильпотентные БРСТ операторы мы вводим духи. Используя связи на спиновое число и духи, мы конструируем калибровочно-инвариатный лагранжиан.

Ключевые слова: *поля высших спинов, калибровочные теории, метод БРСТ, лагранжева формулировка, когерентность.*

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