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Remark on N=2 supersymmetric extension of l-conformal Galilei algebra

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l-conformal Galilei algebra is considered. $\mathcal{N} = 2$ supersymmetric extension of this algebra is constructed. A relation between its representations in flat spacetime and in Newton-Hooke spacetime is discussed.

Keywords: *conformal Galilei algebra, supersymmetry.*

1 Introduction

The non-relativistic version of the Anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence [1] stimulates extensive investigation of non-relativistic conformal algebras [2]- [4]. Algebras relevant for physical applications in flat non-relativistic spacetime as well as in Newton-Hooke spacetime (i.e. spacetime with cosmological repulsion/attraction [5]) belong to the family of the l-conformal Galilei algebras [7, 16] l being a positive integer or half-integer.

Supersymmetric extension of the l-conformal Galilei algebras and their dynamical realizations have been studied in detail for $l = \frac{1}{2}$ (the Schródinger algebra) and $l = 1$ (the conformal Galilei algebra). In [8] an $\mathcal{N} = 1$ supersymmetric extension of the Schrödinger algebra was identified with the symmetry algebra of the non–relativistic spin-$\frac{1}{2}$ particle. In [9] it was shown that the non–relativistic limit of the Chern-Simons matter system in $(2+1)$ dimensions is invariant under $\mathcal{N} = 2$ Schrödinger supersymmetry. Many-body quantum mechanics invariant under $\mathcal{N} = 2$ Schrödinger supersymmetry was studied in [10]. Relations between the Schrödinger superalgebra and relativistic superconformal algebras were discussed in [11]– [12]. More recently, supersymmetric extensions of the conformal Galilei algebra were extensively investigated by applying various non-relativistic contractions [13]– [14].

The purpose of this work is to construct an $\mathcal{N} = 2$ supersymmetric extension of the l-conformal Galilei algebra for the case of arbitrary l [15]. We do this in section 2. Representations of this superalgebra in flat and Newton-Hooke spacetimes are considered in section 3. We also give a coordinate transformation, which relates the representations.

2 N=2 supersymmetric extension of the l-conformal Galilei algebra

First let us recall the structure of the l-conformal Galilei algebra. It involves the generator of time translations H, the generator of dilatations D, the generator of special conformal transformations K, the generators of space rotations M_{ij}, and a chain of vector generators $C_i^{(n)}$, $n = 0, 1, .., 2l$. In particular, for $n = 0$ one obtains the generator of space translations, $n = 1$ gives the generator of Galilei boosts, while higher n describe accelerations. The non-vanishing structure relations read [7]

$$\begin{aligned}
&[H, D] = H, \quad [H, C_i^{(n)}] = nC_i^{(n-1)},\\
&[H, K] = 2D, \ [D, C_i^{(n)}] = (n-l)C_i^{(n)},\\
&[D, K] = K, \quad [K, C_i^{(n)}] = (n-2l)C_i^{(n+1)},\\
&[M_{ij}, C_k^{(n)}] = -\delta_{ik}C_j^{(n)} + \delta_{jk}C_i^{(n)},\\
&[M_{ij}, M_{kl}] = -\delta_{ik}M_{jl} - \delta_{jl}M_{ik} + \delta_{il}M_{jk} + \delta_{jk}M_{il}.
\end{aligned} \tag{1}$$

Note that H, D and K form the conformal algebra in one dimension $so(2,1)$.

In order to construct an $\mathcal{N} = 2$ supersymmetric extension of this algebra, we introduce a pair of supersymmetry generators Q^+ and Q^-, the superconformal generators S^+ and S^-, fermionic partners of the vector generators $L_i^{(n)+}$ and $L_i^{(n)-}$ with $n = 0, 1, .., 2l-1$, extra bosonic vector generators $P_i^{(n)}$ with $n = 0, 1, .., 2l-2$, and the bosonic generator J which corresponds to $u(1)$–R–symmetry. It is assumed that the odd generators are antihermitian conjugates of each other

$$\begin{aligned}
&\left(Q^+\right)^\dagger = -Q^-, \quad \left(S^+\right)^\dagger = -S^-,\\
&\left(L_i^{(n)+}\right)^\dagger = -L_i^{(n)-}.
\end{aligned} \tag{2}$$

The bosonic operators J and $P_i^{(n)}$ are taken to be antihermitian as well.

In addition to (5) we impose the following structure relations [15]

$$\{Q^+, Q^-\} = 2iH,\ \{Q^\pm, S^\mp\} = 2iD \pm J,$$
$$\{S^+, S^-\} = 2iK,\ [Q^\pm, C_i^{(n)}] = nL_i^{(n-1)\pm},$$
$$[H, S^\pm] = Q^\pm, \quad [Q^\pm, P_i^{(n)}] = iL_i^{(n)\pm},$$
$$[K, Q^\pm] = -S^\pm, \quad [S^\pm, C_i^{(n)}] = (n-2l)L_i^{(n)\pm},$$
$$[D, Q^\pm] = -\frac{1}{2}Q^\pm,\ [S^\pm, P_i^{(n)}] = iL_i^{(n+1)\pm},$$
$$[D, S^\pm] = \frac{1}{2}S^\pm, \quad [H, L_i^{(n)\pm}] = nL_i^{(n-1)\pm},$$
$$[J, Q^\pm] = \pm iQ^\pm, \quad [H, P_i^{(n)}] = nP_i^{(n-1)},$$
$$[J, S^\pm] = \pm iS^\pm, \quad [J, L_i^{(n)\pm}] = \pm iL_i^{(n)\pm},$$
$$\{Q^\pm, L_i^{(n)\mp}\} = iC_i^{(n)} \mp nP_i^{(n-1)},$$
$$\{S^\pm, L_i^{(n)\mp}\} = iC_i^{(n+1)} \mp (n-2l+1)P_i^{(n)},$$
$$[D, L_i^{(n)\pm}] = (n-l+1/2)L_i^{(n)\pm},$$
$$[D, P_i^{(n)}] = (n-l+1)P_i^{(n)},$$
$$[K, L_i^{(n)\pm}] = (n-2l+1)L_i^{(n+1)\pm},$$
$$[K, P_i^{(n)}] = (n-2l+2)P_i^{(n+1)},$$
$$[M_{ij}, L_k^{(n)\pm}] = -\delta_{ik}L_j^{(n)\pm} + \delta_{jk}L_i^{(n)\pm},$$
$$[M_{ij}, P_k^{(n)}] = -\delta_{ik}P_j^{(n)} + \delta_{jk}P_i^{(n)}. \qquad (3)$$

Note that $l = 1/2$ reproduces the well known $\mathcal{N} = 2$ Schrödinger superalgebra [10].

3 Realizations in superspace

First let us construct a realization of the superalgebra (3) in flat superspace. Introducing two Grassmann variables θ^+ and θ^-, which are complex conjugates of each other $(\theta^+)^\dagger = \theta^-$, one finds [15] (see also a related work [16])

$$D = t\partial_t + lx_i\partial_i + \frac{1}{2}\theta^-\partial_{\theta^-} + \frac{1}{2}\theta^+\partial_{\theta^+},$$
$$K = t^2\partial_t + 2ltx_i\partial_i + t\theta^-\partial_{\theta^-} + t\theta^+\partial_{\theta^+},$$
$$S^\pm = t\theta^\pm\partial_t + it\partial_{\theta^\mp} + 2l\theta^\pm x_i\partial_i + \theta^\pm\theta^\mp\partial_{\theta^\mp},$$
$$Q^\pm = i\partial_{\theta^\mp} + \theta^\pm\partial_t,$$
$$H = \partial_t, \quad J = i\theta^+\partial_{\theta^+} - i\theta^-\partial_{\theta^-},$$
$$C_i^{(n)} = t^n\partial_i \quad n = 0,..,2l,$$
$$P_i^{(n)} = \theta^-\theta^+t^n\partial_i \quad n = 0,..,2l-2,$$
$$L_i^{(n)\pm} = \theta^\pm t^n\partial_i, \quad n = 0,..,2l-1,$$
$$M_{ij} = x_i\partial_j - x_j\partial_i. \qquad (4)$$

Discarding the fermions one reproduces a realization of the l-conformal Galilei algebra in [7].

In order to construct a realization of the superalgebra (3) in Newton-Hooke spacetime extended by fermionic variables, we introduce an analogue of Niederer's transformation. Guided by the analysis in [10], we first consider a coordinate transformation [15]

$$t' = R\tan(t/R), \quad t' = R\tanh(t/R),$$
$$x_i' = (\cos(t/R))^{-2l}x_i, \quad x_i' = (\cosh(t/R))^{-2l}x_i,$$
$$(\theta^\pm)' = (\cos(t/R))^{-1}\theta^\pm,$$
$$(\theta^\pm)' = (\cosh(t/R))^{-1}\theta^\pm, \qquad (5)$$

where the prime denotes coordinates parameterizing flat superspace. Here the left/right column corresponds to Newton–Hooke spacetime with negative/positive cosmological constant. Then we consider a linear change of the basis in the l-conformal Galilei algebra

$$H \to H \pm \frac{1}{R^2}K \mp \frac{1}{R}J, \quad Q^\pm \to Q^\pm \pm \frac{i}{R}S^\pm, \qquad (6)$$

where the upper/lower sign in the generator of time translations corresponds to negative/positive cosmological constant. The operators of realization (3) in Newton-Hooke superspacetime are constructed by using (5) in paper [15].

4 Conclusion

To summarize, in this paper we have constructed an $\mathcal{N} = 2$ supersymmetric extension of the l-conformal Galilei algebra and its realizations in flat spacetime and in Newton-Hooke spacetime. A coordinate transformation which links the realizations was given. An infinite–dimensional extension was proposed.

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ЗАМЕТКА ОБ N=2 СУПЕРСИММЕТРИЧНОМ РАСШИРЕНИИ l-КОНФОРМНОЙ АЛГЕБРЫ ГАЛИЛЕЯ

Построено N=2 суперсимметричное расширение l-конформной алгебры Галилея и ее представления в плоском суперпространстве и суперпространстве Ньютона-Гука. Установлены координатные преобразования, связывающие построенные представления.

Ключевые слова: *алгебра Галилея, конформная симметрия, N=2 суперсимметрия.*

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