НАУЧНЫЕ СТАТЬИ

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CONFORMAL ANOMALY OF DUAL QFT FROM HIGHER DIMENSIONAL DILATONIC GRAVITY

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Five-dimensional gauged supergravity (SG) plays an important role in AdS/GFT correspondence [1]. It is known that different versions of d5 gauged SG (for example, N=8 d5 gauged SG [2] with forty-two scalars and non-trivial scalar potential) may appear as a result of truncation of d10 IIB SG. In particular, $AdS_5 \times S_5$ deformed truncation of IIB SG (with non-trivial scalars) corresponds to some specific solution of d5 gauged SG. Hence, it is often enough to study 5d gauged SG classical solutions in AdS/CFT set-up instead of the investigation of non-linear IIB SG solutions. Such (deformed) solutions describe RG flows in dual boundary field theory (for a very recent discussion of such flows, see [16, 3, 4] and refs. therein). It is very interesting that even 4d curvature or non-zero temperature effects may be taken into account in bulk description of such RG flows [5]. In consideration of extended d5 gauged SG solutions there are often more symmetric (special) RG flows where scalars lie in one-dimensional submanifold of complete scalars space. (Then such theory corresponds to d5 dilatonic gravity with non-trivial dilaton potential.) Such flows may also correspond to certain (D3)-brane distributions [6]. However, note that it is extremely difficult to make the explicit identification of deformed gauged SG solution with the corresponding non-conformal dual gauge theory.

The important characteristic of boundary (gauged) theory in AdS/CFT correspondence is the conformal anomaly which may be found from the bulk side (see paper by Witten in ref. [1]). The calculation of conformal anomaly in d5 gauged SG with single scalar and constant scalar potential (dilatonic gravity) on dilaton-gravitational background via AdS/CFT correspondence has been initiated in ref. [12]. It was shown that N=4 super YM theory covariantly coupled with N=4 conformal SG [7] is actual dual of d5 dilatonic gravity (see also derivation of anomaly in gravity-complex scalar background in refs. [8, 13]). From holographic SG description (see refs. [9, 10] for introduction) it is known that dilaton (or in more general case, scalars) describe the coupling of dual (gauge) theory, say, masses, scalars or coupling constants. Hence, it is extremely interesting to get the conformal anomaly for gauged SG with non-trivial scalar potential. This may give much better understanding of RG flows in dual (non-conformal) boundary theory and also in the definition of analog of central charge (c-function) away of conformity. Even more, considering the conformal anomaly of dual general boundary theory with radiative corrections and comparing it with the one from bulk gauged SG may help in correct identification of dual boundary theory with correspondent bulk identification (which is currently noneasy task). Note also that conformal anomaly plays an important role in the construction of local surface counterterm for gauged SGs with non-constant scalar potential [14].

In the present letter we find the AdS/CFT conformal anomaly from d3 and d5 gauged SG with single scalar (dilaton) and arbitrary dilaton potential. This situation corresponds to special RG flow in dual description. The candidates for analogs of central charge (or more exactly, of c-function) away of conformity are presented. We believe that even for non-conformal theory this analogous c-function may be of importance as it measures the dilatonic deviation from conformity. The numerical study of c-function for two explicit choices of dilaton potential indicates to non-monotonic behaviour as expected.

We start with the bulk action of d+1-dimensional dilatonic gravity with potential Φ:

$$S = \frac{1}{16\pi G}\int_{M_{d+1}} d^{d+1}x\sqrt{-\hat{G}}\left\{\hat{R} + X(\phi)(\hat{\nabla}\phi)^2 + \right. \\ \left. + Y(\phi)\hat{\nabla}\phi + \Phi(\phi) + 4\lambda^2\right\}. \qquad (1)$$

Here M_{d+1} is d+1 dimensional manifold whose boundary is d dimensional manifold M_d and we choose $\phi(0) = 0$. Such action corresponds to (bosonic sector) of gauged SG with single scalar (special RG flow). Note also that classical vacuum stability restricts the form of dilaton potential [15]. As wellknown, we also need to add the surface terms [11] to the bulk action in order that the variational principle to be well-defined. We should only note here that the surface terms become irrelevant finally in the calculation for the Weyl anomaly given in this work. The equations of motion given by variation of (1) with respect to ϕ and $G^{\mu\nu}$ are

$$0=-\sqrt{-\hat{G}}\,\Phi'(\phi)-\sqrt{-\hat{G}}\,V'(\phi)\hat{G}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+$$
$$+2\partial_\mu\left(-\sqrt{-\hat{G}}\,\hat{G}^{\mu\nu}V(\phi)\partial_\nu\phi\right), \tag{2}$$

$$0=\frac{1}{3}\hat{G}^{\mu\nu}\left(\Phi(\phi)+\frac{12}{l^2}\right)+\hat{R}_{\mu\nu}+V(\phi)\partial_\mu\phi\partial_\nu\phi. \tag{3}$$

Here

$$V(\phi)\equiv X(\phi)-Y'(\phi). \tag{4}$$

We chose the metric $\hat{G}^{\mu\nu}$ on M_{d+1} and the metric $\hat{g}^{\mu\nu}$ on M_d in the following form

$$ds^2\equiv\hat{G}^{\mu\nu}dx^\mu dx^\nu=\frac{l^2}{4}\rho^{-2}d\rho\, d\rho+$$
$$+\sum_{i=1}^{d}\hat{g}_{ij}dx^i dx^j,\quad \hat{g}_{ij}=\rho^{-1}g_{ij}. \tag{5}$$

Here l is related with λ^2 by $4\lambda^2=d(d-1)/l^2$. If $g=\eta_{ij}$, the boundary of AdS lies at g_{ij}. Note that we follow the method of calculation in ref. [12, 13] where dilatonic gravity with constant dilaton potential has been considered.

The action (1) diverges in general since the action contains the infinite volume integration on M_{d+1}. The action is regularized by introducing the infrared cutoff e and replacing

$$\int d^{d+1}x\to\int d^dx\int_\varepsilon d\rho,\quad \int_{M_d}d^dx(\cdots)\to\int d^dx(\cdots)_{\rho=\varepsilon}. \tag{6}$$

We also expand g_{ij} and ϕ with respect to ρ:

$$g_{ij}=g(0)_{ij}+\rho\, g(1)_{ij}+\rho^2 g(2)_{ij}+\cdots,$$

$$\phi=\phi(0)+\rho\phi(1)+\rho^2\phi(2)+\cdots. \tag{7}$$

Then the action is also expanded as a power series on ρ. The subtraction of the terms proportional to the inverse power of ε does not break the invariance under the scale transformation $\delta g_{\mu\nu}=2\delta\sigma g_{\mu\nu}$ and $\delta\varepsilon=2\delta\sigma\varepsilon$. When d is even, however, the term proportional to ln ε appears. This term is not invariant under the scale transformation and the subtraction of the ln e term breaks the invariance. The variation of the ln e term under the scale transformation is finite when $\varepsilon\to 0$ and should be canceled by the variation of the finite term (which does not depend on ε) in the action since the original action (1) is invariant under the scale transformation. Therefore the ln e term Sln gives the Weyl anomaly T of the action renormalized by the subtraction of the terms which diverge when $\varepsilon\to 0$ (d = 4)

$$S_{\ln}=-\frac{1}{2}\int d^4x\sqrt{-g}T. \tag{8}$$

First we consider the case of d = 2, i.e. three-dimensional gauged SG. Then anomaly term is found as follows:

$$T=\frac{1}{8\pi G}\frac{l}{2}\left\{R_{(0)}+X(\phi_{(0)})(\nabla\phi_{(0)})^2+Y(\phi_{(0)})\Delta\phi_{(0)}+\right.$$
$$+\frac{1}{2}\left\{\frac{3\Phi'(\phi_{(0)})}{l^2}\left(\Phi''(\phi_{(0)})\left(\Phi(\phi_{(0)})+\frac{3}{l^2}\right)-\right.\right.$$
$$\left.-(\Phi'(\phi_{(0)})^2)^{-1}-\Phi(\phi_{(0)})\right)\times$$
$$\times(R_{(0)}+V(\phi_{(0)})g^{ij}_{(0)}\partial_i(\phi_{(0)})\partial_j(\phi_{(0)}))\times$$
$$\times\left(\Phi(\phi_{(0)})+\frac{3}{l^2}\right)^{-1}+\frac{3\Phi'(\phi_{(0)})}{l^2}\times$$
$$\times\left(\Phi''(\phi_{(0)})\left(\Phi(\phi_{(0)})+\frac{3}{l^2}\right)-\Phi'(\phi_{(0)})^2\right)^{-1}\times$$
$$\times\left(V'(\phi_{(0)})g^{ij}_{(0)}\partial_i(\phi_{(0)})\partial_j(\phi_{(0)})+\right.$$
$$\left.\left.\left.+2\frac{V(\phi_{(0)})}{\sqrt{-g_{(0)}}}\partial_i(\sqrt{-g_{(0)}}g^{ij}_{(0)}\partial_j\phi_{(0)})\right)\right\}\right\}. \tag{9}$$

For $F(\phi)=0$ case, the central charge of the conformal field theory is given by the coefficient of R. Then it might be natural to introduce the analog of central charge c, i.e. c-function for the case when the conformal symmetry is broken by the deformation as follows:

$$c=\frac{3}{2G}\left[l+\frac{1}{2}\left\{\frac{3\Phi'(\phi_{(0)})}{l^2}\times\left(\Phi''(\phi_{(0)})\left(\Phi(\phi_{(0)})+\frac{3}{l^2}\right)-\right.\right.\right.$$
$$\left.\left.\left.\Phi'(\phi_{(0)})^2\right)^{-1}-\Phi(\phi_{(0)})\right\}\left(\Phi(\phi_{(0)})+\frac{3}{l^2}\right)^{-1}\right]. \tag{10}$$

Comparing this with radiatively-corrected central charge of boundary QFT may help in correct bulk description of such theory.

For the case of $d=4$, we obtain the following expression for the anomaly:

$$T=-\frac{1}{8\pi G}[hR^2_{(0)}+kR_{(0)ij}R^{ij}_{(0)}+\cdots]. \tag{11}$$

$$h=[3\{(24-10\Phi)\Phi'^6+(62208+22464\Phi+$$
$$+2196\Phi^2+72\Phi^3+\Phi^4)\Phi''(\Phi''+8V)^2+\ ,$$

$$2\Phi'^4\{(108+162\Phi+7\Phi^2)\Phi''+72(-8+14\Phi+\Phi^2)V\}-2\Phi'^2\{(6912+2736\Phi+192\Phi^2+\Phi^3)\Phi''^2+4(11232+6156\Phi+552\Phi^2+13\Phi^3)\Phi''V+32(-2592+468\Phi+96\Phi^2+5\Phi^3)V^2\}-3(-24+\Phi)(6+\Phi)^2\times\Phi'^3(\Phi'''+8V')\}]/[16(6+\Phi)^2\{-2\Phi'^2+](24+\Phi)\Phi''\}\{-2\Phi'^2+(18+\Phi)(\Phi''+8V)\}^2$$

$$k=-3\frac{\{(12-5\Phi)\Phi'^2+(288+72\Phi+\Phi^2)\Phi''\}}{\{8(6+\Phi)^2\{-2\Phi'^2+(24+\Phi)\Phi''\}\}}. \tag{12}$$

Here ... expresses the terms containing the derivative with respect to x_i, whose explicit forms is given in [22]. In case of the dilaton gravity in [12] corresponding to Φ=0 (or more generally in case that the axion is included as in [13]), we have the following expression:

$$S_{\ln}=\frac{l^3}{16\pi G}\int d^4x\sqrt{-g(0)}\times\left[\frac{1}{8}R_{(0)ij}R^{ij}_{(0)}-\frac{1}{24}R^2_{(0)}+\frac{1}{2}R^{ij}_{(0)}\partial_i\varphi(0)\partial_j\varphi(0)-\frac{1}{6}R_{(0)}g^{ij}_{(0)}\partial_i\varphi(0)\partial_j\varphi(0)+\frac{1}{4}\left\{\frac{1}{\sqrt{-g(0)}}\times\partial_i\left(\sqrt{-g(0)}g^{ij}_{(0)}\partial_j\varphi(0)\right)\right\}^2+\frac{1}{3}\left(g^{ij}_{(0)}\partial_i\varphi(0)\partial_j\varphi(0)\right)^2\right]. \tag{13}$$

Here φ can be regarded as dilaton. When Φ is not trivial, of course, there appear extra terms which are denoted by ... in (11). When Φ is not trivial, for example, the coefficient of $(g^{ij}_{(0)}\partial_i\varphi(0)\partial_j\varphi(0))^2$ becomes dilaton dependent. And there would be appear the terms like $R_{(0)}g^{ij}_{(0)}\partial_i\varphi(0)\partial_j\varphi(0)$ and $R^{ij}_{(0)}\partial_i\varphi(0)\partial_j\varphi(0)$ and their dilaton dependent coefficients are quite complicated.

We should also note that the expression (11) cannot be rewritten as a sum of the Gauss-Bonnet invariant and the square of the Weyl tensor F, which are

$$\tilde{G}=R^2-4R_{ij}R^{ij}+R_{ijkl}R^{ijkl},$$
$$F=\frac{1}{3}R^2-2R_{ij}R^{ij}+R_{ijkl}R^{ijkl}. \tag{14}$$

This is the signal that the conformal symmetry is broken. In the limit of $\Phi\to 0$, we obtain

$$h\to\frac{3\times 62208\Phi''(8V)^2}{16\times 6^2\times 24\times 18\Phi''(8V)^2}=\frac{1}{24},$$
$$k\to-\frac{3\times 288\Phi''}{8\times 6^2\times 24\Phi''}=-\frac{1}{8}. \tag{15}$$

and we can find that the standard result (conformal anomaly of $N = 4$ super YM theory) is reproduced. In order that the region near the boundary at $\rho = 0$ is asymptotically AdS, we need to require $\Phi\to 0$ and $\Phi'\to 0$ when $\rho\to 0$. We can also confirm that $h\to 1/24$ and $k\to -1/8$ in the limit of $\Phi\to 0$ and $\Phi'\to 0$ even if $\Phi''\neq 0$ and $\Phi'''\neq 0$. In the AdS/CFT correspondence, k and h should be related with the central charge c of the conformal field theory (or its analog for nonconformal theory). Since we have two functions h and k, there are two natural ways to define the c-function when the conformal field theory is deformed:

$$c_1=\frac{24\pi h}{G},\quad c_2=-\frac{8\pi k}{G}. \tag{16}$$

If we put $V(\phi)=4\lambda^2+\Phi(\phi)$, we have $l=(12/V(0))^{\frac{1}{2}}$. We should note that we have chosen $l = 1$ in the expressions in (16). We can restore l-dependence by changing $h\to l^3h$ and $k\to l^3k$ and $\Phi'\to l\Phi'$, $\Phi''\to l^2\Phi''$ and $\Phi'''\to l^3\Phi'''$ in (11). Then in the limit of $\Phi\to 0$, we obtain

$$c_1,\ c_2\to\frac{\pi}{G}\left(\frac{12}{V(0)}\right)^{\frac{3}{2}}, \tag{17}$$

which agrees with the definition used in the works [17, 18] in above limit. The c_1- or c_2-functions give the new candidate for c-function away of conformity.

We now consider some examples. In [6] and [16], the following dilaton potentials appeared:

$$4\lambda^2+\Phi_{FGPW}(\phi)=4\left(\exp\left[\left(\frac{4\phi}{\sqrt{6}}\right)\right]+2\exp\left[-\left(\frac{2\phi}{\sqrt{6}}\right)\right]\right), \tag{18}$$

$$4\lambda^2+\Phi_{GPPZ}(\phi)=\frac{3}{2}\left(3+\left(\cosh\left[\left(\frac{2\phi}{\sqrt{3}}\right)\right]\right)^2+4\cosh\left[\left(\frac{2\phi}{\sqrt{3}}\right)\right]\right) \tag{19}$$

In both cases V is a constant and V=–2. In the classical solutions for the both cases, ϕ is the monotonically decreasing function of the energy scale ($\sim\rho^{-\frac{1}{2}}$) and ϕ =0 at the UV limit corresponding to the boundary and $\phi\to+\infty$ in the UV limit. Then in order to know the energy scale dependences of c_1 and c_2, we only need to investigate the f dependences of h and k in (11). By the numerical calculations, their behaviors are given in [21]. In any case, h and k are increasing functions when ϕ is small as expected but the monotonities are broken when ϕ is not small. That proves that such bulk regime corresponds to non-conformal boundary gauge theory. Furthermore there appear singularities coming from $0\sim -2\Phi'^2+(24+\Phi)\Phi''$ which are included in the denominators in h and k. In h for Φ_{FGPW}, there also appears a singularities coming from $0\sim -2\Phi'^2+(18+\Phi)(\Phi''+8V)$, which is also included in the denominator of h. Hence, our candidate c-functions may be seriously considered as realistic ones only in the region with small dilaton.

In summary, we found the conformal anomaly from d3 and d5 gauged supergravity with single scalar and arbitrary scalar potential on the scalar-gravitational background. It corresponds to the conformal anomaly of dual boundary theory. The attempt to define c-function away of conformity is also presented. Our work may be extended for d5 gauged SG with bigger number of scalars (say N=8 gauged SG) and arbitrary scalar potential. The final result appears in really complicated and lengthy form as it will be shown in another place. This opens the possibility of explicit check if the results on RG flows in dual gauge theory (deformed N=4 super Yang-Mills) presented in ref. [4, 16] from bulk side indeed describe 4d gauge Yang-Mills theory with lesser supersymmetry and the correspondent identification is correct. From another side, our conformal anomaly in the spirit of ref. [19] may be used to calculate the Casimir energy in dilatonic gravity. As the final remark let us note that dilaton-dependent conformal anomaly found in this work may be used for calculation of anomaly induced effective action of non-conformal boundary QFT in the presence of scalars (see ref. [20] for related example of dilaton dependent induced effective action in SUSY Yang-Mills theory).

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НЕКОТОРЫЕ АСПЕКТЫ СОВРЕМЕННОЙ КВАНТОВОЙ ХИМИИ

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Когда вы пьете кофе, вы глотаете молекулы. Когда вы сидите в комнате, вас непрерывно бомбардирует целый рой молекул. Вы одеты в молекулы, вы едите и выделяете молекулы. В конце концов, вы сами построены из молекул. И тем не менее многие люди даже не подозревают о существовании молекул. В сущности, до начала этого века считалось, что молекулы – это всего лишь символы, употребляемые химиками для описания превращения веществ, и до настоящего времени никому не удавалось увидеть молекулу.

На рубеже XIX и XX вв. в результате плодотворного сотрудничества физиков, химиков и биологов было показано, что молекулы действи-