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ON MASSIVE SPIN 3/2 INTERACTIONS IN FRAME-LIKE FORMALISM

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In this paper we consider massive spin 3/2 field and study its gravitational interaction. We use frame-like formulation for higher spin fields ($s \geq 3/2$) in terms of gauge invariant field strengths. It is shown that as for massless higher spin field the gravitational interaction for massive spin 3/2 field can be constructed as strength deformation procedure.

Keywords: *frame-like formalism, higher spins, gauge symmetries.*

1 Introduction

In last three decades for massless higher spin field significant progress has been achieved in the problem of construction of interaction. As is well known non-linear theory for massless spin 3/2 is associated with gauge theories of extended Poincare (AdS) superalgebras. At the same time for spins higher than 3/2 non-linear theory is associated with gauge theories of extended higher spin superalgebras [1, 2]. Less progress has been made in the theory of interacting massive fermion fields. Among the available results one can distinguish Metsaev classification of cubic vertices using the light cone approach [3, 4]. In this paper, using massive spin $\frac{3}{2}$ as a simple but physically interesting and non-trivial example of massive fermionic higher spin fields, we apply the so-called Fradkin-Vasiliev formalism [1, 2] to the construction of gravitational cubic vertices. Unlike Metsaev classification of cubic vertices these ones are constructed in explicitly covariant form.

2 Frame-like gauge invariant approach

It is known that different approaches to higher-spin theory formulation have been developed. We will use the frame-like formalism [5]. The advantage of such a formulation is a natural geometric interpretation. Moreover within of such formulation it has been able to realize non-linear theory for higher-spin fields. We briefly recall the main features of this formulation and focus only on the massless fields. Let us just say that for massive fields we must introduce auxiliary Stueckelberg fields with appropriate symmetries. Concrete massive spin-3/2 example will be shown below. So the frame-like formulation of fields with spin $s \geq 3/2$ is just a generalization of the well-known frame formulation of gravity in terms of the tetrad $\bar{e}_\mu{}^a$ and Lorentz connection $\bar{\omega}_\mu{}^{a,b}$. We have to consider the full bunch of tensor fields for bosons

$$\bar{e}_\mu{}^a, \bar{\omega}_\mu{}^{a,b} \Rightarrow \quad \Phi_\mu{}^{a_1 \dots a_{s-1}, b_1 \dots b_k}, \quad 0 \leq k \leq s-1 \qquad (1)$$

and spin-tensor[1] fields for fermions

$$\psi_\mu \Rightarrow \Psi_\mu{}^{a_1 \dots a_{s-3/2}, b_1 \dots b_k}, \quad 0 \leq k \leq s-3/2. \qquad (2)$$

Note that for $k = 0$ we have generalized tetrad field and for other values of k we have the so-called extra fields that are physically expressed through derivatives of tetrad field

$$\Phi_\mu{}^{a_1 \dots a_{s-1}, b_1 \dots b_k} \sim \partial^k \Phi_\mu{}^{a_1 \dots a_{s-1}}.$$

It is very important that all these fields are the gauge ones so that each field has a gauge transformation with its own parameter

$$\delta \Phi_\mu{}^{a_1 \dots a_{s-1}, b_1 \dots b_k} = \partial_\mu \xi^{a_1 \dots a_{s-1}, b_1 \dots b_k} + \dots$$

Here dots denote terms without derivatives. Moreover each field has its own gauge-invariant field strength (we will call them the curvatures)

$$\mathcal{R}_{\mu\nu}{}^{a_1 \dots a_{s-1}, b_1 \dots b_k} = \partial_{[\mu} \Phi_{\nu]}{}^{a_1 \dots a_{s-1}, b_1 \dots b_k} + \dots \qquad (3)$$

It is remarkable that free Lagrangian in terms of these curvatures can be rewritten as follows

$$\mathcal{L}_0 = \sum \mathcal{R} \wedge \mathcal{R}.$$

This is very similar to usual Yang-Mills theory.

Now let us discuss the general scheme of constructing of cubic interaction vertices. Universal method in gauge-invariant approach is the decomposition of the Lagrangian and gauge transformations in powers of fields

$$\mathcal{L} = \mathcal{L}_0 + \kappa \mathcal{L}_1 + \dots, \qquad \delta = \delta_0 + \kappa \delta_1 + \dots,$$

where

$$\mathcal{L}_1 = \Phi\Phi\Phi, \qquad \delta_1 \Phi = \Phi\xi,$$

[1] We omit all spinor indices.

then condition of gauge invariance $\delta\mathcal{L} = 0$ also decomposes in powers of the fields

$$\kappa^0 \quad \delta_0\mathcal{L}_0 = 0,$$
$$\kappa^1 \quad \delta_0\mathcal{L}_1 + \delta_1\mathcal{L}_0 = 0,$$
$$\ldots$$

In the zero-order we have the invariance condition for free theory. In the first-order we have condition for cubic verteces. Note that this scheme is universal and works in general gauge invariant approach including frame-like formalism.

However in frame-like gauge invariant approach there are additional features. Using the linearized curvatures (3) the general structure for cubic interaction will have form

$$\mathcal{L}_1 = \mathcal{R}\mathcal{R}\mathcal{R} + \mathcal{R}\mathcal{R}\Phi + \mathcal{R}\Phi\Phi. \qquad (4)$$

Here first term gives trivial vertices because they are constructed in terms of explicitly gauge invariant curvatures. We will not consider them. Second and third terms in (4) give us abelian and non-abelian vertices respectively. For massless field it was shown that non-abelian vertices are obtained from deformation of curvatures [6]. There exist quadratic deformation for all curvatures

$$\mathcal{R} \to \hat{\mathcal{R}} = \mathcal{R} + \Delta\mathcal{R}$$

such that deformed curvatures transform covariantly

$$\delta\hat{\mathcal{R}} = \delta_0\Delta\mathcal{R} + \delta_1\mathcal{R} = \mathcal{R}\xi,$$

where $\Delta\mathcal{R} = \Phi\Phi, \delta_1\Phi = \Phi\xi$. Thus non-trivial interacting Lagrangian has form

$$\mathcal{L} = \hat{\mathcal{R}}\hat{\mathcal{R}} + \mathcal{R}\mathcal{R}\Phi \quad \to \quad \delta\mathcal{L} = \mathcal{R}\mathcal{R}\xi = 0.$$

Further we apply this procedure for investigation of massive spin 3/2 gravitational interaction.

3 Free massive spin 3/2 field

Firstly consider the free massive spin 3/2 field and formulate for it frame-like gauge invariant description. As the field variables we have master spin-vector field ψ_μ and auxiliary Stuckelberg spinor field ϕ. Note that in accordance with (2) there are no extra fields. The free Lagrangian and gauge transformations in four dimensional AdS space have form[2]

$$\begin{aligned}\mathcal{L}_0 &= -\frac{i}{2}\left\{ {}^{\mu\nu\alpha}_{abc} \right\} \bar\psi_\mu \Gamma^{abc} D_\nu \psi_\alpha + \frac{i}{2} e^\mu{}_a \bar\phi \gamma^a D_\mu \phi \\ &+3ime^\mu{}_a \bar\psi_\mu \gamma^a \phi \\ &-\frac{3M}{2}\left\{ {}^{\mu\nu}_{ab} \right\} \bar\psi_\mu \Gamma^{ab} \Psi_\nu - M\bar\phi\phi, \end{aligned} \qquad (5)$$

$$\delta_0\psi_\mu = D_\mu\xi + \frac{iM}{2}\gamma_\mu\xi, \qquad \delta_0\phi = 3m\xi,$$

where M is determined as $M^2 = m^2 + \lambda^2$. Small m is the mass parameter, and λ is associated with the cosmological Λ as follows $\lambda^2 = -\Lambda/3$. Note also that in the massless limit, when $m = 0$ the cross term drops out and Lagrangian describes massless spin-3/2 and a massive spin 1/2 fields.

For both fields ψ_μ and ϕ one can construct a gauge-invariant strengths (curvatures)

$$\begin{aligned}\Psi_{\mu\nu} &= D_{[\mu}\psi_{\nu]} + \frac{m}{6}\Gamma_{\mu\nu}\phi + \frac{iM}{2}\gamma_{[\mu}\psi_{\nu]}, \\ \Phi_\mu &= D_\mu\phi - 3m\psi_\mu + \frac{iM}{2}\gamma_\mu\phi\end{aligned}$$

which also will be the equations of motion. The general expression for the Lagrangian in terms of curvatures will contain three terms

$$\begin{aligned}\mathcal{L}_0 &= c_1 \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} \bar\Psi_{\mu\nu}\Gamma^{abcd}\Psi_{\nu\alpha} \\ &+ic_2\left\{ {}^{\mu\nu\alpha}_{abc} \right\} \bar\Psi_{\mu\nu}\Gamma^{abc}\Phi_\alpha \\ &+c_3\left\{ {}^{\mu\nu}_{ab} \right\} \bar\Phi_\mu\Gamma^{ab}\Phi_\nu. \end{aligned} \qquad (6)$$

The requirement to reproduce the original Lagrangin (5) partially fixes parameters c_{1-3} but one-parameter ambiguity say c_2 remains

$$3c_3 = -8c_1, \qquad 32c_1 M = 1 - 12c_2 m.$$

In principle there is a simple solution $c_2 = 0$. However we would like to consider the general situation and to explore the role of c_2 in the construction of interaction.

4 Gravitational interaction of massive spin 3/2 field

Let us consider the example of gravitational coupling for massive spin 3/2 field. And at first, for clarity, we illustrate the general scheme. Let us for a while denote fields and curvatures for massive spin-3/2 by

$$\psi \equiv (\psi_\mu, \phi), \quad \Psi \equiv (\Psi_{\mu\nu}, \Phi_\mu)$$

fields and curvatures for gravity (massless spin 2) by

$$\omega \equiv (\omega_\mu{}^{ab}, h_\mu{}^a), \quad R \equiv (R_{\mu\nu}{}^{ab}, T_{\mu\nu}{}^a).$$

Gravitational coupling for spin 3/2 field corresponds to cubic vertex $\frac{3}{2} - \frac{3}{2} - 2$. For this type of vertex deformations to curvatures $\hat{\mathcal{R}} = \mathcal{R} + \Delta\mathcal{R}$ will look like these

$$\Delta\Psi = \omega\psi, \qquad \Delta R = \psi\psi$$

[2] In AdS space we have non-dynamical background velbein $e_\mu{}^a$ and covariant derivatives normalized as follows $D_{[\mu}D_{\nu]}\xi = \frac{\lambda^2}{2}\Gamma_{\mu\nu}\xi$. For antisymmetric combinations of $e_\mu{}^a$ and γ^a-matrices we use notations $\Gamma^{a_1\cdots a_n} = \frac{1}{n!}\gamma^{[a_1}\ldots\gamma^{a_n]}$ and $\left\{ {}^{\mu_1\cdots\mu_n}_{a_1\cdots a_n} \right\} = e^{[\mu_1}{}_{a_1}\ldots e^{\mu_n]}{}_{a_n}$, where in four dimensions $n = 1, 2, 3, 4$.

while corrections to gauge transformations and curvature transformations as follows

$$\delta_1\psi = \omega\xi \oplus \psi\hat{\xi}, \qquad \delta_1\omega = \psi\xi,$$
$$\delta\hat{\Psi} = \Psi\hat{\xi} \oplus R\xi, \qquad \delta\hat{R} = \Psi\xi. \qquad (7)$$

Interacting Lagrangian will have a sum of free Lagrangians for spin-2, spin 3/2 plus Abelian verteces

$$\mathcal{L} = \hat{R}\hat{R} \oplus \hat{\Psi}\hat{\Psi} \oplus \Psi\Psi\omega \quad \Rightarrow$$
$$\Rightarrow \quad \delta\mathcal{L} = R\Psi\xi \oplus \Psi\Psi\hat{\xi} = 0. \qquad (8)$$

Two conditions (7), (8) will fix all arbitrariness.

Proceeding to above procedure let us explicitly write out pieces for massive spin 3/2 deformation: deformations to curvatures

$$\begin{aligned}
\Delta\Psi_{\mu\nu} &= g_0(\omega_{[\mu}{}^{ab}\Gamma_{ab}\psi_{\nu]} \\
&\quad + 2Mih_{[\mu}{}^a\gamma_a\psi_{\nu]} - \frac{2m}{3}h_{[\mu}{}^a\Gamma_{\nu]}{}^a\phi), \\
\Delta\Phi_\mu &= g_0(\omega_\mu{}^{ab}\Gamma_{ab}\phi + 2Mih_\mu{}^a\gamma_a\phi)
\end{aligned}$$

corrections to gauge transformations

$$\begin{aligned}
\delta_1\psi_\mu &= -g_0(\Gamma^{ab}\psi_\mu\hat{\eta}_{ab} + 2iM\gamma^a\psi_\mu\hat{\xi}_a \\
&\quad - \frac{2m}{3}\Gamma_\mu{}^a\phi\hat{\xi}_a - \omega_\mu{}^{ab}\Gamma_{ab}\xi - 2iMh_\mu{}^a\gamma_a\xi), \\
\delta_1\phi &= -g_0(\Gamma^{ab}\phi\hat{\eta}_{ab} + 2iM\gamma^a\phi\hat{\xi}_a)
\end{aligned}$$

and curvature transformations

$$\begin{aligned}
\delta\hat{\Psi}_{\mu\nu} &= -g_0(\Gamma^{ab}\Psi_{\mu\nu}\hat{\eta}_{ab} + 2iM\gamma^a\Psi_{\mu\nu}\hat{\xi}_a \\
&\quad + \frac{2m}{3}\Gamma_{[\mu}{}^a\Phi_{\nu]}\hat{\xi}_a - R_{\mu\nu}{}^{ab}\Gamma_{ab}\xi \\
&\quad - 2iMT_{\mu\nu}{}^a\gamma_a\xi), \\
\delta\hat{\Phi}_\mu &= -g_0(\Gamma^{ab}\Phi_\mu\eta_{ab} + 2iM\gamma^a\Phi_\mu\hat{\xi}_a).
\end{aligned}$$

Note that here we have only one free parameter g_0 being identified with the gravitational coupling constant. Now let us present in explicit form the pieces for massless spin-2 deformation: deformations to curvatures

$$\begin{aligned}
\Delta R_{\mu\nu}{}^{ab} &= b_1\bar{\psi}_{[\mu}\Gamma^{ab}\psi_{\nu]} + ib_2e_{[\mu}{}^{[a}\bar{\psi}_{\nu]}\gamma^{b]}\phi \\
&\quad + ib_3\bar{\psi}_{[\mu}\Gamma_{\nu]}{}^{ab}\phi + b_4e_{[\mu}{}^ae_{\nu]}{}^b\bar{\phi}\phi \\
&\quad + b_5\bar{\phi}\Gamma_{\mu\nu}{}^{ab}\phi, \\
\Delta T_{\mu\nu}{}^a &= ib_6\bar{\psi}_{[\mu}\gamma^a\psi_{\nu]} + b_7e_{[\mu}{}^a\bar{\psi}_{\nu]}\phi \\
&\quad + b_8\bar{\psi}_{[\mu}\Gamma_{\nu]}{}^a\phi + ib_9\bar{\phi}\Gamma_{\mu\nu}{}^a\phi
\end{aligned}$$

corrections to gauge transformations

$$\begin{aligned}
\delta_1\omega_\mu{}^{ab} &= 2b_1\bar{\psi}_\mu\Gamma^{ab}\xi - ib_2e_\mu{}^{[a}\bar{\phi}\gamma^{b]}\xi - ib_3\bar{\phi}\Gamma_\mu{}^{ab}\xi, \\
\delta_1h_\mu{}^a &= 2ib_6\bar{\psi}_\mu\gamma^a\xi + b_7e_\mu{}^a\bar{\phi}\xi + b_8\bar{\phi}\Gamma_\mu{}^a\xi
\end{aligned}$$

and curvature transformations

$$\begin{aligned}
\delta\hat{R}_{\mu\nu}{}^{ab} &= 2b_1\bar{\Psi}_{\mu\nu}\Gamma^{ab}\xi + ib_2e_{[\mu}{}^{[a}\bar{\Phi}_{\nu]}\gamma^{b]}\xi \\
&\quad - ib_3\bar{\Phi}_{[\mu}\Gamma_{\nu]}{}^{ab}\xi, \\
\delta\hat{T}_{\mu\nu}{}^a &= 2ib_6\bar{\Psi}_{\mu\nu}\gamma^a\xi - b_7e_{[\mu}{}^a\bar{\Phi}_{\nu]}\xi \\
&\quad + b_8\bar{\Phi}_{[\mu}\Gamma_{\nu]}{}^a\xi. \qquad (9)
\end{aligned}$$

General expressions for deformations to curvature and torsion will contain nine terms. In this one can verify that only b_1 will be as free parameter. Part of them is fixed from the requirement of curvature transformations (9) and another part is removed by field redefinitions.

At the last the interacting Lagrangian has form

$$\begin{aligned}
\mathcal{L} &= c_0\left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\}\hat{R}_{\mu\nu}{}^{ab}\hat{R}_{\alpha\beta}{}^{cd} \\
&\quad + c_1\left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\}\hat{\bar{\Psi}}_{\mu\nu}\Gamma^{abcd}\hat{\Psi}_{\nu\alpha} \\
&\quad + ic_2\left\{ {}^{\mu\nu\alpha}_{abc} \right\}\hat{\bar{\Psi}}_{\mu\nu}\Gamma^{abc}\hat{\Phi}_\alpha + c_3\left\{ {}^{\mu\nu}_{ab} \right\}\hat{\bar{\Phi}}_\mu\Gamma^{ab}\hat{\Phi}_\nu \\
&\quad + ic_4\left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\}\bar{\Psi}_{\mu\nu}\Gamma^{abc}\Phi_\alpha h_\beta{}^d \\
&\quad + c_5\left\{ {}^{\mu\nu\alpha}_{abc} \right\}\bar{\Phi}_\mu\Gamma^{ab}\Phi_\nu h_\alpha{}^c.
\end{aligned}$$

At non-interacting level the first four terms corresponds to spin 2, and massive spin 3/2. The last two terms are Abelian verteces. Here we have three free parameters $c_{2,4,5}$. Gauge invariance for Lagrangian imposes the following restrictions

$$b_1 = \frac{6c_1}{c_0}g_0, \qquad c_4 = 4c_2g_0, \qquad c_5 = 4c_3g_0$$

expressing all throug gravitational coupling constant g_0. From the second conditon we see arbitrariness of c_2 is related to construction abelian vertex at coefficient c_4.

5 Conclusion

Using frame-like gauge invariant approach we show that like for massless higher spin fields (i) massive spin 3/2 theory can be rewritten in terms of gauge invariant curvature (6) (ii) cubic vertices can be constructed as curvature deformation procedure.

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References

[1] Fradkin E. S., Vasiliev M. A. 1987 *Phys. Lett.* B **189** 89.

[2] Fradkin E. S., Vasiliev M. A. 1987 *Nucl. Phys.* B **291** 141.

[3] Metsaev R. R. 2006 *Nucl. Phys.* B **759** 147.

[4] Metsaev R. R. 2012 *Nucl. Phys.* B **859** 13.

[5] Vasiliev M. A. 1980 *Yad. Fiz.* **32** 855.

[6] Vasiliev M. 2012 *Nucl. Phys.* B **862** 341.

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МАССИВНОЕ ПОЛЕ СО СПИНОМ 3/2 И ЕГО ВЗАИМОДЕЙСТВИЯ В РЕПЕРНОМ ФОРМАЛИЗМЕ

Рассматривается массивное поле со спином 3/2 и изучается его взаимодействие с гравитационным полем. Используется реперная формулировка полей высших спинов ($s \geq 3/2$) в терминах калибровочно-инвариантных напряженностей. Показано, что, как и для безмассовых полей высших спинов, гравитационное взаимодействие массивного поля со спином 3/2 может быть построено с помощью процедуры деформации этих напряженностей.

Ключевые слова: *реперная формулировка, высшие спины, калибровочные симметрии.*

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