

EXCEPTIONAL FIELD THEORY FOR  $E_{6(6)}$  SUPERGRAVITY**E. Musaev**National Research University Higher School of Economics, Faculty of Mathematics  
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A brief description of the supersymmetric and duality covariant approach to supergravity is presented. The formalism is based on exceptional geometric structures and turns the hidden U-duality group into a manifest gauge symmetry. Tensor hierarchy of gauged supergravity appears naturally here as a consequence of covariance of the construction. Finally, the full supersymmetric Lagrangian is explicitly constructed. This work was presented on the International Conference “Quantum Field Theory and Gravity (QFTG’14)” in Tomsk.

**Keywords:** supergravity, extended geometry, dualities, exceptional field theory.

**1 Introduction***1.1 Dualities in supergravity*

Since the seminal work of Cremmer and Julia [1] it is well known that 11-dimensional supergravity compactified on a torus  $\mathbb{T}^d$  enjoys a hidden symmetry  $E_{d(d)}$ . From the point of view of the underlying M-theory these are the so-called U-duality transformations that unify the perturbative T-duality, that relates Type IIA and Type IIB theories, and S-duality of Type IIB string theory.

To get the basic idea of the construction it is the most instructive to start with  $D = 11$ ,  $\mathcal{N} = 1$  supergravity, whose field content is very simple. This introduction mainly follows the paper [2] by Cremmer and Julia that contains very clear and detailed review of their results presented in the letter [1]. The field content of eleven-dimensional supergravity is very simple: graviton, 11-dimensional gravitino and the 3-form gauge field. Upon reduction on a  $d$ -dimensional torus  $\mathbb{T}^d$ , parametrised by the coordinates  $\{x^n\}$ , the theory fits into the maximal supergravity in  $D = 11 - d$  dimensions. Decomposing the 11-dimensional fields under the split  $11 = D + d$  one gets the following field content in  $d$  dimensions. From the vielbein we get one  $D$ -dimensional vielbein  $e_{\mu}^{\alpha}$ ,  $d$  vector fields  $A_{\mu}^m$  and  $d(d + 1)/2$  scalar fields  $g_{mn}$ . The 3-form field reduces into a 3-form  $C_{\mu\nu\rho}$ ,  $d$  number of 2-forms  $B_{\mu\nu m}$ ,  $d(d - 1)/2$  vectors  $A_{\mu mn}$  and  $q = d(d - 1)(d - 2)/6$  scalar fields  $C_{mnk}$ .

Such constructed effective theory has in general  $SL(d) \times \mathbb{R}^q$  global (rigid) symmetry group, where the  $SL(d)$  part comes from the diffeomorphisms of the internal space of the form  $\delta x^m = \Lambda_n^m x^n$ . The abelian group  $\mathbb{R}^q$ , that is the remnant of the gauge symmetry, acts on the axions  $C_{mnk}$  as constant shifts

$$\delta C_{mnk} = c_{mnk} (= \text{const}). \quad (1)$$

In addition, in dimensions  $D = 3, 4, 5$  one can dualise 1, 2 and 3-forms respectively to obtain additional scalars when the  $p$ -forms enter the Lagrangian only by their derivatives. There are certain subtleties when this procedure is applied to the 11-dimensional supergravity because of the Chern-Simons-like terms  $F[C] \wedge F[C] \wedge C$ , which will not be described here. Very detailed inspection of the global rigid symmetries that survive this construction is presented in [3]. To be mentioned is that such dualisations are necessary in  $D \leq 5$  to obtain the full U-duality group  $E_{d(d)}$  in the scalar sector.

Hence, the scalar fields can be nicely packaged into a matrix  $\mathcal{V}$  that is an element of the coset  $E_{d(d)}/K(E_{d(d)})$ . By choosing a correct parametrisation of the coset the scalar potential can be written in the following form that is globally invariant under  $E_{d(d)}$  [3]

$$\mathcal{L}_{\text{scalar}} = \frac{1}{4} e \text{Tr}[\partial \mathcal{M}^{-1} \partial \mathcal{M}], \quad (2)$$

where  $\mathcal{M} = \mathcal{V}^* \mathcal{V}$  is the metric on the coset space. The involution  $*$  here denotes the usual transposition for  $D \geq 6$  and is replaced by Hermitian conjugate and contraction with a certain symplectic matrix  $\Omega$  for  $D \leq 5$  (this is known as Cartan involution).

It is possible to repeat the same story for the  $p$ -forms sector, taking into account that to have the global symmetry on the level of Lagrangian (not the EOM), in even dimensions  $D = 2n$  one has to add extra “magnetic” duals to  $n$ -forms. This is necessary since on the level of equations of motion the symmetry is realised on the field strengths rather than the gauge potentials. Hence, an  $n$ -form field strength together with its Hodge dual forms a representation of the duality group.

It is important that the hidden symmetries in the described construction are global symmetries of a  $D$ -dimensional effective theory. Following analogy with

General Relativity one may ask what is the geometric origin of the duality symmetries and to what extent do they present in the initial 11-dimensional supergravity. The formalism of Exceptional Field Theory that is an attempt to make sense of these questions and to find a way to answer them is briefly described in this letter. For calculational details and more involved discussion the reader may refer to [4].

### 1.2 Basic conventions

In what follows we focus on the  $E_6$  exceptional field theory and hence it is useful to list few basic conventions and definitions that will be used [5]. A coset representative is denoted as usual by

$$\mathcal{V}_M^{ij} \in \frac{E_{6(6)}}{USp(8)}, \quad (3)$$

where the index convention is the following

$$\begin{aligned} M, N, O, P, \dots &= 1, \dots, 27, & E_{6(6)} \text{ indices} \\ A, B, C, D, \dots &= 1, \dots, 27, & \text{local } USp(8) \\ i, j, k, \dots &= 1, \dots, 8, & \text{local } USp(8) \\ \mu, \nu, \rho, \sigma, \dots &= 1, \dots, 5, & GL(5) \text{ indices} \\ a, b, c, d, \dots &= 1, \dots, 5, & \text{local } SO(1,4). \end{aligned} \quad (4)$$

The scalar matrix  $\mathcal{V}_M^{ij}$  and the symplectic  $USp(8)$  matrix  $\Omega_{ij}$  satisfy a set of constraint

$$\begin{aligned} \mathcal{V}_M^{ij} \mathcal{V}_{ij}^N &= \delta_M^N, & \mathcal{V}_M^{kl} \mathcal{V}_{ij}^m &= \delta_{ij}^{kl} - \frac{1}{8} \Omega_{ij} \Omega^{kl}, \\ \mathcal{V}_M^{kl} \Omega_{kl} &= 0, & \mathcal{V}_{Mij} &= (\mathcal{V}_M^{ij})^* = \mathcal{V}_M^{kl} \Omega_{ki} \Omega_{lj}, \\ \Omega_{kl} \Omega^{lm} &= -\delta_k^m, \end{aligned} \quad (5)$$

where the star denotes complex conjugation and the Kronecker symbol for pairs of antisymmetric indices is defined as  $\delta_{kl}^{ij} = 1/2(\delta_k^i \delta_l^j - \delta_k^j \delta_l^i)$ . In addition we use the convention that all (anti)symmetrisations of  $n$  indices are performed with a prefactor of  $1/n!$ , i.e.

$$A_{[i_1, \dots, i_n]} \equiv \frac{1}{n!} (A_{i_1 \dots i_n} + \text{permutations}). \quad (6)$$

For the spinor sector we use symplectic Majorana spinors  $\psi^i$  subject to the reality constraint

$$C^{-1} \bar{\psi}_i^T = \Omega_{ij} \psi^j, \quad \psi^{iT} C = \Omega^{ij} \bar{\psi}_j, \quad (7)$$

where the charge conjugation matrix  $C$  is defined by the following relations

$$\begin{aligned} C \gamma_a C^{-1} &= \gamma_a^T, & C^T &= -C, & C^\dagger &= C^{-1} \\ \gamma_{abcde} &= \mathbf{1} \varepsilon_{abcde}. \end{aligned} \quad (8)$$

This implies the following relation for fermionic bilinears with spinor fields  $\psi^i$  and  $\varphi^i$

$$\bar{\psi}_i \Gamma \varphi^j = -\Omega_{ik} \Omega^{jl} \bar{\psi}_l (C^{-1} \Gamma^T C) \psi^k \quad (9)$$

for any expression of gamma matrices  $\Gamma$ .

### 1.3 Extended geometry

Following the construction of Cremmer and Julia the hidden exceptional symmetries of lower dimensional maximal supergravities most straightforwardly can be reproduced in toroidal reductions of 11-dimensional supergravity. The formalism of extended geometry provides more geometric background to the exceptional groups in terms of extended geometric structures on an extended space (for review see [6–8]).

The extended space is constructed by adding extra directions to the would-be internal manifold that correspond to winding modes of M-branes [9, 10]

$$\mathbb{X}^M = \{x^m, y_{mn}, z_{mnklp}, \dots\}. \quad (10)$$

Infinitesimal coordinate transformations on this space consistent with the exceptional groups are defined as a generalisation of the well-known Hitchin's construction. Hence, one defines generalised tensors that live on the extended space as objects with the following transformation rule [11, 12]

$$\begin{aligned} (\mathcal{L}_\Lambda T)^M &= \Lambda^N \partial_N T^M - 6 \mathbb{P}^M{}_L{}^N{}_K \partial_N \Lambda^K T^L \\ &\quad + \lambda_T (\partial_K \Lambda^K) T^M \equiv [\Lambda, T]_D^M. \end{aligned} \quad (11)$$

The first and the last terms play the roles of translation and a weight term respectively. The second term reflects the exceptional group symmetry and involves the projection of the matrix  $\partial_N \Lambda^K$  on the U-duality algebra, since in general it does not belong to the structure group  $E_{6(6)}$  [13]. This is very similar to General Relativity where, however, the group is  $GL(n)$  and any non-degenerate matrix belongs to its algebra. Hence, in the case of the  $GL$  geometry the projector will be just trivial.

In addition one introduces a differential constraint on all fields in the theory that restricts dependence on the extended coordinates  $\mathbb{X}^M$

$$d^{PMN} \partial_M \otimes \partial_N = 0. \quad (12)$$

This extra condition in particular implies existence of a trivial transformation given by  $\Lambda_0^M = d^{MNK} \partial_N \Xi_K$  which itself transforms as a generalised vector. The Jacobi identity and closure of the algebra hold up to a trivial transformation as well. The latter leads to the notion of E-bracket that is an antisymmetrisation of the Dorfman bracket

$$\begin{aligned} [\mathcal{L}_{\Lambda_1}, \mathcal{L}_{\Lambda_2}] &= \mathcal{L}_{[\Lambda_1, \Lambda_2]_E}, \\ [\Lambda_1, \Lambda_2]_E &\equiv [\Lambda_{[1}, \Lambda_{2]}]_D. \end{aligned} \quad (13)$$

It is important to mention that in contrast to the E-bracket, the Dorfman bracket  $[\cdot]_D$  is not antisymmetric nor symmetric. This will play a crucial role in construction of tensor hierarchy starting from the covariant derivative to be defined in the next section.

## 2 $E_{6(6)}$ covariant exceptional field theory

### 2.1 Covariant derivative for D-bracket and tensor hierarchy

In the formalism of Extended Geometry generalised tensors and the corresponding transformations are considered to be independent of space-time coordinates  $x^\mu$ , that decouples the would be scalar sector.

In order to naturally incorporate the tensor and fermionic sector into the formalism the fields and all the gauge parameters are now allowed to depend on the external space-time coordinates. In the spirit of the ordinary Yang-Mills construction this implies that one has to introduce a long space-time derivative, that is covariant with respect to D-bracket [13]

$$\begin{aligned} \delta_\Lambda D_\mu T^M &= \mathcal{L}_\Lambda D_\mu T^M, \\ D_\mu &= \partial_\mu - \mathcal{L}_{A_\mu^M} = \partial_\mu - [A_\mu, ]_D, \\ \delta_\Lambda A_\mu^M &= \partial_\mu \Lambda^M - [A_\mu, \Lambda]_D^M = \mathcal{D}_\mu \Lambda^M, \end{aligned} \quad (14)$$

where the gauge field  $A_\mu^M$  is identified with the vector field of the corresponding maximal supergravity (with all necessary dualisations).

Since the E-bracket does not satisfy the Jacobi identity, one has to deform the usual field strength by a trivial transformation

$$\begin{aligned} [\mathcal{D}_\mu, \mathcal{D}_\nu] &= -\mathcal{L}_{\mathcal{F}_{\mu\nu}}, \\ \mathcal{F}_{\mu\nu}^M &= 2\partial_{[\mu} A_{\nu]}^M - [A_\mu, A_\nu]_E^M + 10d^{MNK} \partial_N B_{K\mu\nu}. \end{aligned} \quad (15)$$

In the spirit of tensor hierarchy, gauge transformations of the 1- and 2-forms naturally have the following form

$$\begin{aligned} \delta A_\mu^M &= \mathcal{D}_\mu \Lambda^M - 10d^{MNK} \partial_N \Xi_{K\mu}, \\ \Delta B_{M\mu\nu} &= 2\mathcal{D}_{[\mu} \Xi_{M\nu]} + d_{MNK} \Lambda^N \mathcal{F}_{\mu\nu}^K. \end{aligned} \quad (16)$$

This construction nicely utilises the  $p$ -forms of the  $D = 5$  maximal supergravity and naturally leads to tensor hierarchy as a consequence of generalised covariance.

### 2.2 Geometry and connections

The structure of EFT explicitly distinguishes between the two sets of coordinates: space-time  $\{x^\mu\}$  and the extended space  $\mathbb{X}^M$ . Respectively, one has two local groups  $SO(1,4)$  and  $USp(8)$ . Hence, there are four types of connections listed in the following table

	$\mathcal{D}_\mu$	$\nabla_M$
$SO(1,4)$	$\omega_\mu^{ab}$	$\omega_M^{ab}$
$USp(8)$	$Q_\mu^{ij}$	$Q_M^{ij}$

The  $SO(1,4)$  connection  $\omega_\mu^{ab}$  is defined by the usual vanishing torsion condition  $\mathcal{D}_{[\mu} e_{\nu]}^a = 0$ . The

$USp(8)$  connection  $Q_\mu$  is defined according to the group properties of the matrix  $\mathcal{V}_M^{ij}$  as usual (see [5])

$$\begin{aligned} \mathcal{V}_{kl}^M D_\mu \mathcal{V}_M^{ij} &= 2\delta_k^{[i} Q_{\mu l}^{j]} + P_\mu^{ijmn} \Omega_{mk} \Omega_{nl}, \\ Q_\mu &\in usp(8), \quad P_\mu \in \mathfrak{e}_6 \oplus usp(8). \end{aligned} \quad (17)$$

Explicit form of the  $SO(1,4)$  connection  $\omega_M^{ab}$  can be found following the same story but for the space-time vielbein  $e_\mu^a$ , i.e.

$$e^{a\mu} \partial_M e_\mu^b = \omega_M^{ab} + \pi_M^{ab}, \quad (18)$$

where  $\pi_M^{ab} = \pi_M^{ba}$ . Finally, the internal  $USp(8)$  connection  $Q_M^{ij}$  is derived from an analogue of the vanishing torsion condition for the extended space vielbein  $\mathcal{V}_M^{ij}$ . The generalised torsion is given by

$$\begin{aligned} \mathcal{T}_{NK}^M &= \Gamma_{NK}^M - 6\mathbb{P}^M_{K^P L} \Gamma_{PN}^L \\ &+ \frac{3}{2} \mathbb{P}^M_{K^Q N} \Gamma_{PQ}^P, \end{aligned} \quad (19)$$

that follows from the usual definition  $\mathcal{T}(V, W)^M = \mathcal{L}_V^\nabla W^M - \mathcal{L}_W V^M$ , where  $\mathcal{L}_V^\nabla$  is the covariant generalised Lie derivative. Hence one may write for the vanishing torsion condition

$$\begin{aligned} \mathcal{V}_{\bar{M}}^M \mathcal{D}_N \mathcal{V}_K^{\bar{M}} - 6\mathbb{P}^M_{K^P L} \mathcal{V}_{\bar{M}}^L \mathcal{D}_P \mathcal{V}_N^{\bar{M}} \\ + \frac{3}{2} \mathbb{P}^M_{K^Q N} \Gamma_{PQ}^P = 0. \end{aligned} \quad (20)$$

This equation has the form of the familiar expression  $\nabla_{[\mu} e_{\nu]}^a = 0$  however deformed in accordance to the algebraic structure of the duality group  $E_6$ .

## 3 Supersymmetry transformations

Supersymmetry transformations of the fields of  $E_{6(6)}$  covariant supergravity are taken to be of the following form

$$\begin{aligned} \delta_\epsilon e_\mu^a &= \frac{1}{2} \bar{\epsilon}_i \gamma^a \psi_\mu^i, \\ \delta_\epsilon \psi_\mu^i &= \mathcal{D}_\mu \epsilon^i - i\sqrt{2} \mathcal{V}^{Mij} \left( \tilde{\nabla}_M (\gamma_\mu \epsilon^k) \right. \\ &\quad \left. - \frac{1}{3} \gamma_\mu \tilde{\nabla}_M \epsilon^k \right) \Omega_{jk}, \\ \delta_\epsilon \chi^{ijk} &= \frac{i}{2} P_\mu^{ijkl} \gamma^\mu \Omega_{lm} \epsilon^m - \frac{3}{\sqrt{2}} \left( \mathcal{V}^{M[ij} \Omega^{k]m} \right. \\ &\quad \left. - \frac{1}{3} \mathcal{V}^{Mm[i} \Omega^{jk]} \right) \Omega_{mr} \tilde{\nabla}_M \epsilon^r, \\ \delta_\epsilon \mathcal{V}_M^{ij} &= i \mathcal{V}_M^{kl} \left[ 4\Omega_{p[k} \bar{\chi}_{l]mn} \epsilon^p + 3\Omega_{[kl} \bar{\chi}_{mn]p} \epsilon^p \right] \Omega^{mi} \Omega^{nj}, \\ \delta_\epsilon A_\mu^M &= \sqrt{2} \left[ i \Omega^{ik} \bar{\epsilon}_k \psi_\mu^j + \bar{\epsilon}_k \gamma_\mu \chi^{ijk} \right] \mathcal{V}_{ij}^M, \\ \delta_\epsilon B_{\mu\nu M} &= \frac{1}{\sqrt{5}} \mathcal{V}_M^{ij} \left[ 2\bar{\psi}_{i[\mu} \gamma_{\nu]} \epsilon^k \Omega_{jk} - i \bar{\chi}_{ijk} \gamma_{\mu\nu} \epsilon^k \right] \\ &\quad + d_{MNP} A_{[\mu}^N \delta_\epsilon A_{\nu]}^P, \end{aligned} \quad (21)$$

where we define the full covariant derivative as

$$\tilde{\nabla}_M \epsilon^i = \nabla_M \epsilon^i - \frac{1}{8} \mathcal{F}_M^{\rho\sigma} \gamma_{\rho\sigma} \epsilon^i. \quad (22)$$

Closure of supersymmetry transformations on the fields of EFT has the following structure

$$\begin{aligned} &= \xi^\mu \mathcal{D}_\mu + \delta_{so(1,4)}(\Omega^{ab}) + \delta_{usp(8)}(\Lambda^{ij}) \\ &+ \delta_{gauge}(\Lambda^M) + \delta_{gauge}(\Xi_{M\mu}) \\ &+ \delta_{gauge}(\Xi^\alpha{}_{\mu\nu}) + \delta_{susy}(\epsilon_3) + \delta(\mathcal{O}_{M\mu\nu}), \end{aligned} \quad (23)$$

that is the same as for the five-dimensional theory. Parameters of the transformation on the RHS are given by the following expressions made of the spinors  $\epsilon_{1,2}$  and the scalar matrix  $\mathcal{V}_M^{ij}$

$$\begin{aligned} \xi^\mu &= \frac{1}{2} \bar{\epsilon}_{2i} \gamma^\mu \epsilon_1^i, \quad \Lambda^M = -\frac{i}{\sqrt{2}} (\bar{\epsilon}_{2i} \epsilon_1^k \mathcal{V}^{Mij} \Omega_{jk}), \\ \Omega^{ab} &= -\frac{i\sqrt{2}}{3} (\bar{\epsilon}_{1i} \gamma^{ab} \tilde{\nabla}_M \epsilon_2^k - \tilde{\nabla}_M \bar{\epsilon}_{1i} \gamma^{ab} \epsilon_2^k) \mathcal{V}^{Mij} \Omega_{jk} \\ &- \Lambda^M \omega_M^{ab} + \frac{1}{2} \Lambda^M \mathcal{F}_M^{ab}, \\ \Xi_{M\mu} &= -\frac{1}{\sqrt{5}} \mathcal{V}_M^{kl} \Omega_{lm} (\bar{\epsilon}_{2k} \gamma_\mu \epsilon_1^m) \\ \Xi^\alpha{}_{\mu\nu} &= -\frac{3i}{\sqrt{10}} (t^\alpha)^M{}_N \mathcal{V}_{Msi} \mathcal{V}^{Nki} (\bar{\epsilon}_{2k} \gamma_{\mu\nu} \epsilon_1^s), \\ \mathcal{O}_{M\mu\nu} &= \frac{i}{\sqrt{10}} (\bar{\epsilon}_{2k} \gamma_{\mu\nu} \partial_M \epsilon_1^k - \partial_M \bar{\epsilon}_{2k} \gamma_{\mu\nu} \epsilon_1^k \\ &- (\bar{\epsilon}_{2k} \epsilon_1^s) e_{[\mu}^a \partial_M e_{\nu]a} + \frac{2}{3} \mathcal{V}_{Nsi} \partial_M \mathcal{V}^{Nki} (\bar{\epsilon}_{2k} \gamma_{\mu\nu} \epsilon_1^s)). \end{aligned}$$

Here  $\xi^\mu$  and  $\Lambda^M$  are the diffeomorphism parameters,  $\Omega^{ab}$  parametrizes the Lorentz rotations,  $\Xi_{M\mu}$  and  $\Xi_M^\alpha$  are the gauge transformation parameters of the 2-form  $B_{\mu\nu K}$  and the extra 3-form  $C^\alpha{}_{\mu\nu\rho}$ . Finally, as a consequence of the section condition the tensor  $\mathcal{O}_{M\mu\nu}$  is constrained by

$$d^{MNK} \partial_N \mathcal{O}_{K\mu\nu} = 0. \quad (24)$$

The operator  $\delta(\mathcal{O}_{M\mu\nu})$  leaves invariant the field  $\mathcal{F}_{\mu\nu}^M$  that is the only way of how the 2-form field enters the Lagrangian. The same is true for the gauge transformation generated by  $\Xi^\alpha{}_{\mu\nu}$ . Hence, the superalgebra is closed up to the section condition.

#### 4 Invariant Lagrangian

Given the definitions of the covariant derivatives that respect the  $E_{6(6)}$  structure of the extended space the full supersymmetric Lagrangian for the covariant

Exceptional Field Theory takes the following form

$$\begin{aligned} e^{-1} \mathcal{L} &= R - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}_{\mu\nu}^M \mathcal{F}^{\mu\nu N} - \frac{1}{6} |\mathcal{P}_\mu^{ijkl}|^2 \\ &- \bar{\psi}_{\mu i} \gamma^{\mu\nu\rho} \mathcal{D}_\nu \psi_\rho^i - 2\sqrt{2} i \mathcal{V}^M{}_{ij} \Omega^{ik} \bar{\psi}_{\mu k} \gamma^{[\mu} \tilde{\nabla}'_M (\gamma^{\nu]} \psi_{\nu}^j) \\ &- \frac{4}{3} \bar{\chi}_{ijk} \gamma^\mu \mathcal{D}_\mu \chi^{ijk} - 4\sqrt{2} i \mathcal{V}^M{}_{mn} \Omega^{np} \bar{\chi}_{pkl} \tilde{\nabla}'_M \chi^{mkl} \\ &+ \frac{4i}{3} \mathcal{P}_\mu^{ijkl} \bar{\chi}_{ijk} \gamma^\nu \gamma^\mu \psi_\nu^m \Omega_{lm} \\ &+ 4\sqrt{2} \mathcal{V}^{Mij} \bar{\chi}_{ijk} \gamma^\mu \tilde{\nabla}_M \psi_\mu^k + \mathcal{L}_{\text{top}} - V(\mathcal{M}, g). \end{aligned}$$

Here  $\tilde{\nabla}'_M[\mathcal{F}] = \tilde{\nabla}_M[-\mathcal{F}]$  encodes switch of the sign of the gauge field flux,  $\mathcal{L}_{\text{top}}$  is the topological term that includes the covariant version of the Chern-Simons Lagrangian and  $V$  is the scalar potential of [13, 14]. Due to the lack of space we do not check explicitly supersymmetry invariance of the above Lagrangian. For more detailed consideration the reader is referred to the paper [4]. However, the  $E_{6(6)}$  invariance is manifest since all the objects in the Lagrangian are covariant.

#### 5 Outlook

In this note the U-duality covariant approach to supergravity is briefly described. The essential feature of the Exceptional Field Theory approach is the notion of extended space and the structure of extended geometry defined on it. We describe the construction of  $E_{6(6)}$  covariant derivatives in both the space-time and the extended space directions. The corresponding vanishing torsion and algebraic conditions give necessary expressions for the  $SO(1,4)$  and  $USp(8)$  connections.

The final result is the supersymmetric manifestly  $E_{6(6)}$ -covariant Lagrangian that includes all the fields of the maximal  $D = 5$  supergravity. The 11-dimensional diffeomorphism symmetry is not manifest in this construction, however upon solution of the section constraint one is able to restore the full 11- or 10-dimensional Lagrangian.

As it was shown in [13], decomposition of the 27 extended space coordinates under the  $GL(6)$  subgroup of  $E_{6(6)}$  and leaving only the coordinates in the **6**, provides a consistent solution of the section constraint. This corresponds to the Kaluza-Klein decomposition of the full 11-dimensional supergravity.

An alternative solution is given by decomposition of the **27** under the  $GL(5) \times SL(2)$  subgroup. This leads to Type IIB supergravity with manifest  $SL(2)$  duality symmetry.

Relation between the described formalism and the embedding tensor approach to gauged supergravities is given by generalised Scherk-Schwarz reductions [15]. As it was shown in [14, 16] the reduction naturally provides all the gaugings in terms of generalised twist

matrices and their derivatives with respect to the full set of extended coordinates.

### Acknowledgement

The author expresses his gratitude to theoretical dpt of CERN for warm hospitality during complection

of this letter. In addition I would like to thank ENS de Lyon and personally Henning Samtleben for generous financial support and productive collaboration. Finally, I thank Tomsk State Pedagogical University and personally Vladimir Epp and Joseph Buchbinder for creating a wonderful atmosphere during the QFTG'2014 conference.

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Received 12.11.2014

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## ИСКЛЮЧИТЕЛЬНАЯ ТЕОРИЯ ПОЛЯ ДЛЯ $E_{6(6)}$ СУПЕРГРАВИТАЦИИ

Представлено краткое описание суперсимметричного дуальность-ковариантного подхода к супергравитации. В основе описываемого формализма лежит структура обобщенной геометрии, при этом скрытая симметрия U-дуальности рассматривается в качестве локальной калибровочной симметрии. Тензорная иерархия полей деформированной супергравитации появляется естественным образом как следствие ковариантности подхода. Полностью суперсимметричный дуальность-ковариантный Лагранжиан выписан в явном виде. Эта работа была доложена на международной конференции «Quantum Field Theory and Gravity (QFTG'14)» в Томске.

**Ключевые слова:** супергравитаци, расширенная геометрия, дуальности, исключительная теория поля.

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