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One-loop divergences for the field of spin 3 on a de Sitter background in nonminimal gauge

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We consider the field of spin 3 on the de Sitter background in two dimensions and calculate the the divergent part of the one-loop effective action in a nonminimal gauge. For this purpose we construct a simple formula for the b_2 coefficient of an arbitrary second order differential operator with small nonminimal terms. Using this formula we find that the divergent part of one-loop effective action for the considered theory is gauge independent.

Keywords: *higher spins, Minakshisundaram-De Witt-Seeley coefficients, (anti)-de Sitter space.*

1 Introduction

Higher spins are described by the fields $\phi_{\mu_1\mu_2\ldots\mu_s}$ which are totally symmetric and satisfy the condition

$$\phi_\alpha{}^\alpha{}_\beta{}^\beta{}_{\mu_5\ldots\mu_s} = 0. \tag{1}$$

The action for these fields on the flat background was obtained in [1] for bosons and in [2] for fermions. It is also possible to describe the higher spin fields on the (anti)-de Sitter background:

$$R_{\mu\nu\alpha\beta} = \frac{1}{D(D-1)}(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})R, \tag{2}$$

where $R = \text{const}$. Then the action for bosons has the form:

$$S = \frac{(-1)^s}{2}\int d^Dx\sqrt{-g}\Big[(\nabla_\alpha\phi_{\mu_1\ldots\mu_s})^2 - \frac{1}{2}s(s-1) \times(\nabla_\alpha\phi_\beta{}^\beta{}_{\mu_3\ldots\mu_s})^2 - s(\nabla^\alpha\phi_{\alpha\mu_2\ldots\mu_s})^2 + s(s-1) \times\nabla_\alpha\phi^{\alpha\beta\mu_3\ldots\mu_s}\nabla_\beta\phi_\gamma{}^\gamma{}_{\mu_3\ldots\mu_s} - \frac{1}{4}s(s-1)(s-2) \times(\nabla_\alpha\phi^{\alpha\beta}{}_{\beta\mu_4\ldots\mu_s})^2 +c_1R(\phi_{\mu_1\ldots\mu_s})^2 + c_2R(\phi_{\gamma\gamma\mu_3\ldots\mu_s})^2\Big], \tag{3}$$

where the coefficients c_1 and c_2 are defined by the invariance of the action under the transformations

$$\delta\phi_{\mu_1\ldots\mu_s} = \frac{1}{s}(\nabla_{\mu_1}\alpha_{\mu_2\ldots\mu_s} + \nabla_{\mu_2}\alpha_{\mu_1\mu_3\ldots\mu_s} + \ldots). \tag{4}$$

The parameter $\alpha_{\mu_1\mu_2\ldots\mu_{s-1}}$ is totally symmetric and traceless $\alpha_{\beta\beta\mu_3\ldots\mu_{s-1}} = 0$.

In [3] it was shown that under certain assumptions the consistent Lagrangian formulation for free bosonic totally symmetric higher spin fields is possible only in constant curvature Riemann space.

In order to calculate quantum corrections for the considered theory (as for all gauge theories), it is necessary to fix a gauge. It is well-known that the effective action is gauge independent on shell. However, explicit calculations show that off shell the effective action depends on the gauge choice, see e.g. [4]. The choice of the minimal gauge considerably simplifies the calculations, especially on the curved background.

We will investigate the gauge dependence of the effective action for higher spin fields on the (A)dS background. For this purpose it is convenient to choose the λ, β gauge

$$S_{gf} = \frac{(-1)^s}{2}\int d^Dx\,\sqrt{-g}\,s(1+\lambda)[\nabla_\alpha\phi_{\alpha\mu_2\ldots\mu_s} -\frac{1}{2}(s-1)(1+\beta)\nabla_{(\mu_2}\phi_{\alpha\alpha\mu_3\ldots\mu_s)}]^2. \tag{5}$$

In this paper we calculate the divergent part of the one-loop effective action in this gauge and show that the result does not depend on the parameters λ and β.

2 Algorithm for calculating the one-loop divergences

In the one-loop approximation the effective action is given by

$$\Gamma[\phi] = S[\phi] + \frac{i}{2}\hbar\ \text{Tr}\ \ln\hat{D} + O(\hbar^2), \tag{6}$$

where Tr includes $\int d^Dx$ and the differential operator $\hat{D}$ is defined by

$$\hat{D} = \frac{\delta^2 S}{\delta\varphi_i\delta\varphi_j}. \tag{7}$$

Gauge (5) is called nonminimal, because the corresponding second variation of the classical action (with the gauge fixing terms) is given by a nonminimal operator

$$\hat{D} \sim \nabla_\mu^2 + K^{\mu\nu}\nabla_\mu\nabla_\nu + S^\mu\nabla_\mu + W \tag{8}$$

in which terms with the largest number of derivatives differ from the Laplace operator.

From the mathematical point of view, calculation of the divergent part of the one-loop effective action corresponds to obtaining the Minakshisundaram-De Witt-Seeley coefficients [5]- [10]. For the second order operator $\hat{D}$ these coefficients are defined by

$$\mathrm{tr}\langle x|\exp(is\hat{D})|x\rangle = \frac{\sqrt{-g}}{(4\pi is)^{D/2}}\Big(b_0 + b_2(is) + b_4(is)^2 + O(s^3)\Big). \qquad (9)$$

Using the dimensional regularization one-loop divergences can be related with certain Minakshisundaram-De Witt-Seeley coefficients. Really, taking into account that

$$\ln\hat{D} = -i\int\limits_0^\infty \frac{ds}{s}\exp(is(\hat{D}+i0)), \qquad (10)$$

it is easy to see, that, for example, for $D\approx 2$ the divergent part of the one-loop effective action can be written as

$$\Gamma^{(\infty)}_{1-loop} = \frac{1}{4\pi(D-2)}\int d^Dx\sqrt{-g}\,b_2, \qquad (11)$$

Similar formulas can be also written for other dimensions. For various differential operators Minakshisundaram-De Witt-Seeley coefficients can be found using the Schwinger—De Witt technique and its generalizations [11]- [14]. For a minimal operator heat kernel coefficients on the (A)dS background can be found using harmonic analysis on homogeneous spaces [15]. Using this method the one-loop effective potential for fields of arbitrary spin on the (A)dS background in four dimensions was calculated in [16] in the minimal gauge.

Here we use the generalization of the method proposed by G.t'Hooft and M.Veltman [17]. Using this technique b_4 coefficient (without terms which are integrals of total derivatives) has been calculated for an arbitrary differential operator [18]. However, on the (A)dS background this algorithm does not work, because terms containing total derivatives are essential. In order to take into account terms with total derivatives we consider the simplest case: b_2 coefficient, which gives the divergences in two dimensions. Moreover, the nonminimal terms are considered to be small. Namely, we consider the operator

$$\widehat{D} = \nabla_\mu^2 + \varepsilon K^{\mu\nu}\nabla_\mu\nabla_\nu + S^\mu\nabla_\mu + W, \qquad (12)$$

assuming $\varepsilon\to 0$.

The divergent diagrams are constructed using the expansion

$$\begin{aligned}&\ln(\widehat{D})\\ &= \ln(\partial_\mu^2+\varepsilon K_0^{\mu\nu}\partial_\mu\partial_\nu) + \ln\Big(1+\frac{1}{\partial_\mu^2+\varepsilon K_0^{\mu\nu}\partial_\mu\partial_\nu}\widehat{V}\Big)\\ &+\frac{1}{2}\Big[\ln(\partial_\mu^2+\varepsilon K_0^{\mu\nu}\partial_\mu\partial_\nu), \ln\Big(1+\frac{1}{\partial_\mu^2+\varepsilon K_0^{\mu\nu}\partial_\mu\partial_\nu}\widehat{V}\Big)\Big]\\ &+\frac{1}{12}\Big[\ln(\partial_\mu^2+\varepsilon K_0^{\mu\nu}\partial_\mu\partial_\nu), \Big[\ln(\partial_\mu^2+\varepsilon K_0^{\mu\nu}\partial_\mu\partial_\nu),\\ &\ln\Big(1+\frac{1}{\partial_\mu^2+\varepsilon K_0^{\mu\nu}\partial_\mu\partial_\nu}\widehat{V}\Big)\Big]\Big]+\ldots. \end{aligned} \qquad (13)$$

After constructing the divergent diagrams we extract the logarithmically divergent terms and replace them according to the prescription

$$\int\frac{d^Dk}{(2\pi)^2k^2}\to\frac{1}{2\pi(D-2)}. \qquad (14)$$

In the curved space we use the expansion with respect to

$$h_{\mu\nu}\equiv g_{\mu\nu}-\eta_{\mu\nu} \qquad (15)$$

and after the calculation of the divergent part restore the covariant result, using the equations:

$$\begin{aligned}&R_{\mu\nu} = \frac{1}{2}\Big(\partial_\mu\partial_\alpha h_{\nu\alpha}+\partial_\nu\partial_\alpha h_{\mu\alpha}-\partial_\mu\partial_\nu h-\partial^2 h_{\mu\nu}\Big)\\ &+O(h^2);\\ &R = \partial_\mu\partial_\nu h_{\mu\nu}-\partial^2 h+O(h^2).\end{aligned} \qquad (16)$$

The result is

$$\begin{aligned}&b_2 = \mathrm{tr}\Big(W-\frac{1}{2}\nabla_\mu S^\mu-\frac{1}{4}S_\mu S^\mu+\frac{1}{6}R\\ &-\frac{1}{2}\varepsilon K^{\alpha\alpha}W-\frac{1}{2}\varepsilon\nabla_\mu S^\mu+\frac{1}{4}\varepsilon(K^{\alpha\alpha}\nabla_\mu S^\mu\\ &+2K^{\mu\nu}\nabla_\mu S_\nu)+\frac{1}{8}\varepsilon(K^{\alpha\alpha}S_\mu^2+2K^{\mu\nu}S_\mu S_\nu)\\ &-\frac{1}{12}\varepsilon K^{\alpha\alpha}R+\frac{1}{6}\varepsilon K^{\mu\nu}R_{\mu\nu}\Big).\end{aligned} \qquad (17)$$

For $\varepsilon = 0$ this formula gives the result for the minimal operator.

3 One-loop divergences for the spin 3 field in nonminimal gauge

In order to understand if the one-loop divergences for higher spin fields are gauge dependent, we investigate the simplest case $D = 2$, $s = 3$, considering the parameters β and λ to be small. Then the classical action with the gauge fixing terms in the lowest order in β and λ is written as

$$S+S_{gf}=-\frac{1}{2}\int d^2x\sqrt{-g}\Big((\nabla_\alpha\phi_{\mu_1\mu_2\mu_3})^2$$
$$-3\Big[1-\frac{1}{2}(1+\lambda+2\beta)\Big](\nabla_\alpha\phi_{\beta\beta\mu_3})^2$$
$$+3\lambda(\nabla_\alpha\phi_{\alpha\mu_1\mu_2})^2-6(\lambda+\beta)\nabla_\alpha\phi_{\alpha\beta\mu_3}\nabla_\beta\phi_{\gamma\gamma\mu_3}$$
$$+\frac{3}{2}(\lambda+2\beta)(\nabla_\alpha\phi_{\alpha\beta\beta})^2+\frac{R}{2}(\phi_{\mu_1\mu_2\mu_3})^2$$
$$+\Big[3-\frac{3}{4}(1+\lambda+2\beta)\Big]R(\phi_{\alpha\alpha\mu_3})^2\Big). \quad (18)$$

The second variation of this expression with respect to the field $\phi_{\alpha_1\alpha_2\alpha_3}$ is the differential operator, corresponding to one-loop diagrams with a loop of the spin 3 field. From this operator we construct matrixes $\varepsilon K^{\mu\nu}$, S^μ, and W and substitute them into Eq. (17). Then we obtain the main part of the result

$$b_{2(main)}=\frac{1}{3}\Big(20+36\beta-o(\lambda,\beta)\Big)R. \quad (19)$$

However, it is also necessary to take into account diagrams with a ghost loop. For the considered gauge the ghost Lagrangian is

$$L_{gh}=\bar{c}^{\mu\nu}(\nabla^2_\alpha c_{\mu\nu}-\frac{\beta}{2}(\nabla_\mu\nabla_\nu+\nabla_\alpha\nabla_\mu)c_{\alpha\nu}$$
$$-\frac{\beta}{2}(\nabla_\nu\nabla_\alpha+\nabla_\alpha\nabla_\nu)c_{\alpha\mu}+(2+\beta)Rc_{\mu\nu}). \quad (20)$$

Substituting the corresponding matrixes in formula (17) for the b_2 coefficient we obtain the result for the ghost contribution:

$$b_{2(ghost)}=\frac{1}{3}\Big(13+18\beta+o(\beta)\Big)R. \quad (21)$$

Combining the results for $b_{2(ghost)}$ and $b_{2(main)}$ one obtain the one-loop divergences

$$\Gamma^{(\infty)}_{1-loop}$$
$$=\frac{1}{4\pi(D-2)}\int d^2x\sqrt{-g}\left(b_{2(main)}-2b_{2(ghost)}\right)$$
$$=\frac{1}{4\pi(D-2)}\int d^2x\sqrt{-g}\Big(-2R+o(\lambda,\beta)\Big). \quad (22)$$

This expression does not contain terms of the first order in λ and β. Therefore, in the considered approximation the result is gauge invariant.

4 Conclusion

We present a simple formula for the b_2 coefficient of an arbitrary second order differential operator with small nonminimal terms. In particular, this formula allows to calculate terms with total derivatives. By the same method it is possible to find a coefficient b_4. Now this work is in progress. Using the constructed formula in two dimensions we calculated a divergent part of the one-loop effective action for the field of spin 3 on the (anti)- de Sitter background in a nonminimal gauge. The result appeared to be gauge independent in the considered approximation (first order in the small parameters λ and β).

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ОДНОПЕТЛЕВЫЕ РАСХОДИМОСТИ ДЛЯ ПОЛЯ СО СПИНОМ 3 В ПРОСТРАНСТВЕ ДЕ-СИТТЕРА В НЕМИНИМАЛЬНОЙ КАЛИБРОВКЕ

Мы рассматриваем поле со спином 3 в 2-мерном пространстве де-Ситтера и вычисляем расходящуюся часть однопетлевого эффективного действия. С этой целью мы конструируем простую формулу для коэффициента b_2 при произвольном дифференциальном операторе второго порядка. Используя эту формулу, мы показали, что расходящаяся часть однопетлевого эффективного действия для рассматриваемой теории не зависит от калибровки.

Ключевые слова: *высшие спины, эффективное действие, пространство (анти)-де-Ситтера.*

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