

# QUANTUM GRAVITY AND COSMOLOGY

Setare M.R.

## THE CARDY-VERLINDE FORMULA AND ENTROPY OF BLACK HOLES IN DE SITTER SPACES

Institute for Theoretical Physics and Mathematics, Tehran, Iran

### Introduction

The holographic duality which connects  $n+1$ -dimensional gravity in Anti-de Sitter (AdS) background with  $n$ -dimensional conformal field theory (CFT) has been discussed vigorously for some years [1]. But it seems that we live in a universe with a positive cosmological constant which will look like de Sitter space-time in the far future. Therefore, we should try to understand quantum gravity or string theory in de Sitter space preferably in a holographic way. Of course, physics in de Sitter space is interesting even without its connection to the real world; de Sitter entropy and temperature have always been mysterious aspects of quantum gravity [2]. While string theory successfully has addressed the problem of entropy for black holes, dS entropy remains a mystery. One reason is that the finite entropy seems to suggest that the Hilbert space of quantum gravity for asymptotically de Sitter space is finite dimensional, [3,4]. Another, related, reason is that the horizon and entropy in de Sitter space have an obvious observer dependence. For a black hole in flat space (or even in AdS) we can take the point of view of an outside observer who can assign a unique entropy to the black hole. The problem of what an observer venturing inside the black hole experiences, is much more tricky and has not been given a satisfactory answer within string theory. While the idea of black hole complementarity provides useful clues, [5], rigorous calculations are still limited to the perspective of the outside observer. In de Sitter space there is no way to escape the problem of the observer dependent entropy. This contributes to the difficulty of de Sitter space.

More recently, it has been proposed that defined in a manner analogous to the AdS/CFT correspondence, quantum gravity in a de Sitter (dS) space is dual to a certain Euclidean CFT living on a spacelike boundary of the dS space [6] (see also earlier works [7]-[9]). Following this proposal, some investigations on the dS space have been carried out recently [8]-[25]. According to the dS/CFT correspondence, it might be expected that as in the case of AdS black holes [26], the thermodynamics of cosmological horizon in asymptotically dS spaces can be identified with that of

a certain Euclidean CFT residing on a spacelike boundary of the asymptotically dS spaces.

One of the remarkable outcomes of the AdS/CFT and dS/CFT correspondence has been the generalization of Cardy's formula (Cardy-Verlinde formula) for arbitrary dimensionality, as well as a variety of AdS and dS backgrounds. In this paper, we will show that the entropy of a cosmological horizon in the topological Reissner-Nordström-de Sitter (TRNdS) and topological Kerr-Newman-de Sitter spaces (TKNdS) can also be rewritten in the form of the Cardy-Verlinde formula. We then show that if one uses the Abbott and Deser (AD) prescription [27], the entropy of black hole horizons in dS spaces can also be expressed by the Cardy-Verlinde formula.

### Topological Reissner-Nordström-de Sitter Black Holes

We start with an  $(n+2)$ -dimensional TRNdS black hole solution, whose metric is

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2\gamma_{ij}dx^i dx^j,$$

$$f(r) = k - \frac{\omega_n M}{r^{n-1}} + \frac{n\omega_n^2 Q^2}{8(n-1)r^{2n-2}} - \frac{r^2}{l^2}, \quad (1)$$

where

$$\omega_n = \frac{16\pi G_{n+2}}{n \text{Vol}(\Sigma)}, \quad (2)$$

where  $\gamma_{ij}$  denotes the line element of an  $n$ -dimensional hypersurface  $\Sigma$  with constant curvature  $n(n-1)k$  and volume  $\text{Vol}(\Sigma)$ ,  $G_{n+2}$  is the  $(n+2)$ -dimensional Newtonian gravity constant,  $M$  is an integration constant,  $Q$  is the electric/magnetic charge of Maxwell field. When  $k=1$ , the metric Eq. (1) is just the Reissner-Nordström-de Sitter solution. For general  $M$  and  $Q$ , the equation  $f(r)=0$  may have four real roots. Three of them are real, the largest one is the cosmological horizon  $r_c$ , the smallest is the inner (Cauchy) horizon of black hole, the one in between is the outer horizon  $r_+$  of the black hole. And the fourth is negative and has no physical meaning. The case  $M=Q=0$  reduces to the de Sitter space with a cosmological horizon  $r_c=l$ .

When  $k=0$  or  $k<0$ , there is only one positive real root of  $f(r)$ , and this locates the position of cosmological horizon  $r_c$ .

In the case of  $k=0$ ,  $\gamma_{ij}dx^i dx^j$  is an  $n$ -dimensional Ricci flat hypersurface, when  $M=Q=0$  the solution Eq.(1) goes to pure de Sitter space

$$ds^2 = \frac{r^2}{l^2} dt^2 - \frac{l^2}{r^2} dr^2 + r^2 dx_n^2 \quad (3)$$

in which  $r$  becomes a timelike coordinate.

When  $Q=0$ , and  $M \rightarrow -M$  the metric Eq. (1) is the TdS (topological de Sitter) solution [28,29], which have a cosmological horizon and a naked singularity, for this type of solution, the Cardy-Verlinde formula also work well.

Here we review the BBM prescription [18] for computing the conserved quantities of asymptotically de Sitter spacetimes briefly. In a theory of gravity, mass is a measure of how much a metric deviates near infinity from its natural vacuum behavior; i.e, mass measures the warping of space. Inspired by the analogous reasoning in AdS space [30,31] one can construct a divergence-free Euclidean quasilocal stress tensor in de Sitter space by the response of the action to variation of the boundary metric we have

$$T^{\mu\nu} = \frac{2}{\sqrt{h}} \frac{\delta I}{\delta h_{\mu\nu}} = \frac{1}{8\pi G} \left[ K^{\mu\nu} - K h^{\mu\nu} + \frac{n}{l} h^{\mu\nu} + \frac{1}{n} G^{\mu\nu} \right] \quad (4)$$

where  $h^{\mu\nu}$  is the metric induced on surfaces of fixed time,  $K^{\mu\nu}$ ,  $K$  are respectively extrinsic curvature and its trace,  $G^{\mu\nu}$  is the Einstein tensor of the boundary geometry. To compute the mass and other conserved quantities, one can write the metric  $h^{\mu\nu}$  in the following form

$$h_{\mu\nu} dx^\mu dx^\nu = N_\rho^2 d\rho^2 + \sigma_{ab} (d\phi^a + N_\xi^a d\rho) \times (d\phi^b + N_\xi^b d\rho) \quad (5)$$

where the  $\xi^\mu$  are angular variables parametrizing closed surfaces around the origin. When there is a Killing vector field  $\xi^\mu$  on the boundary, then the conserved charge associated to  $\xi^\mu$  can be written as [30,31]

$$\oint \Sigma d^n \phi \sqrt{\sigma} n^\mu \xi^\mu T_{\mu\nu} \quad (6)$$

where  $n^\mu$  is the unit normal vector on the boundary,  $\sigma$  is the determinant of the metric  $\sigma_{ab}$ . Therefore the mass of an asymptotically de Sitter space is as

$$M = \oint \Sigma d^n \phi \sqrt{\sigma} N_\rho \varepsilon, \quad \varepsilon \equiv n^\mu n^\nu T_{\mu\nu} \quad (7)$$

where Killing vector is normalized as  $\xi^\mu = N_\rho n^\mu$ .

Using this prescription [18], the gravitational mass, having subtracted the anomalous Casimir energy, of the TRNdS solution is

$$E = -M = \frac{r_c^{n-1}}{\omega_n} \left( k - \frac{r_c^2}{l^2} + \frac{n\omega_n^2 Q^2}{8(n-1)r_c^{2n-2}} \right) \quad (8)$$

Some thermodynamic quantities associated with the cosmological horizon are

$$T_c = \frac{1}{4\pi r_c} \left( -(n-1)k + (n+1) \frac{r_c^2}{l^2} + \frac{\omega_n^2 Q^2}{8r_c^{2n-2}} \right),$$

$$S_c = \frac{r_c^n \text{Vol}(\Sigma)}{4G},$$

$$\phi_c = -\frac{n}{4n-1} \frac{\omega_n Q}{r_c^{n-1}}, \quad (9)$$

where  $\phi_c$  is the chemical potential conjugate to the charge  $Q$ .

The Casimir energy  $E_C$ , defined as

$$E_C = (n+1)E - nTS - n\phi Q$$

in this case, is found to be

$$E_C = -\frac{2nkr_c^{n-1} \text{Vol}(\Sigma)}{16\pi G}, \quad (10)$$

when  $k=0$ , the Casimir energy vanishes, as the case of asymptotically AdS space. When  $k = \pm 1$ , we see from Eq. (10) that the sign of energy is just contrast to the case of TRNAdS space [32].

Thus we can see that the entropy Eq. (9) of the cosmological horizon can be rewritten as

$$S = \frac{2\pi l}{n} \sqrt{\frac{E_C}{k} (2(E - E_q) - E_C)}, \quad (11)$$

where

$$E_q = \frac{1}{2} \phi_c Q = -\frac{n}{8(n-1)} \frac{\omega_n Q^2}{r_c^{n-1}}. \quad (12)$$

We note that the entropy expression (11) has a similar form as the case of TRNAdS black holes [32].

For the black hole horizon, which is only for the case  $k=1$ , associated thermodynamic quantities are

$$T_b = \frac{1}{4\pi r_b} \left( (n-1) - (n+1) \frac{r_b^2}{l^2} - \frac{n\omega_n^2 Q^2}{8r_b^{2n-2}} \right),$$

$$S_b = \frac{r_b^n \text{Vol}(\Sigma)}{4G},$$

$$\phi_b = \frac{n}{4(n-1)} \frac{\omega_n Q}{r_b^{n-1}}. \quad (13)$$

Now if we use the BBM mass Eq.(8) the black hole horizon entropy cannot be expressed in a form like Cardy-Verlinde formula [28]. The other way for computing conserved quantities of asymptotically de Sitter space is the Abbott and Deser (AD) prescription [27]. According to this prescription, the gravitational mass of asymptotically de Sitter space coincides with the ADM mass in asymptotically flat space, when the cosmological constant goes to zero. Using the AD prescription for calculating conserved quantities the black hole horizon entropy of TKNdS space can be expressed in term of the Cardy-Verlinde formula [28]. The AD mass of TRNdS solution can be expressed in terms of black hole horizon radius  $r_b$  and charge  $Q$ ,

$$E' = M = \frac{r_b^{n-1}}{\omega_n} \left( 1 - \frac{r_b^2}{l^2} + \frac{n\omega_n^2 Q^2}{8(n-1)r_b^{2n-2}} \right). \quad (14)$$

In this case, the Casimir energy, defined as

$$E'_C = (n+1)E' - nT_b S_b - n\phi_b Q, \text{ is}$$

$$E'_C = \frac{2nr_b^{n-1} \text{Vol}(\Sigma)}{16\pi G}, \quad (15)$$

and the black hole entropy  $S_b$  can be rewritten as

$$S_b = \frac{2\pi l}{n} \sqrt{E'_C |2(E' - E'_q) - E'_C|}, \quad (16)$$

where

$$E'_q = \frac{1}{2} \phi_b Q = \frac{n\omega_n Q^2}{8(n-1)r_b^{n-1}}, \quad (17)$$

which is the energy of electromagnetic field outside the black hole horizon. Thus we demonstrate that the black hole horizon entropy of TRNdS solution can be expressed in a form as the Cardy-Verlinde formula. However, if one uses the BBM mass Eq. (8), the black hole horizon entropy  $S_b$  cannot be expressed by a form like the Cardy-Verlinde formula.

### Topological Kerr-Newman-de Sitter Black Holes

The line element of TKNdS black holes in 4-dimension case is given by

$$ds^2 = -\frac{\Delta_r}{\rho^2} \left( dt - \frac{a}{\Xi} \sin^2 \theta d\phi \right)^2 + \frac{\rho^2}{\Delta_r} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2 + \frac{\Delta_\theta \sin^2 \theta}{\rho^2} \left[ a dt - \frac{(r^2 + a^2)}{\Xi} d\phi \right]^2, \quad (18)$$

where

$$\begin{aligned} \Delta_r &= (r^2 + a^2) \left( k - \frac{r^2}{l^2} \right) - 2Mr + q^2, \\ \Delta_\theta &= 1 + \frac{a^2 \cos^2 \theta}{l^2}, \\ \Xi &= 1 + \frac{a^2}{l^2}, \\ \rho^2 &= r^2 + a^2 \cos^2 \theta. \end{aligned} \quad (19)$$

Here the parameters  $M$ ,  $a$ , and  $q$  are associated with the mass, angular momentum, and electric charge parameters of the space-time, respectively. The topological metric Eq. (18) will only solve the Einstein equations if  $k=1$ , which is the spherical topology. In fact when  $k=1$ , the metric Eq. (18) is just the Kerr-Newman-de Sitter solution. Three real roots of the equation  $\Delta_r = 0$ , are the locations of three horizons, the largest being the cosmological horizon  $r_c$ , the smallest is the inner horizon of black hole, the one in between is the outer horizon  $r_b$  of the black hole.

If we want in the  $k=0, -1$  cases to solve the Einstein equations, then we must set  $\sin \theta \rightarrow \theta$ , and  $\sin \theta \rightarrow \sinh \theta$  respectively [35]-[38]. When  $k=0$  or  $k=-1$ , there is only one positive real root of  $\Delta_r$ , and this locates the position of cosmological horizon  $r_c$ .

In the BBM prescription [18], the gravitational mass, subtracted the anomalous Casimir energy, of the 4-dimensional TKNdS solution is

$$E = \frac{-M}{\Xi}. \quad (20)$$

Where the parameter  $M$  can be obtained from the equation  $\Delta_r = 0$ . On this basis, the following relation for the gravitational mass can be obtained

$$E = \frac{-M}{\Xi} = \frac{(r^2 + a^2)(r_c^2 - kl^2) - q^2 l^2}{2\Xi r_c l^2}. \quad (21)$$

The Hawking temperature of the cosmological horizon is given by

$$\begin{aligned} T_c &= \frac{-1}{4\pi} \frac{\Delta'_r(r_c)}{(r_c^2 + a^2)} \\ &= \frac{3r_c^4 + r_c^2(a^2 - kl^2) + (ka^2 + q^2)l^2}{4\pi r_c l^2 (r_c^2 + a^2)}. \end{aligned} \quad (22)$$

The entropy associated with the cosmological horizon can be calculated as

$$S_c = \frac{\pi(r_c^2 + a^2)}{\Xi}. \quad (23)$$

The angular velocity of the cosmological horizon is given by

$$\Omega_c = \frac{-a\Xi}{(r_c^2 + a^2)}. \quad (24)$$

The angular momentum  $J_c$ , the electric charge  $Q$ , and the electric potentials  $\phi_{qc}$  and  $\phi_{qc0}$  are given by

$$J_c = \frac{Ma}{\Xi^2},$$

$$Q = \frac{q}{\Xi},$$

$$\phi_{qc} = \frac{qr_c}{r_c^2 + a^2},$$

$$\phi_{qc0} = -\frac{q}{r_c}. \quad (25)$$

The obtained above quantities of the cosmological horizon satisfy the first law of thermodynamics:

$$dE = T_c dS_c + \Omega_c dJ_c + (\phi_{qc} + \phi_{qc0}) dQ. \quad (26)$$

Using the Eqs. (23,25) for the cosmological horizon entropy, angular momentum and charge, and also the equation  $\Delta_r(r_c) = 0$ , we can obtain the metric parameters  $M$ ,  $a$ ,  $q$  as a function of  $S_c$ ,  $J_c$  and  $Q$ , and after that we can write  $E$  as a function of these thermodynamical quantities:  $E(S_c, J_c, Q)$  (see [40]). Then one can define the quantities conjugate to  $S_c$ ,  $J_c$  and  $Q$ , as

$$T_c = \left( \frac{\partial E}{\partial S_c} \right)_{J_c, Q}, \quad \Omega_c = \left( \frac{\partial E}{\partial J_c} \right)_{S_c, Q},$$

$$\phi_{qc} = \left( \frac{\partial E}{\partial Q} \right)_{S_c, J_c}, \quad \phi_{qc0} = \lim_{a \rightarrow 0} \left( \frac{\partial E}{\partial Q} \right)_{S_c, J_c}. \quad (27)$$

Making use of the fact that the metric for the boundary CFT can be determined only up to a conformal factor, we rescale the boundary metric for the CFT to the following form:

$$ds_{CFT}^2 = \lim_{r \rightarrow \infty} \frac{R^2}{r^2} ds^2, \quad (28)$$

Then the thermodynamic relations between the boundary CFT and the bulk TKNdS are given by

$$E_{CFT} = \frac{1}{R} E, \quad T_{CFT} = \frac{1}{R} T, \quad J_{CFT} = \frac{1}{R} J,$$

$$\phi_{CFT} = \frac{1}{R} \phi, \quad \phi_{0CFT} = \frac{1}{R} \phi_0. \quad (29)$$

The Casimir energy  $E_C$ , defined as

$$E_C = (n+1)E - n(T_c S_c + J_c \Omega_c + Q/2\phi_{qc} + Q/2\phi_{qc0}),$$

and  $n=2$  in this case, is found to be

$$E_C = -\frac{k(r_c^2 + a^2)l}{R\Xi r_c}, \quad (30)$$

in KNdS space case [33] the Casimir energy  $E_C$  is always negative, but in TKNdS space case the Casimir energy can be positive, negative or vanishing depending on the choice of  $k$ . Thus we can see that the entropy Eq. (23) of the cosmological horizon can be rewritten as

$$S = \frac{2\pi R}{n} \sqrt{\left| \frac{E_c}{k} \right| (2(E - E_q) - E_c)}, \quad (31)$$

where

$$E_q = \frac{1}{2} \phi_{c0} Q. \quad (32)$$

We note that the entropy expression (31) has a similar form as in the case of TRNdS black holes Eq. (11).

For the black hole horizon, which only exists for the case  $k=1$  the associated thermodynamic quantities are

$$T_b = \frac{1}{4\pi} \frac{\Delta'_r(r_b)}{(r_b^2 + a^2)} = \frac{3r_b^4 + r_b^2(a^2 - l^2) + (a^2 + q^2)l^2}{4\pi r_b l^2 (r_b^2 + a^2)}, \quad (33)$$

$$S_b = \frac{\pi(r_b^2 + a^2)}{\Xi}. \quad (34)$$

$$\Omega_b = \frac{a\Xi}{(r_b^2 + a^2)}. \quad (35)$$

$$J_b = \frac{Ma}{\Xi^2}, \quad (36)$$

$$Q = \frac{q}{\Xi}, \quad (37)$$

$$\phi_{qb} = \frac{qr_b}{r_b^2 + a^2}, \quad (38)$$

$$\phi_{qb0} = \frac{q}{r_b}. \quad (39)$$

The AD mass of TKNdS solution can be expressed in terms of the black hole horizon radius  $r_b$ ,  $a$  and charge  $q$ :

$$E' = \frac{M}{\Xi} = \frac{(r_b^2 + a^2)(r_b^2 - l^2) - q^2 l^2}{2\Xi r_b l^2}. \quad (40)$$

The quantities obtained above of the black hole horizon also satisfy the first law of thermodynamics:

$$dE' = T_b dS_b + \Omega_b dJ_b + (\phi_{qb} + \phi_{qb0}) dQ. \quad (41)$$

The thermodynamics quantities of the CFT must be rescaled by a factor  $\frac{1}{R}$  similar to the pervious case. In this case, the Casimir energy, defined by  $E'_C = (n+1)E' - n(T_b S_b + J_b \Omega_b + Q/2\phi_{qb} + Q/2\phi_{qb0})$ ,

is

$$E'_C = \frac{(r_b^2 + a^2)l}{R \Xi r_b}, \quad (42)$$

and the black hole entropy  $S_b$  can be rewritten as

$$S_b = \frac{2\pi R}{n} \sqrt{E'_C [(2(E' - E'_q) - E'_C)]}, \quad (43)$$

where

$$E'_q = \frac{1}{2} \phi_{qb0} Q. \quad (44)$$

This is the energy of an electromagnetic field outside the black hole horizon. Thus we demonstrate that the black hole horizon entropy of the TKNdS solution can be expressed in the form of the Cardy-Verlinde formula. However, if one uses the BBM mass Eq. (21) the black hole horizon entropy  $S_b$  cannot be expressed in a form like the Cardy-Verlinde formula. Our result is in favour of the dS/CFT correspondence.

## Conclusion

The Cardy-Verlinde formula recently proposed by Verlinde [39], relates the entropy of a certain CFT to its total energy and Casimir energy in arbitrary dimensions. In the spirit of dS/CFT correspondence, this formula has been shown to hold exactly for the cases of dS Schwarzschild, dS topological, dS Reissner-Nordström, dS Kerr, and dS Kerr-Newman black holes. In this paper we have further checked the Cardy-Verlinde formula with topological Reissner-Nordström- de Sitter and topological Kerr-Newman de Sitter black holes.

It is well-known that there is no black hole solution whose event horizon is not a sphere, in a de Sitter background, although there are such solutions in an anti-de Sitter background; then in TRNdS, TKNdS

spaces for the case  $k=0,-1$  the black hole does not have an event horizon, however the cosmological horizon geometry is spherical, flat and hyperbolic for  $k=1,0,-1$ , respectively. As we have shown there exist two different temperatures and entropies associated with the cosmological horizon and black hole horizon, in TRNdS, TKNdS spacetimes. If the temperatures of the black hole and cosmological horizon are equal, then the entropy of the system is the sum of the entropies of cosmological and black hole horizons. The geometric features of the black hole temperature and entropy seem to imply that the black hole thermodynamics is closely related to nontrivial topological structure of spacetime. In [41] Cai, et al in order to relate the entropy with the Euler characteristic  $\chi$  of the corresponding Euclidean manifolds have presented the following relation:

$$S = \frac{\chi_1 A_{BH}}{8} + \frac{\chi_2 A_{CH}}{8}, \quad (45)$$

in which the Euler number of the manifolds is divided into two parts; the first part comes from the black hole horizon and the second part come from the cosmological horizon (see also [42,43,44]). If one uses the BBM mass of the asymptotically dS spaces, the black hole horizon entropy cannot be expressed in a form like the Cardy-Verlinde formula [28]. In this paper, we have found that if one uses the AD prescription to calculate conserved charges of asymptotically dS spaces, the black hole horizon entropy can also be rewritten in the form of the Cardy-Verlinde formula, which indicates that the thermodynamics of the black hole horizon in dS spaces can also be described by a certain CFT. Our result is also reminiscent of Carlip's claim [45] (to see a new formulation which is free of the inconsistencies encountered in Carlip's in [46] that for black holes of any dimensionality the Bekenstein-Hawking entropy can be reproduced using the Cardy formula [47]). Also we have shown that the Casimir energy for a cosmological horizon in TKNdS space case can be positive, negative or vanishing, depending on the choice of  $k$ ; by contrast, the Casimir energy for a cosmological horizon in KNdS space is always negative [33].

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