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EARLY-TIME COSMIC DYNAMICS IN $F(R)$ AND $F(|\hat{\Omega}|)$ EXTENSIONS OF BORN-INFELD GRAVITY

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We consider two types of modifications of Born-Infeld gravity in the Palatini formulation and explore their dynamics in the early universe. One of these families considers $f(R)$ corrections to the Born-Infeld Lagrangian, which can be seen as modifications of the dynamics produced by the quantum effects of matter, while the other consists on different powers of the elementary building block of the Born-Infeld Lagrangian, which we denote by $|\hat{\Omega}|$. We find that the two types of nonsingular solutions that arise in the original Born-Infeld theory are also present in these extensions, being bouncing solutions a stable and robust branch. Singular solutions with a period of approximate de Sitter inflation are found even in universes dominated by radiation.

Keywords: *Cosmology, nonsingular universes, modified gravity, Palatini formalism.*

1 Introduction

Understanding the dynamical laws of nature at very high energies is a challenge for theoretical physics. The idea of our Universe being born from a big bang singularity is disturbing and alternative nonsingular scenarios are desirable. In this respect, a high-energy extension of General Relativity (GR) constructed in analogy with the Born-Infeld theory of nonlinear electrodynamics [1] has been recently considered with very positive results. In this theory, formulated in a metric-affine manifold, nonsingular solutions exist that prevent the big bang. These solutions are of two types: bouncing solutions, characterized by $H = 0$ and $dH/d\rho \neq 0$ at the density of the bounce, and unstable minimal volume solutions with $H = 0$ and $dH/d\rho = 0$. In recent works we have studied the stability of these solutions under small perturbations of the action and also under large deformations. In this talk we summarize the main results of our analyses and the main conclusions.

2 Perturbations of BI via $f(R)$ corrections

The action of the Born-Infeld theory of gravity takes the form

$$S_{BI} = \frac{1}{\kappa^2\epsilon}\int d^4x \times \left[\sqrt{-|g_{\mu\nu}+\epsilon R_{\mu\nu}(\Gamma)|}-\lambda\sqrt{-|g_{\mu\nu}|}\right]. \quad (1)$$

This action was originally introduced [2] and reconsidered in [3] within the Palatini formulation, i.e., assuming that the metric and affine geometric structures are independent. For clarifications on the notation see [4] (for reviews on modified gravity in Palatini formulation, see [5, 6]). It admits a power series expansion in the parameter ϵ of the form

$$S_{BI} \approx \int \frac{d^4x}{2\kappa^2}\sqrt{-g} \times \left[R-2\Lambda_{eff}+\frac{\epsilon R^2}{4}-\frac{\epsilon}{2}R_{\mu\nu}R^{\mu\nu}+\dots\right], \quad (2)$$

where $\Lambda_{eff} = \frac{\lambda-1}{\epsilon}$. Clearly, it recovers GR at the zeroth order and quadratic gravity with specific coefficients at the next-to-leading order. It is important to note, however, that if quantum effects of matter are taken into account, quadratic curvature corrections arise that depend on the kind and number of matter fields, which induces deviations from this effective low-energy Lagrangian. It is thus interesting to explore their potential effect on the dynamics as a test of the robustness of the predictions of this theory for the early universe.

In order to be as general as possible, we consider a family of extensions of the original Born-Infeld theory of the form

$$S_{BI-f(R)} = S_{BI} + \frac{\alpha}{2\kappa^2}\int d^4x\sqrt{-g}f(R) + S_m \,. \quad (3)$$

In the limit $\epsilon \to 0$ this theory can be seen as a typical $f(R)$ theory (for general review of $f(R)$ gravity, see [7]), whereas for $\alpha \to 0$ it recovers the original Born-Infeld theory. The analysis and discussion of the field equations of this type of Born-Infeld-$f(R)$ theories was presented in [4] (see also [8] and [9]). Here we simply summarize the relevant results regarding the Hubble function in cosmologies with a perfect fluid with constant equation of state $P = w\rho$.

In Fig. 1, we represent the (dimensionless) Hubble function $|\epsilon|H^2$ as a function of the (dimensionless) energy density $|\epsilon|\kappa^2\rho$ in the original BI theory for different equations of state. The blue curves, which end at $|\epsilon|\kappa^2\rho = 1$ represent bouncing solutions and occur for $w > -1$. The other curves are nonsingular if $w > 0$ and represent unstable states of minimum volume.

In Fig. 2, we show that bouncing solutions exist in the original BI theory (solid blue) and in two quadratic modifications of the form $f(R) = aR^2$, with $a = 1/2$ (dashed orange) and $a = 1$ (dashed red), for different equations of state ($w = -1/5, 0,$ and $1/3$). The existence of a bounce appears as a robust property of the $\epsilon < 0$ branch of the theory.

In Fig. 3, we find the Hubble function in a radiation universe ($\omega = 1/3$) in the cases $a = 0$ (solid blue), $a = 1/10$ (dashed brown), $a = 1/3$ (dashed green), $a = 1/2$ (dashed orange), and $a = 1$ (dashed red). We see that a plateau follows a maximum around $\epsilon\kappa^2\rho \approx 0.6$ in the case $a = 1/3$, which could support a period of inflation generated by the radiation fluid.

The plots in Figs $1-3$ put forward that the bouncing solutions of Born-Infeld gravity are robust against modifications of the R^2 coefficient, whereas those in the unstable branch undergo significant changes. Remarkably, the modifications experienced by these solutions may lead to a period of de Sitter-like expansion after the big bang singularity, as is apparent from the case $a = 1/3$ in Fig. 3.

3 Deformations of Born-Infeld gravity

The Lagrangian density in the action (1) can be rewritten in a more standard form by noting that one can introduce an auxiliary metric $q_{\mu\nu} \equiv g_{\mu\nu} + \epsilon R_{\mu\nu}$ such that $q_{\mu\nu} \equiv g_{\mu\alpha}\Omega^{\alpha}{}_{\nu}$, which allows to write (1) as

$$S_{BI} = \frac{1}{\kappa^2\epsilon}\int d^4x\sqrt{-g}\left[\sqrt{|\hat{\Omega}|} - \lambda\right] + S_m \ . \quad (4)$$

This representation suggests the following family of theories:

$$S_f = \frac{1}{\kappa^2\epsilon}\int d^4x\sqrt{-g}\left[f(|\hat{\Omega}|) - \lambda\right] + S_m \ , \quad (5)$$

being $f(|\hat{\Omega}|) = |\hat{\Omega}|^{1/2}$ the original Born-Infeld theory. A detailed exploration of the field equations of these models and their representation in cosmological models was provided in [10]. Here we simply discuss the results through their graphical representation. In Fig. 4 we see that for a radiation fluid ($w = 1/3$) the two types of nonsingular solutions, those of the bouncing type (dashed curves) and those representing unstable minimum volume states (solid curves), persist for small deviations of the parameter n from the original Born-Infeld case $n = 1/2$.

For the case $\omega = -1/5$ (see Fig. 5) we find that the solid lines are divergent for small values of n, which indicates that only the bouncing branch is able to yield non-singular solutions.

For larger values of the deformation parameter n and $\omega > 0$, these theories are able to yield nonsingular solutions even in the unstable branch (solid curves in Fig. 6). In these solutions, the curve H^2 hits the axis forming a non-zero angle, which indicates that they are closer to the bouncing solutions of the original Born-Infeld theory than to the unstable type, characterized by $dH/d\rho = 0$ at the maximum density.

4 Conclusions

By exploring different extensions of the Born-Infeld theory of gravity we have been able to conclude that the avoidance of the big bang singularity by means of a bounce is a very robust result in theories of the Born-Infeld type. This property is stable against perturbations of the R^2 term of the low-energy expansion of the Lagrangian, which suggests that the predictions of the theory might be quite insensitive to matter loop corrections. We have also found that periods of inflationary behavior can be induced by means of R^2 corrections. On the other hand, large deformations of the Lagrangian, encoded in the power of the function $|\hat{\Omega}|$, also preserve the bouncing branch of the original theory. The unstable solutions become more stable when $n > 1$. The study of cosmological perturbations and black hole solutions in these models are subjects of future work.

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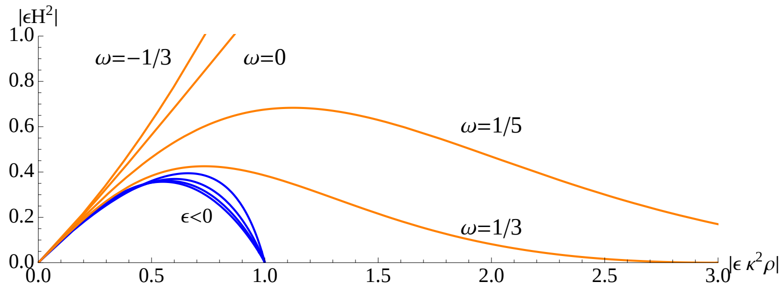

Figure 1. H^2 in Born-Infeld theory

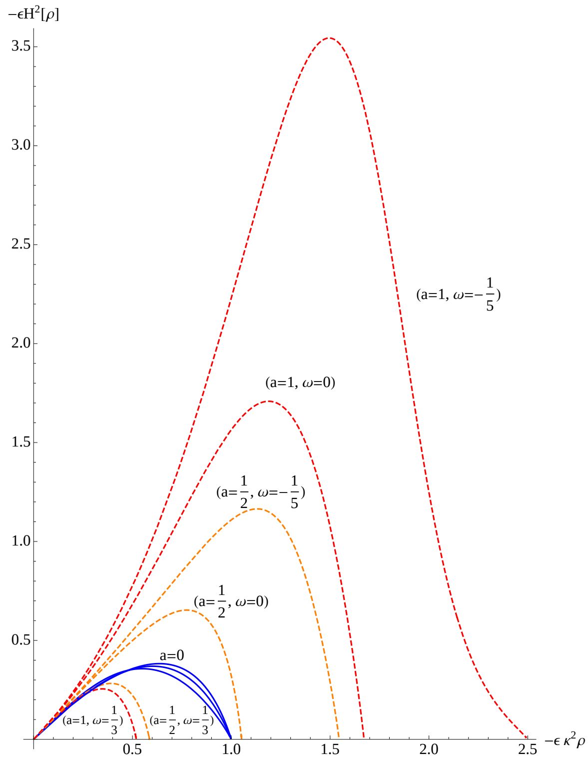

Figure 2. Hubble function in BI with $f(R) = aR^2$ corrections

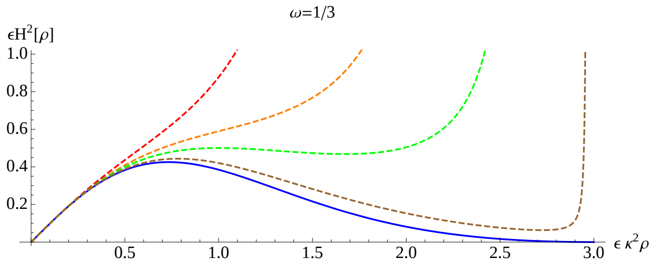

Figure 3. Inflationary behavior for $a = 1/3$ (dashed green curve). The solid blue curve is $a = 0$

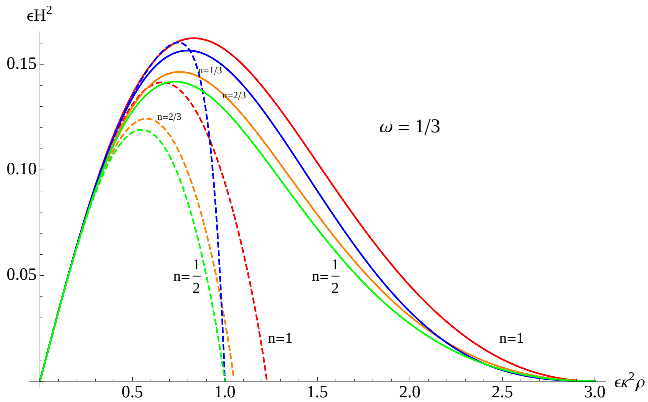

Figure 4. H^2 for radiation in $f(|\hat{\Omega}|) = |\hat{\Omega}|^n$

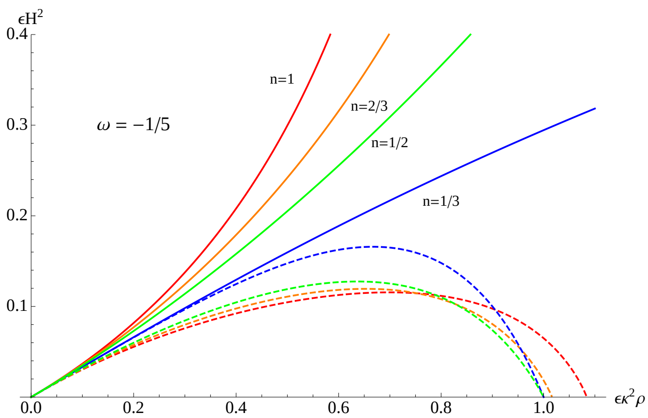

Figure 5. H^2 for $w = -1/5$ and $n < 1$

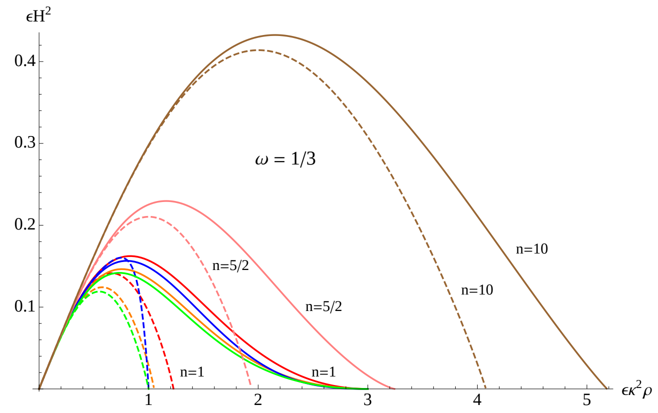

Figure 6. H^2 for $w = 1/3$ and $n > 1$

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КОСМИЧЕСКАЯ ДИНАМИКА РАННЕЙ ВСЕЛЕННОЙ В РАСШИРЕННОЙ $f(R)$ И $f(|\hat{\Omega}|)$ ГРАВИТАЦИИ БОРНА-ИНФЕЛЬДА

Рассмотрено два типа модификации гравитации Борна-Инфельда в формализме Палатини и изучена их динамика для ранней Вселенной. Модификация, содержащая $f(R)$ поправки к лагранжиану Борна-Инфельда, изменяет динамику квантовых эффектов материи. Второй тип рассмотренных модификаций состоит из различных степеней элементарных строительных блоков лагранжиана Борна-Инфельда, которые обозначаются следующим образом: $|\hat{\Omega}|$. Показано, что два типа несингулярных решений, возникающих в теории Борна-Инфельда, также присутствуют в этих расширениях в виде, так называемых, решений с отскоком, образующих устойчивое ответвление. Сингулярные решения с периодом, близким к инфляции де Ситтера, найдены для вселенных с преобладанием излучения.

Ключевые слова: *космология, несингулярная Вселенная, модифицированные теории гравитации, формализм Палатини.*

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