

13. This is a natural ansatz given that the  $OZ$ -direction is the only distinguished direction in the  $S^1 \times R^3$  space under consideration, and that the  $A_\mu$  field obeys periodic boundary conditions along the  $OZ$ -direction.
14. In the considered low-frequency approximation the group and phase velocities coincide and can be found by the same proposed formula.
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## ON THE STABILITY OF RANDALL-SUNDRUM BRANEWORLDS WITH CONFORMAL BULK FIELDS

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### 1 Introduction

In the Randall-Sundrum (RS) scenario [1,2] the observable Universe is a 3-brane world of a  $Z_2$  symmetric 5-dimensional anti-de Sitter (AdS) space. In the RS1 model [1] the AdS orbifold has a compactified fifth dimension and two brane boundaries. The gravitational field is bound to the hidden positive tension brane and decays towards the visible negative tension brane. In this model the hierarchy problem is reformulated as an exponential hierarchy between the weak and Planck scales [1]. In the RS2 model [2] the AdS orbifold is non-compact with an infinite fifth dimension and a single positive tension brane. Gravity is localized on the positive tension brane now interpreted as the visible brane.

At low energies the theory of gravity on the observable brane is 4-dimensional general relativity and the cosmology may be Friedmann-Robertson-Walker [1]-[10]. In the RS1 model this is only possible if the radion mode is stabilized and this has been achieved using a scalar field in the bulk [3,6,9,10]. The

gravitational collapse of matter was also analyzed in the RS scenario [11]-[16]. Using an extended black string solution (first discussed in a different context by Myers and Perry [17]) Chamblin, Hawking and Reall showed that it was possible to induce on the brane the Schwarzschild black hole metric [11]. However, this solution is divergent at the AdS horizon and at the black string singularity. Consequently, it is expected to be unstable [11,18]. A black cylinder localized near the brane which is free from naked singularities was conjectured to be its stable decay product. This exact 5-dimensional solution has not yet been found. The only known static black holes localized on a brane remain to be those found for a 2-brane in a 4-dimensional AdS space [12]. The problem lies in the simultaneous non-singular localization of gravity and matter in the vicinity of the brane [11], [13]-[16]. This has led to another conjecture stating that  $D+1$ -dimensional black hole solutions localized on a  $D-1$ -brane should correspond to quantum corrected  $D$ -dimensional black holes on the brane [15]. This is an extra motivation to look for 5-dimensional collapse solutions localized on

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a brane. In addition, the effective covariant Gauss-Codazzi approach [19,20] has permitted the discovery of many braneworld solutions which have not yet been associated with exact 5-dimensional spacetimes [21]-[24].

In this paper we continue the research on the dynamics of a spherically symmetric RS 3-brane when the bulk is filled with conformal matter fields [16,25] (see also [26]). In our previous work [16,25] we have found a new class of exact 5-dimensional dynamical solutions for which gravity is localized near the brane by the exponential RS warp. These solutions were shown to be associated with conformal bulk fields characterized by a stress-energy tensor  $\tilde{T}_\mu^\nu$  of weight -4 and by the equation of state  $T_a^a = 2T_5^5$ . They were also shown to describe on the brane the dynamics of inhomogeneous dust, generalized dark radiation and homogeneous polytropic matter. However, the density and pressures of the conformal bulk fluid increase with the coordinate of the fifth dimension. In the RS2 model this generates a divergence at the AdS horizon as in the Schwarzschild black string solution. In the RS1 model this is not a difficulty because the space is cut before the AdS horizon is reached. However, the solutions must be stable under radion field perturbations. In this paper we analyze this problem using a saddle point expansion procedure based on the action [28,29]. In section 2 we present a brief review of the conformal analysis performed on the 5-dimensional Einstein equations which leads to the set of braneworld backgrounds supported by the class of conformal bulk matter fields with weight -4. In section 3 we consider a dimensional reduction on the RS action and determine the radion effective potential. In section 4 we identify our braneworld solutions as its extrema and show that the exponential RS warp and a radion independent equation of state for the conformal bulk fields lead to unstable solutions. In section 5 we present our conclusions and a new class of stable braneworld solutions which describe on the brane the dynamics of inhomogeneous dust, generalized dark radiation and homogeneous polytropic matter.

## 2 Einstein Equations and Conformal Bulk Fields

Let  $(t, r, \theta, \phi, z)$  be a set of comoving coordinates in the RS orbifold. The most general metric consistent with the  $Z_2$  symmetry in  $z$  and with 4-dimensional spherical symmetry on the brane may be written as

$$d\tilde{s}^2 = \Omega^2 (-e^{-2A} dt^2 + e^{2B} dr^2 + R^2 d\Omega_2^2 + dz^2), \quad (1)$$

where the metric functions  $\Omega = \Omega(t, r, z)$ ,  $A = A(t, r, z)$ ,  $B = B(t, r, z)$  and  $R = R(t, r, z)$  are  $Z_2$  symmetric.  $\Omega$  is the warp factor and  $R$  the physical radius of the 2-spheres.

When the 5-dimensional space is filled with bulk matter fields characterized by a lagrangian  $\tilde{L}_B$  the dynamical RS action is given by

$$\tilde{S} = \int d^4x \sqrt{-\tilde{g}} \left\{ \frac{\tilde{R}}{2\kappa_5^2} - \Lambda_B - \frac{1}{\sqrt{\tilde{g}_{55}}} [\lambda \delta(z - z_0) + \lambda' \delta(z - z'_0)] + \tilde{L}_B \right\}. \quad (2)$$

The Planck brane with tension  $\lambda$  is assumed to be located at  $z = z_0$  and the visible brane with tension  $\lambda'$  at  $z'_0$ .  $\Lambda_B$  is the negative bulk cosmological constant. A Noether variation on the action (2) gives the Einstein field equations

$$\tilde{G}_{\mu\nu} = -\kappa_5^2 \left\{ \Lambda_B \delta_\mu^\nu + \frac{1}{\sqrt{\tilde{g}_{55}}} [\lambda \delta(z - z_0) + \lambda' \delta(z - z'_0)] (\delta_\mu^\nu - \delta_5^\nu \delta_\mu^5) - \tilde{T}_\mu^\nu \right\}, \quad (3)$$

where the stress-energy tensor associated with the bulk fields is defined by

$$\tilde{T}_\mu^\nu = \tilde{L}_B \delta_\mu^\nu - 2 \frac{\delta \tilde{L}_B}{\delta \tilde{g}^{\mu\alpha}} \tilde{g}^{\alpha\nu} \quad (4)$$

and is conserved in the bulk,  $\tilde{\nabla}_\mu \tilde{T}_\nu^\mu = 0$ .

To find exact solutions of the 5-dimensional Einstein equations (3) we need simplifying assumptions. Let us first consider that under the conformal transformation  $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ , the bulk stress-energy tensor has conformal weight  $s$ ,  $\tilde{T}_\mu^\nu = \Omega^{s+2} T_\mu^\nu$ . Then Eq. (3) may be re-written as

$$G_\mu^\nu = -6\Omega^{-2} (\nabla_\mu \Omega) g^{\nu\rho} \nabla_\rho \Omega + 3\Omega^{-1} g^{\nu\rho} \nabla_\rho \nabla_\mu \Omega - 3\Omega^{-1} \delta_\mu^\nu g^{\rho\sigma} \nabla_\rho \nabla_\sigma \Omega - \kappa_5^2 \Omega^2 \{ \Lambda_B \delta_\mu^\nu + \Omega^{-1} [\lambda \delta(z - z_0) + \lambda' \delta(z - z'_0)] (\delta_\mu^\nu - \delta_5^\nu \delta_\mu^5) - \Omega^{s+2} T_\mu^\nu \}. \quad (5)$$

Similarly under the conformal transformation the conservation equation becomes

$$\nabla_\mu T_\nu^\mu + \Omega^{-1} [(s+7) T_\nu^\mu \partial_\mu \Omega - T_\mu^\mu \partial_\nu \Omega] = 0. \quad (6)$$

If in addition it is assumed that  $\tilde{T}_\mu^\nu = \Omega^{-2} T_\mu^\nu$  then equation (5) may be separated in the following way

$$G_\mu^\nu = \kappa_5^2 T_\mu^\nu, \quad (7)$$

$$6\Omega^{-2} \nabla_\mu \Omega \nabla_\rho \Omega g^{\rho\nu} - 3\Omega^{-1} \nabla_\mu \nabla_\rho \Omega g^{\rho\nu} + 3\Omega^{-1} \nabla_\rho \nabla_\sigma \Omega g^{\rho\sigma} \delta_\mu^\nu = -\kappa_5^2 \Omega^2 \{ \Lambda_B \delta_\mu^\nu + \Omega^{-1} [\lambda \delta(z - z_0) + \lambda' \delta(z - z'_0)] (\delta_\mu^\nu - \delta_5^\nu \delta_\mu^5) \}. \quad (8)$$

Because of the Bianchi identity we must also have

$$\nabla_\mu T_\nu^\mu = 0, \quad (9)$$

$$3T_\nu^\mu \partial_\mu \Omega - T \partial_\nu \Omega = 0. \quad (10)$$

Equations (7) and (9) are 5-dimensional Einstein equations with matter fields present in the bulk but without a brane or bulk cosmological constant. They do not depend on the conformal warp factor which is dynamically defined by equations (8) and (10). Consequently, only the warp reflects the existence of the brane or of the bulk cosmological constant. Note that this is only possible for the special class of conformal bulk fields which have a stress-energy tensor with weight -4.

Because the equations still depend non-linearly on the metric functions  $A$ ,  $B$  and  $R$  let us further assume that  $A = A(t, r)$ ,  $B = B(t, r)$ ,  $R = R(t, r)$  and  $\Omega = \Omega(z)$ . Then we obtain

$$G_a^b = \kappa_5^2 T_a^b, \quad \nabla_a T_a^b = 0, \tag{11}$$

$$G_5^5 = \kappa_5^2 T_5^5, \tag{12}$$

$$6\Omega^{-2}(\partial_z \Omega)^2 = -\kappa_5^2 \Omega^2 \Lambda_B, \tag{13}$$

$$3\Omega^{-1} \partial_z^2 \Omega = -\kappa_5^2 \Omega^2 \{\Lambda_B + \tag{14}$$

$$+\Omega^{-1}[\lambda \delta(z - z_0) + \lambda' \delta(z - z'_0)]\}$$

and (see also [5] and [27])

$$2T_5^5 = T_c^c, \tag{15}$$

where the latin indices represent the coordinates  $t, r, \theta$  and  $\phi$ . Our braneworld geometries [16,25] are solutions of equations (11)-(15) when the stress-tensor is diagonal,

$$T_\mu^\nu = \text{diag}(-\rho, p_r, p_T, p_T, p_5), \tag{16}$$

where  $\rho, p_r, p_T$  and  $p_5$  denote the bulk matter density and pressures, and  $\Omega$  is the exponential RS warp,

$$\Omega_{\text{RS}}(z) = \frac{l}{|z - z_0| + z_0}, \tag{17}$$

where  $z_0 = l$  and  $l$  is the AdS radius given by  $l = 1/\sqrt{-\Lambda_B \kappa_5^2 / 6}$  with  $\kappa_5^2 = 8\pi M_5^3$  defined by the fundamental 5-dimensional Planck mass  $M_5$ . Then the Planck brane is located at  $z_0 = l$  and the observable brane is located at  $z'_0 = le^{\pi r_c / l}$  where  $r_c$  is the RS compactification scale [1]. The former has a positive tension  $\lambda$  and the latter a negative tension  $\lambda' = -\lambda$  where  $\lambda = -\Lambda_B l$ . They are twin Universes with identical collapse or cosmological dynamics.

### 3 The Radion Potential

To analyze the behaviour of these solutions under radion field perturbations we apply a saddle point expansion procedure based on the action [28,29]. As a starting point this requires the determination of the

radion effective potential. For the calculation it is convenient to work with the coordinate  $y$  related to  $z$  by  $z = le^{y/l}$  for  $y > 0$ . Let us write the most general metric consistent with the  $Z_2$  symmetry in  $y$  and with 4-dimensional spherical symmetry on the brane in the form

$$d\tilde{s}^2 = a^2 ds_4^2 + b^2 dy^2, \tag{18}$$

$$ds_4^2 = -dt^2 + e^{2B} dr^2 + R^2 d\Omega_2^2,$$

where the metric functions  $a = a(t, r, y)$ ,  $B = B(t, r, y)$ ,  $R = R(t, r, y)$  and  $b = b(t, r, y)$  are  $Z_2$  symmetric. Now  $a$  is the warp factor,  $R$  is still the physical radius of the 2-spheres and  $b$  is related to the radion field.

The 5-dimensional dynamical RS action is now given by

$$\tilde{S} = \int d^4 x dy \sqrt{-\tilde{g}} \left\{ \frac{\tilde{R}}{2\kappa_5^2} - \Lambda_B - \frac{1}{\sqrt{\tilde{g}_{55}}} [\lambda \delta(y) + \lambda' \delta(y - \pi r_c)] + \tilde{L}_B \right\}. \tag{19}$$

In the new coordinates the Planck brane is located at  $y = 0$  and the visible brane at  $\pi r_c$ . Our braneworld backgrounds correspond to the metric functions  $b = 1$ ,  $B = B(t, r)$ ,  $R = R(t, r)$  and  $a = \Omega_{\text{RS}}(y)$  where

$$\Omega_{\text{RS}}(y) = e^{-y/l}. \tag{20}$$

To calculate the radion potential we consider the dimensional reduction of the action (19). Using the metric (18) we obtain  $\sqrt{-\tilde{g}} = a^4 b \sqrt{-g_4}$  and

$$\begin{aligned} \tilde{R} = & \frac{1}{a^2} \left( R_4 - \frac{6}{a} g_4^{cd} \nabla_c \nabla_d a - \right. \\ & \left. - \frac{2}{b} g_4^{cd} \nabla_c \nabla_d b - \frac{4}{ab} g_4^{cd} \nabla_c a \nabla_d b \right) - \\ & - \frac{4}{b^2} \left[ 3 \left( \frac{\partial_y a}{a} \right)^2 + 2 \frac{\partial_y^2 a}{a} \right], \end{aligned} \tag{21}$$

where  $g_4^{cd}$  is the inverse metric associated with the 4-dimensional line element  $ds_4^2$ ,  $R_4$  is the 4-dimensional Ricci scalar and the covariant derivatives are 4-dimensional. Consider the particular metric setting defined by  $a = \Omega e^{-\beta}$  and  $b = e^\beta$  where  $\Omega = \Omega(y)$  and  $\beta = \beta(t, r)$ . Then in the Einstein frame the dimensional reduction leads to

$$\tilde{S} = \int d^4 x \sqrt{-g_4} \left( \frac{R_4}{2\kappa_4^2} - \frac{1}{2} \nabla_c \gamma \nabla_d \gamma g_4^{cd} - \tilde{V} \right), \tag{22}$$

where  $\gamma = \beta / \kappa_4 \sqrt{2/3}$  is the canonically normalized radion field. The function  $\tilde{V} = \tilde{V}(\gamma)$  is the radion potential and it may be written in the form

$$\begin{aligned} \tilde{V} = & \frac{2}{\kappa_5^2} \chi^3 \left[ 3 \int dy \Omega^2 (\partial_y \Omega)^2 + 2 \int dy \Omega^3 \partial_y^2 \Omega \right] + \\ & + \chi \int dy \Omega^4 (\Lambda_B - \tilde{L}_B) + \\ & + \chi^2 \int dy \Omega^4 [\lambda \delta(y) + \lambda' \delta(y - \pi r_c)], \end{aligned} \quad (23)$$

where the field  $\chi$  is defined as  $\chi = e^{-\sqrt{(2/3)\kappa_4} \gamma}$ . Note that the integration in the fifth dimension is performed in the interval  $[-\pi r_c, \pi r_c]$  and that we have chosen

$$\int dy \Omega^2 = \frac{\kappa_5^2}{\kappa_4^2}. \quad (24)$$

#### 4 The Radion Field Instability

To analyze the stability of our braneworld solutions we consider a saddle point expansion of the radion field potential  $V$  [28,29]. This procedure requires the determination of its first and second variations. To do so we consider the integral of the radion potential written in the form

$$\begin{aligned} \tilde{V} = & \int d^4 x \sqrt{-g_4} \tilde{V}(\gamma) = \\ = & \int d^5 x \sqrt{-\tilde{g}} \left\{ \frac{2}{\kappa_5^2} \chi^2 \left[ 3 \left( \frac{\partial_y \Omega}{\Omega} \right)^2 + 2 \frac{\partial_y^2 \Omega}{\Omega} \right] \right\} + \\ & + \int d^5 x \sqrt{-\tilde{g}} \{ \Lambda_B - \tilde{L}_B + \chi [\lambda \delta(y) + \lambda' \delta(y - \pi r_c)] \}. \end{aligned} \quad (25)$$

Taking into account that an integration by parts and the  $Z_2$  symmetry leads to

$$\int dy \Omega^2 (\partial_y \Omega)^2 = -\frac{1}{3} \int dy \Omega^3 \partial_y^2 \Omega \quad (26)$$

we find that the first variation of the integral of the radion potential is given by

$$\begin{aligned} \frac{\delta \tilde{V}}{\delta \gamma} = & -\sqrt{\frac{8}{3}} \kappa_4 \int d^5 x \sqrt{-\tilde{g}} \chi [\lambda \delta(y) + \lambda' \delta(y - \pi r_c)] - \\ & - \sqrt{\frac{2}{3}} \kappa_4 \int d^5 x \sqrt{-\tilde{g}} \Lambda_B + \\ & + \frac{\kappa_4}{\sqrt{6}} \int d^5 x \sqrt{-\tilde{g}} \left( \tilde{T}_a^a - 2\tilde{T}_5^5 - \frac{12}{\kappa_5^2} \chi^2 \frac{\partial_y^2 \Omega}{\Omega} \right). \end{aligned} \quad (27)$$

For our solutions  $\Omega$  is taken to be the exponential RS warp factor  $\Omega_{RS}$  and satisfies the following warp equation in the  $y$  coordinate

$$-\frac{6}{\kappa_5^2} \frac{\partial_y^2 \Omega}{\Omega} = \Lambda_B + 2[\lambda \delta(y) + \lambda' \delta(y - \pi r_c)]. \quad (28)$$

On the other hand the bulk matter fields which must have a stress-energy tensor of conformal weight  $-4$  must also obey the equation of state (15). Integrating in the fifth dimension with  $\Omega = \Omega_{RS}$  we find

$$\frac{\delta \tilde{V}}{\delta \gamma} = \frac{\kappa_4}{\sqrt{6}} \Lambda_B l (1 - e^{-4\pi r_c l}) \chi (4\chi - 1 - 3\chi^2). \quad (29)$$

The critical extrema of the radion potential are the non-zero finite roots of the polynomial equation  $\chi(3\chi^2 - 4\chi + 1) = 0$ . They are  $\chi_1 = 1$  and  $\chi_2 = 1/3$ . Our braneworld solutions correspond to the first root  $\chi_1 = 1$ . The same happens if the bulk matter is absent as in the RS vacuum solutions. The other extremum is not a solution of the Einstein equations for the RS1 model. Indeed, a rescaling of the coordinates shows that the point  $\chi_2 = 1/3$  corresponds to an exponential warp  $\Omega = e^{-|y|/(3l)}$  which is different from  $\Omega_{RS}$  and does not satisfy the RS warp equations (28) and

$$6 \left( \frac{\partial_y \Omega}{\Omega} \right)^2 = -\kappa_5^2 \Lambda_B. \quad (30)$$

The stability of the extrema depends on the sign of the second variation of the radion potential. This variation defines the radion mass. Consequently, stable background solutions must be associated with a positive sign. Consider the first variation of the 4-dimensional integral of  $V$  given in Eq. (27). If the equation of state (15) of the conformal bulk fields is independent of the radion perturbation and  $\Omega = \Omega_{RS}$  we find

$$\frac{\delta^2 V}{\delta \gamma^2} = \frac{\kappa_4^2}{3} \Lambda_B l (1 - e^{-4\pi r_c l}) \chi (9\chi^2 - 8\chi + 1). \quad (31)$$

For  $\chi = \chi_1 = 1$  we conclude that the second variation of  $V$  is negative. This implies that the radion is a negative mass tachyon and that our braneworld solutions are unstable.

#### 5 Conclusions

In this paper we have considered the set of exact 5-dimensional dynamical solutions with gravity localized near the brane which are associated with conformal bulk fields of weight  $-4$  and which describe the dynamics of inhomogeneous dust, generalized dark radiation and homogeneous polytropic matter on the brane. We have studied their behaviour under radion field perturbations. We have shown that these solutions are extrema of the radion potential. We have also shown that if the metric function responsible for the localization of gravity is the exponential RS warp and the equation of state characterizing the conformal bulk fluid is independent of the radion field then the braneworld solutions are unstable. We have also found that the radion potential has another extremum which is not a solution of the complete set of Einstein equations. This point is connected with a different warp factor and its existence suggests that stable braneworlds associated with the same state of the conformal bulk matter should correspond to warp functions other than the standard exponential RS warp. This is indeed true. Stable solutions are defined by a new set of warp functions given by

$$\Omega(y) = e^{-y/l} \left( 1 + \frac{p_5^s}{4\Lambda_B} e^{2y/l} \right), \quad (32)$$

where  $p_5^s < 0$  is a negative constant fraction of the 5-dimensional pressure of the conformal bulk fields of weight-4. On the brane these solutions also describe the dynamics of inhomogeneous dust, generalized dark radiation and homogeneous polytropic matter. More details will be presented in a forthcoming publication [30].

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