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# BRST-BFV LAGRANGIAN FORMULATIONS FOR HS FIELDS SUBJECT TO TWO-COLUMN YOUNG TABLEAUX

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The details of Lagrangian description of irreducible integer higher-spin representations of the Poincare group with an Young tableaux $Y[\hat{s}_1, \hat{s}_2]$ having 2 columns are considered for Bose particles propagated on an arbitrary dimensional Minkowski space-time. The procedure is based, first, on using of an auxiliary Fock space generated by Fermi oscillators (antisymmetric basis), second, on construction of the Verma module and finding auxiliary oscillator realization for $sl(2)\oplus sl(2)$ algebra which encodes the second-class operator constraints subsystem in the HS symmetry superalgebra. Application of an universal BRST-BFV approach permits to reproduce gauge-invariant Lagrangians with reducible gauge symmetries describing the free dynamics of both massless and massive mixed-antisymmetric bosonic fields of any spin with appropriate number of gauge and Stukelberg fields. The general prescription possesses by the possibility to derive constrained Lagrangians with only BRST-invariant extended algebraic constraints which describes the Poincare group irreducible representations in terms of mixed-antisymmetric tensor fields with 2 group indices.

**Keywords:** *higher spins, BRST operator, Lagrangian formulation, Verma module, gauge invariance.*

## 1 Introduction

The belief to reconsider the problems of an unique description of variety of elementary particles and known interactions maybe resolved within higher-spin (HS) field theory whose revealing together with the proof of supersymmetry display, and finding a new insight on origin of Dark Matter remains by the aims in LHC experiment programm ( [1]). Because of the existence of so-called tensionless limit in the (super)string theory [2] which operates with an infinite tower of HS fields with integer and half-integer spins the HS field theory may be considered both of superstring theory part and as an method to study a superstring theory structure. On present state of HS field theory we recommend to know from reviews [3–6]. The paper considers the results of constructing Lagrangian formulations (LFs) for free integer both massless and massive mixed-antisymmetry tensor HS fields on flat $\mathbb{R}^{1,d-1}$-space-time subject to arbitrary Young tableaux (YT) with 2 columns $Y[\hat{s}_1, \hat{s}_2]$ for $\hat{s}_1 \geq \hat{s}_2$ in Fronsdal metric-like formalism on a base of BFV-BRST approach [7], and precesses the results which appear soon in [8] (as well as for fermionic mixed-antisymmetric spin-tensor HS fields on $\mathbb{R}^{1,d-1}$-space-time subject to arbitrary $Y[\hat{n}_1 + \frac{1}{2}, \hat{n}_2 + \frac{1}{2}]$ in [9]).

The irreducible Poincare or (Anti)-de-Sitter ((A)dS) group representations in the constant curvature space-times may be described both by mixed-symmetric HS fields subject to arbitrary YT with $k$ rows, $Y(s_1, ..., s_k)$, (case of symmetric basis) determined by more than one spin-like parameters $s_i$ [10,11] and, equivalently, by mixed-antisymmetric tensor or spin-tensor fields subject to arbitrary YT now with $l$ columns, $Y[\hat{s}_1, ..., \hat{s}_l]$, (case of antisymmetric basis) with integers or half-integers $\hat{s}_1 \geq \hat{s}_2 \geq ... \geq \hat{s}_l$ having a spin-like interpretation [8,9]. Both mixed-symmetric and mixed-antisymmetric HS fields appear for $d > 4$ space-time dimensions, in addition to totally symmetric and antisymmetric irreducible representations of Poincare or (A)dS algebras. Whereas for the latter ones and mixed-symmetric HS fields case the LFs both for massless and massive free higher-spin fields is well enough developed [12–16] as well as on base of BFV-BRST approach, e.g. in [17–24], for the mixed-antisymmetric case the problem of their field-theoretic description has not yet solved except for constrained bosonic fields subject to $Y[\hat{s}_1, \hat{s}_2]$ on the level of the equations of motion only [25] in so-called "frame-like" formulation.

We use, first, the conventions for the metric tensor $\eta_{\mu\nu} = diag(+, -, ..., -)$, with Lorentz indices $\mu, \nu = 0, 1, ..., d-1$, second, the notation $\epsilon(A)$, $gh(A)$ for the respective values of Grassmann parity and ghost number of a quantity $A$, and denote by $[A, B\}$ the supercommutator of quantities $A, B$, which for theirs definite values of Grassmann parities is given by $[A\,, B\} = AB - (-1)^{\epsilon(A)\epsilon(B)}BA$.

## 2 Derivation of integer HS symmetry superalgebra on $\mathbb{R}^{1,d-1}$

We consider a massless integer spin irreducible representation of Poincare group in a Minkowski space $\mathbb{R}^{1,d-1}$ which is described by a tensor field $\Phi_{[\mu^1]_{\hat{s}_1},[\mu^2]_{\hat{s}_2}} \equiv \Phi_{\mu^1_1\dots\mu^1_{\hat{s}_1},\mu^2_1\dots\mu^2_{\hat{s}_2}}$ of rank $\hat{s}_1 + \hat{s}_2$ and generalized spin $\mathbf{s} \equiv (s_1,...,s_{s_2}; s_{s_2+1},...,s_{s_1}) = (2,2,...,2;1,...,1)$, (with omitting later a sign "$\hat{}$" under $\hat{s}_i$ and $s_1 \geq s_2 > 0, s_1 \leq [d/2]$) subject to a YT with 2 columns of height $s_1, s_2$, respectively

$$\Phi_{[\mu^1]_{s_1},[\mu^2]_{s_2}} \longleftrightarrow \begin{array}{|c|c|}\hline \mu^1_1 & \mu^2_1 \\ \hline \cdot & \cdot \\ \hline \mu^1_{s_2} & \mu^2_{s_2} \\ \hline \cdots & \multicolumn{1}{c}{} \\ \cline{1-1} \mu^1_{s_1} & \multicolumn{1}{c}{} \\ \cline{1-1} \end{array} . \qquad (1)$$

This field is antisymmetric with respect to the permutations of each type of Lorentz indices $\mu^i$, and obeys to the Klein-Gordon (2), divergentless (3), traceless (4) and mixed-antisymmetry equations (5):

$$\partial^\mu \partial_\mu \Phi_{[\mu^1]_{s_1},[\mu^2]_{s_2}} = 0, \qquad (2)$$

$$\partial^{\mu^i_{l_i}} \Phi_{[\mu^1]_{s_1},[\mu^2]_{s_2}} = 0, \ \texttt{for} \ 1 \leq l_i \leq s_i, \ i = 1,2, \qquad (3)$$

$$\eta^{\mu^1_{l_1}\mu^2_{l_2}} \Phi_{[\mu^1]_{s_1},[\mu^2]_{s_2}} = 0, \ \texttt{for} \ 1 \leq l_i \leq s_i, \qquad (4)$$

$$\Phi_{[[\mu^1]_{s_1}, \underbrace{\mu^2_1\dots\mu^2_{l_2-1}}\mu^2_{l_2}]\dots\mu^2_{s_2}} = 0, \qquad (5)$$

where the bracket below in (5) denotes that the indices in it are not included into antisymmetrization, i.e. the antisymmetrization concerns only indices $[\mu^1]_{s_1}, \mu^2_{l_2}$ in $[[\mu^1]_{s_1}, \underbrace{\mu^2_1\dots\mu^2_{l_2-1}}\mu^2_{l_2}]$.

Combined description of all integer spin mixed-antisymmetric $ISO(1,d-1)$ group irreps can be reformulated with help of an auxiliary Fock space $\mathcal{H}^f$, generated by 2 pairs of fermionic creation $a^i_{\mu^i}(x)$ and annihilation $a^{j+}_{\nu^j}(x)$ operators (in antisymmetric basis), $i,j = 1,2, \mu^i, \nu^j = 0,1...,d-1$: $a^i_{\mu^i}, a^{j+}_{\nu^j} = -\eta_{\mu^i\nu^j}\delta^{ij}$ and a set of constraints for an arbitrary string-like vector $|\Phi\rangle \in \mathcal{H}^f$,

$$|\Phi\rangle = \sum_{s_1=0}^{[d/2]} \sum_{s_2=0}^{s_1} \Phi_{[\mu^1]_{s_1},[\mu^2]_{s_2}}(x) \prod_{i=1}^{2} \prod_{l_i=1}^{s_i} a_i^{+\mu^i_{l_i}} |0\rangle, \qquad (6)$$

$$\bigl(l_0, l^i, l^{12}, t^{i_1 j_1}\bigr)|\Phi\rangle = 0, \quad l_0 = \partial^\mu \partial_\mu, \qquad (7)$$

$$\bigl(l^i, l^{12}, t^{i_1 j_1}\bigr) = \bigl(-i a^i_\mu \partial^\mu, \tfrac{1}{2} a^1_\mu a^{2\mu}, a^{1+}_\mu a^{2\mu}\bigr). \qquad (8)$$

The set of 3 even and 2 odd, $l^i$, primary constraints (7), (8) with $\{o_\alpha\} = \bigl\{l_0, l^i, l^{12}, t^{12}\bigr\}$, because of the property of translational invariance of the vacuum, $\partial_\mu|0\rangle = 0$, are equivalent to (2)–(5) for all possible heights $s_1 \geq s_2$. In turn, when we impose on $|\Phi\rangle$ the additional to (7), (8) constraints with number particles operators, $g^i_0$,

$$g^i_0|\Phi\rangle = (s_i - \tfrac{d}{2})|\Phi\rangle, \ g^i_0 = -\tfrac{1}{2}[a^{i+}_\mu, a^{\mu i}], \qquad (9)$$

these combined conditions are equivalent to Eqs. (2)–(5) for the field $\Phi_{[\mu^1]_{s_1},[\mu^2]_{s_2}}(x)$ with given spin $\mathbf{s} = (2,2,...,2,1,...,1)$.

The procedure of LF construction implies the property of BFV-BRST operator $Q$, $Q = C^\alpha o_\alpha + more$, to be Hermitian, that is equivalent to the requirements: $\{o_\alpha\}^+ = \{o_\alpha\}$ and closedness for $\{o_\alpha\}$ with respect to the supercommutator multiplication $[\ ,\ \}$. Evidently, the set of $\{o_\alpha\}$ violates above conditions. To provide them we consider in standard manner an scalar product on $\mathcal{H}^f$,

$$\langle\Psi|\Phi\rangle = \int d^d x \sum_{s_1=0}^{[d/2]} \sum_{s_2=0}^{s_1} \sum_{p_1=0}^{[d/2]} \sum_{p_2=0}^{p_1} \langle 0| \Bigl( \prod_{(j,m_j)=(1,1)}^{2,p_j} a_j^{\nu^j_{m_j}+} \Bigr)^+ \times \Psi^*_{[\nu^1]_{p_1},[\nu^2]_{p_2}} \Phi_{[\mu^1]_{s_1},[\mu^2]_{s_2}} \prod_{(i,l_i)=(1,1)}^{(2,s_i)} a_i^{+\mu^i_{l_i}} |0\rangle. \qquad (10)$$

As the result, the set of $\{o_\alpha\}$ extended by means of the operators,

$$\bigl(l^{i+}, l^{12+}, t^{12+}\bigr) = \bigl(-i a^{i+}_\mu \partial^\mu, \tfrac{1}{2} a^{2+}_\mu a^{1\mu+}, a^{2+}_\mu a^{1\mu}\bigr), \qquad (11)$$

is closed with respect to Hermitian conjugation, with taken into account of $(l_0^+, \ g_0^{i\,+}) = (l_0, \ g^i_0)$. It is rather simple exercise to see the second requirement is fulfilled as well if the number particles operators $g^i_0$ will be included into set of all constraints $o_I$ having therefore the structure,

$$\{o_I\} = \{o_\alpha, o^+_\alpha; \ g^i_0\} \equiv \{o_a, o^+_a; \ l_0, \ l^i, \ l^{i+}; \ g^i_0\}. \qquad (12)$$

Together the sets $\{o_a, o^+_a\}$ in the Eq. (12), for $\{o_a\} = \{l^{12}, t^{12}\}$ and $\{o_A\} = \{l_0, \ l^i, \ l^{i+}\}$, may be considered from the Hamiltonian analysis of the dynamical systems as the operator respective 4 second-class and 5 first-class constraints subsystems among $\{o_I\}$ for topological gauge system (i.e. with zero Hamiltonian) because of,

$$[o_a, \ o^+_b\} = f^c_{ab} o_c + \Delta_{ab}(g^i_0), \ [o_I, \ o_B\} = f^C_{IB} o_C. \qquad (13)$$

Here the constants $f^c_{ab}, f^C_{IB}$ are the antisymmetric with respect to permutations of lower indices and quantities $\Delta_{ab}(g^i_0)$ form the non-degenerate $4\times 4$ matrix $\|\Delta_{ab}\|$ in the Fock space $\mathcal{H}^f$ on the surface $\Sigma \subset \mathcal{H}^f$: $\|\Delta_{ab}\|_{|\Sigma} \neq 0$, determined by the equations, $(o_a, l_0, \ l^i)|\Phi\rangle = 0$.

Explicitly, operators $o_I$ satisfy to the Lie-algebra commutation relations, $[o_I, \ o_J] = f^K_{IJ} o_K$, $f^K_{IJ} = -(-1)^{\varepsilon(o_I)\varepsilon(o_J)} f^K_{JI}$ with the structure constants $f^K_{IJ}$ being used in the Eq. (13), and determined from the multiplication Table 1.

Table 1. HS symmetry superalgebra $\mathcal{A}(Y[2],\mathbb{R}^{1,d-1})$

| $[\downarrow,\rightarrow\}$ | $t^{12}$ | $t^+_{12}$ | $l_0$ | $l^i$ | $l^{i+}$ | $l^{12}$ | $l^{12+}$ | $g^i_0$ |
|---|---|---|---|---|---|---|---|---|
| $t^{12}$ | $0$ | $g^1_0-g^2_0$ | $0$ | $l^2\delta^{i1}$ | $-l^{1+}\delta^{2i}$ | $0$ | $0$ | $-F^{12,i}$ |
| $t^+_{12}$ | $g^2_0-g^1_0$ | $0$ | $0$ | $l_1\delta^{i2}$ | $-l^+_2\delta^{i1}$ | $0$ | $0$ | $F^{12,i+}$ |
| $l_0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ |
| $l^j$ | $-l^2\delta^{j1}$ | $-l_1\delta^{j2}$ | $0$ | $0$ | $l_0\delta^{ji}$ | $0$ | $\frac{1}{2}l^{[2+}\delta^{1]j}$ | $l^j\delta^{ij}$ |
| $l^{j+}$ | $l^{1+}\delta^{j2}$ | $l^+_2\delta^{j1}$ | $0$ | $l_0\delta^{ji}$ | $0$ | $\frac{1}{2}l^{[1}\delta^{2]j}$ | $0$ | $-l^{j+}\delta^{ij}$ |
| $l^{12}$ | $0$ | $0$ | $0$ | $0$ | $\frac{1}{2}l^{[2}\delta^{1]i}$ | $0$ | $-\frac{1}{4}(g^1_0+g^2_0)$ | $l^{12}$ |
| $l^{12+}$ | $0$ | $0$ | $0$ | $\frac{1}{2}l^{[1+}\delta^{2]i}$ | $0$ | $\frac{1}{4}(g^1_0+g^2_0)$ | $0$ | $-l^{12+}$ |
| $g^j_0$ | $F^{12,j}$ | $-F^{12j+}$ | $0$ | $-l^i\delta^{ij}$ | $l^{i+}\delta^{ij}$ | $-l^{12}$ | $l^{12+}$ | $0$ |

Note, that in the Table 1, the squared brackets for the indices $i$, $j$ in the quantity $A^{[i}B^{j]k}$ mean the antisymmetrization $A^{[i}B^{j]k}=A^iB^{jk}-A^jB^{ik}$ and $F^{12,i}=t^{12}(\delta^{i1}-\delta^{i2})$, $\quad F^{12,i+}=t^{12+}(\delta^{i1}-\delta^{i2})$. We call the superalgebra of the operators $o_I$ as *integer higher-spin symmetry algebra in Minkowski space with a YT having* 2 *columns* and denote it as $\mathcal{A}(Y[2],\mathbb{R}^{1,d-1})$.

The structure of $\mathcal{A}(Y[2],\mathbb{R}^{1,d-1})$ appears by insufficient to construct BRST operator $Q$ with respect to its elements $o_I$ which should generate correct Lagrangian dynamics due to second-class constraints $\{o_a\}$ presence in it. Therefore, we should to convert into enlarged set of operators $O_I$ with only first-class constraints.

## 3 Deformed HS symmetry superalgebra for YT with 2 columns

We apply an additive conversion procedure developed within BRST method, (see e.g. [17]), implying the enlarging of $o_I$ to $O_I=o_I+o'_I$, with additional parts $o'_I$ supercommuting with all $o_I$ and determined on a new Fock space $\mathcal{H}'$. Now, the elements $O_I$ are given on $\mathcal{H}^f\otimes\mathcal{H}'$ so that a condition for $O_I$, $[O_I,\,O_J\}\sim O_K$, leads to the same algebraic relations for $O_I$ and $o'_I$ as those for $o_I$.

Because of only the generators which do not contain space-timer derivatives, $\partial_\mu$, are the second-class constraints in $\mathcal{A}(Y[2],\mathbb{R}^{1,d-1})$, i.e. $\{o'_a,o'^+_a\}$. Therefore, one should to get new operator realization of this subalgebra. Note, this subalgebra is isomorphic to $sl(2)\oplus sl(2)$.

An auxiliary oscillator realization of $sl(2)\oplus sl(2)$ algebra can be found by using Verma module concept [26] and explicitly derived in the form

$$\begin{aligned} t^{+\prime}_{12} &= b^+_2, \quad l^{+\,\prime}_{12} = b^+_1,\\ g^{i\prime}_0 &= h_1+b^+_1b_1+(-1)^ib^+_2b_2,\\ l'_{12} &= -\tfrac{1}{4}(h_1+h_2+b^+_1b_1)b_1,\\ t'_{12} &= -(h_2-h_1+b^+_2b_2)b_2, \end{aligned} \tag{14}$$

with new 2 pairs of bosonic creation (annihilation) operators $b^+_i(b_i)$, with non-trivial commutation relations, $[b_i,b^+_j]=\delta_{ij}$. The operators $t^{+\prime}_{12}$ and $t'_{12}$; $l^{+\prime}_{12}$ and $l'_{12}$ are respectively Hermitian conjugated to each other, as well as the number particles operators $g^{i\prime}_0$ is Hermitian with help of the Grassmann-even operator $(K')^+=K'$ which should be found from the system of 4 equations,

$$\begin{aligned} \langle\Psi|K't(l)'_{12}|\Phi\rangle &= \langle\Phi|K't(l)^{+\prime}_{12}|\Psi\rangle^*,\\ \langle\Psi|K'g^{i\prime}_0|\Phi\rangle &= \langle\Phi|K'g^{i\prime}_0|\Psi\rangle^*, \end{aligned} \tag{15}$$

whose solution may be presented in the form,

$$K'=\sum_{n_i=0}^{\infty}\frac{(-1)^{n_1+n_2}C_{h_1+h_2}(n_1)C_{h_2-h_1}(n_2)}{4^{n_1}n_1!n_2!(h_1+h_2+n_1)(h_2-h_1+n_2)}$$

$$\times|n_1,n_2\rangle\langle n_1,n_2|,\ \texttt{for}\ C_h(n)=\prod_{i=0}^{n}(h+i), \tag{16}$$

and $|n_1,n_2\rangle=(b^+_1)^{n_1}(b^+_2)^{n_2}|0\rangle$.

## 4 BRST-BFV operator and Lagrangian formulations

Because of algebra of $O_I$ under consideration is a Lie superalgebra $\mathcal{A}(Y[2],\mathbb{R}^{1,d-1})$ the BRST-BFV operator $Q'$ is constructed in the standard way

$$Q'=O_I\mathcal{C}^I+\tfrac{1}{2}\mathcal{C}^I\mathcal{C}^Jf^K_{JI}\mathcal{P}_K(-1)^{\epsilon(O_K)+\epsilon(O_I)} \tag{17}$$

with the constants $f^K_{JI}$ from the Table 1, constraints $O_I=(l_0,l^+_i,\ l_i;\ L_{12},L^+_{12},T_{12},\ T^+_{12},G^i_0)$, fermionic [bosonic] ghost fields and conjugated to them momenta $(C^I,\mathcal{P}_I)=\big((\eta_0,\mathcal{P}_0);(\eta_{12},\mathcal{P}^+_{12});\ (\eta^+_{12},\mathcal{P}_{12j});(\vartheta_{12},\lambda^+_{12});$ $(\vartheta^+_{12},\lambda_{12});(\eta^i_G,\mathcal{P}^i_G)\big)$, $\quad[(q^+_i,p_i),(q_i,p^+_i)]$ with non-vanishing (anti)commutators

$$\{\vartheta_{12},\lambda^+_{12}\}=\{\eta_{12},\mathcal{P}^+_{12}\}=1,[q_i,p^+_j]=\delta_{ij} \tag{18}$$

and for zero-mode ghosts $\{\eta_0,\mathcal{P}_0\}=\imath$, $\{\eta^i_{\mathcal{G}},\mathcal{P}^j_{\mathcal{G}}\}=\imath\delta^{ij}$. The ghosts possess the standard ghost number distribution, $gh(\mathcal{C}^I)=-gh(\mathcal{P}_I)=1\Longrightarrow gh(Q')=1$. Therefore, BRST-BFV operator $Q'$ and $Q$ are determined as

$$Q' = Q + \eta^i_G(\sigma^i + h^i) + \mathcal{B}^i\mathcal{P}^i_G, \quad \texttt{with some } \mathcal{B}^i, \quad (19)$$

$$Q = \eta_0 L_0 + iq_iq_i^+P_0 + \Delta Q,\ \Delta Q = \Big(q_il_i^+ + \eta_{12}L_{12}^+ \quad (20)$$
$$+\vartheta_{12}T_{12}^+ + \tfrac{1}{2}\epsilon_{ij}\eta_{12}q_i^+p_j^+ + \vartheta_{12}(q_2^+p_1 + q_1p_2^+) + h.c.\Big),$$

$$\sigma_i + h_i = G_0^i - q_i^+p_i - q_ip_i^+ + \eta_{12}^+\mathcal{P}_{12} - \eta_{12}\mathcal{P}_{12}^+$$
$$+(-1)^i(\vartheta_{12}^+\lambda_{12} - \vartheta_{12}\lambda_{12}^+), \quad (21)$$

with real $\epsilon_{ij} = -\epsilon_{ji}, \epsilon_{12} = 1$. The property of $Q'$ to be Hermitian in $\mathcal{H}_{tot}$, $\mathcal{H}_{tot} = \mathcal{H}^f \otimes \mathcal{H}' \otimes \mathcal{H}_{gh}$ is determined by the rule

$$Q'^+K \ = \ KQ' \ \texttt{where} \ K \ = \ 1 \otimes K' \otimes 1_{gh}. \quad (22)$$

To construct Lagrangian formulation for bosonic HS fields subject to $Y[s_1, s_2]$ we choose a representation of $\mathcal{H}_{tot}$: $(q_i, p_i, \eta_{12},\ \vartheta_{12}, \mathcal{P}_0,\ \mathcal{P}_{12},\ \lambda_{12}, \mathcal{P}^i_G)|0\rangle = 0$, and suppose that the field vectors $|\chi\rangle$ as well as the gauge parameters $|\Lambda\rangle$ do not depend on ghosts $\eta^i_G$: extend our basic vector $|\Phi\rangle$ (6) given in $\mathcal{H}^f$ to

$$|\chi\rangle = \sum_{\{n\}_b=0}^{\infty}\sum_{\{n\}_f=0}^{1} \eta_0^{n_{\eta_0}}\eta_{12}^{+n_{\eta_{12}}}\vartheta_{12}^{+n_{\vartheta_{12}}}\mathcal{P}_{12}^{+n_{P_{12}}}\lambda_{12}^{+n_{\lambda_{12}}}$$
$$\times\prod_{i=1}^{2}(\eta_i^G)^{n_i}q_i^{+n_{q_i}}p_i^{+n_{p_i}}b_i^{+n_{b_i}}\left|\Phi(a_i{}^+)_{\{n\}_f\{n\}_b}\right\rangle, \quad (23)$$

where the integers $\{n\}_b = n_{q_i}, n_{p_i}, n_{b_i} \in \mathbb{N}$ and $\{n\}_f = n_{\eta_0},\ n_{\eta_{12}}, n_{P_{12}}, n_{\vartheta_{12}}, n_{\lambda_{12}} \in \mathbb{Z}_2$.

From the BRST-like equation, determining the physical vector (23) and from the set of reducible gauge transformations, homogeneous in ghost number $Q'|\chi^0\rangle = 0$ and the BRST complex of the reducible gauge transformations, $\delta|\chi\rangle = Q'|\Lambda^0\rangle$, $\delta|\Lambda^0\rangle = Q'|\Lambda^1\rangle$, $\ldots$, $\delta|\Lambda^{(r-1)}\rangle = Q'|\Lambda^{(r)}\rangle$, for $gh(|\chi\rangle) = gh(|\Lambda^{(k)}\rangle)+k+1 = 0$ the decomposition in $\eta^i_G$ leads to the relations:

$$\big(Q|\chi^0\rangle, \delta|\chi^0\rangle, ..., \delta|\Lambda^{(r-1)}\rangle\big) = \big(0, Q|\Lambda^0\rangle, ..., Q|\Lambda^{(r)}\rangle\big),$$
$$[\sigma^i + h^i]\big(|\chi^0\rangle, |\Lambda^0\rangle, \ldots, |\Lambda^{(r)}\rangle\big) = 0, \quad (24)$$

with $r = s_l + s_2$ being the stage of reducibility both for massless and for the massive bosonic HS field. Resolution the spectral problem from (24) yields the eigenvectors of the operators $\sigma^i$: $|\chi^0\rangle_{[n]_2}$, $|\Lambda^0\rangle_{[n]_2}, \ldots, |\Lambda^r\rangle_{[n]_2}$, for $[n]_2 = [n_1, n_2]$, $n_1 \geq n_2 \geq 0$ and corresponding eigenvalues of the parameters $h^i$ (for massless HS fields $i = 1, 2$),

$$-h^i = n_i - \frac{d}{2} - (-1)^i\ ,\ n_1, \in \mathbb{Z}, n_2 \in \mathbb{N}_0\,. \quad (25)$$

One can show, first, the operator $Q$ is nilpotent on the subspaces determined by the solution for (24), second, to construct Lagrangian for the field corresponding to a definite YT (1) we must put $n_i = s_i$, and, third, one should substitute $h^i$ corresponding to the chosen $n_i$ (25) into $Q$ (19) and relations (24). Thus, the equation of motion (24) corresponding to the field with a given $Y[s_1, s_2]$ has the form

$$Q_{[s]_2}|\chi^0\rangle_{[s]_2} = 0, \text{for}|\chi^0\rangle_{[s]_2(\{n\}_f=\{n\}_b=0)} = |\Phi\rangle_{[s]_2}. \quad (26)$$

Because of commutativity $[Q, \sigma_i\} = 0$ we have joint system of proper eigen-functions $|\chi^l\rangle_{[s_1,s_2]}$ for $l = 0, 1, ..., s_1+s_2+1$ and eigen-values $h^i(s_i)$ so that the sequence of reducible gauge transformations for the field with given $[s_1, s_2]$ are described (for $k = 1, ..., \sum_{i=1}^2 s_i$) by:

$$\delta|\chi^0\rangle_{[s]_2} = Q_{[s]_2}|\Lambda^{(0)}\rangle_{[s]_2},\ \delta|\Lambda^{(0)}\rangle_{[s]_2} = Q_{[s]_2}|\Lambda^{(1)}\rangle_{[s]_2}\,,$$
$$\delta|\Lambda^{(k-1)}\rangle_{[s]_2} = Q_{[s]_2}|\Lambda^{(k)}\rangle_{[s]_2},\ \delta|\Lambda^{(s_1+s_2)}\rangle_{[s]_2} = 0. \quad (27)$$

The equation of motion (26) are Lagrangian with appropriate numbers of auxiliary HS fields and derived from a gauge-invariant Lagrangian action (for $K_{[s]_2} = K|_{h^i=h^i(s)}$)

$$\mathcal{S}_{[s]_2} = \int d\eta_{0\,[s]_2}\langle\chi^0|K_{[s]_2}Q_{[s]_2}|\chi^0\rangle_{[s]_2}. \quad (28)$$

## 5 Constrained lagrangian formulations

Let us list the key points of the derivation of the constrained LF from unconstrained one for the same bosonic field subject to $Y[s_1, s_2]$

1. reduction of HS symmetry algebra $\mathcal{A}(Y[2], \mathbb{R}^{1,d-1}) \rightarrow \mathcal{A}_r(Y[2], \mathbb{R}^{1,d-1}) = \dfrac{\mathcal{A}(Y(k), \mathbb{R}^{1,d-1})}{sl(2) \oplus sl(2} = \{l_0, l_i, l_j^+\}$;
2. absence of the 2nd class constraints for $(m = 0)$ $\Longrightarrow$ absence of the conversion procedure;
3. reduction of $Q'$ (19) to $Q_r = \eta_0 l_0 + \sum_i(q_il_i^+ + q_i^+l_i + \imath q_i^+q^i\mathcal{P}_0)$;
4. presence 2 off-shell BRST extended by $q_i, q_i^+, p^i, p_i^+$ constraints $\mathcal{L}_{12}, \mathcal{T}_{12}$, and spin operator

$$\sigma^i_r = g^i_0 + q_ip_i^+ + q_i^+p_i:\ [\mathcal{A}, Q_r] = 0, \quad (29)$$

for $\mathcal{A} \in \{\mathcal{L}_{12}, \mathcal{T}_{12}, \sigma^i_r\}$ which look explicitly as

$$\mathcal{L}_{12} = l_{12} + \tfrac{1}{2}\epsilon_{ij}q_ip_j,\ \mathcal{T}_{12} = t_{12} + q_2p_1^+ + q_1^+p_2. \quad (30)$$

The proper constrained Lagrangian action is determined by the relations

$$\mathcal{S}_{r[s]_2} = \int d\eta_{0\ [s]_2}\langle\chi^0_r|Q|\chi^0_r\rangle_{[s]_2},\ (\mathcal{L}_{12}, \mathcal{T}_{12})|\chi^k_r\rangle = 0. \quad (31)$$

## 6 Conclusion

Thus, we have constructed gauge-invariant unconstrained and constrained Lagrangian descriptions of free integer HS fields belonging to an irreducible representation of the Poincare group $ISO(1, d-1)$ with the arbitrary Young tableaux having 2 columns in the "metric-like" formulation. The results of this study are the general and obtained on the base of universal method which is applied by the unique way to both massive and massless bosonic HS fields with a mixed antisymmetry in a Minkowski space of any dimension.


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# БРСТ-БФВ ЛАГРАНЖЕВЫ ФОРМУЛИРОВКИ ДЛЯ ПОЛЕЙ ВЫСШИХ СПИНОВ, ПОДЧИНЕННЫХ ДИАГРАММАМ ЮНГА С ДВУМЯ СТОЛБЦАМИ

Рассмотрены детали лагранжева описания неприводимых представлений высшего целого спина группы Пуанкаре с таблицей Юнга $Y[\hat{s}_1, \hat{s}_2]$, имеющих 2 столбца для Бозе-частиц, распространяющихся в пространстве-времени Минковского произвольной размерности. Процедура основана, во-первых, на использовании вспомогательного пространства Фока, порожденного фермионными осцилляторами (антисимметричный базис), во-вторых, на построении модуля Верма и нахождении вспомогательной осцилляторной реализации для алгебры $sl(2)\oplus sl(2)$, которая кодирует подсистему связей второго рода в супер態алгебру симметрии высших спинов. Применение универсального БРСТ-БФВ подхода позволяет воспроизвести калибровочно-инвариантные лагранжианы с приводимыми калибровочными симметриями, описывающие свободную динамику как безмассовых, так и массивных смешанно-антисимметричных бозонных полей любого спина с подходяшим набором калибровочных и Штюкельберговых полей. Общая прескрипция обладает возможностью воспроизвести лагранжианы с БРСТ-инвариантными расширенными алгебраическими связями, которые описывают неприводимые представления группы Пуанкаре в терминах смешанно-антисимметричных тензорных полей с 2 группами индексов.

**Ключевые слова:** *высшие спины, БРСТ оператор, Лагранжева формулировка, модуль Верма, калибровочная инвариантность.*

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