UDC 530.1; 539.1

RENORMALIZATION OF FIELD MODELS WITH ONE-PARAMETER FERMIONIC SYMMETRY

P. M. Lavrov, O. V. Radchenko

Tomsk State Pedagogical University, Kievskaya str., 60, 634061 Tomsk, Russia.

E-mail: lavrov@tspu.edu.ru; radchenko@tspu.edu.ru

We prove that the theories invariant under one-parameter fermionic symmetry after renormalization retain invariance. It is shown that the Ward identity for effective action after renormalization has the same form as non-renormalized one.

Keywords: *renormalization, supersymmetric invariance.*

1 Introduction

As it is known modern quantum field theory considers many different field models with quantum action invariant under supersymmetric transformations. For example the Faddeev-Popov action for Yang-Mills fields [1]. This action is invariant under remarkable BRST–transformations [2, 3]. The next sample is well-known Curci-Ferrari model of non-abelian massive vector fields [4] possesses supersymmetric invariance connected with the modified BRST and modified anti-BRST transformations, but these supersymmetric transformations are not nilpotent (in contrast with the BRST transformations).

In recent years there is also an interest to similar theories. One of such examples is superextension of the sigma models [5], which leads to actions again invariant under supersymmetric transformations. Recent attempts [6, 7] to formulate Yang-Mills fields in a form being free of the Gribov problem [8–10] give another examples of actions invariant under some nilpotent supersymmetric transformations. Quite recently a new realization of supersymmetry, called scalar supersymmetry, has been proposed in [11] when one meets supersymmetric invariant field models as well. In the paper [12] from general point of view properties of field theories for which an action appearing in the generating functional of Green functions is invariant under supersymmetric transformations were studied. Notice, that here the term "supersymmetry" we use as synonym of fermionic symmetry.

In this paper we continue the study of renormalization of the field theories [13–15] in the case of one-parameter global supersymmetry. Our research of renormalization is mainly based on the method proposed in [16].

We employ the DeWitt's condensed notation [17]. Derivatives with respect to fields are taken from the right and those with respect to antifields, from the left. The Grassmann parity of a quantity X is denoted as $\varepsilon(X)$. We use the notation $X_{,i}$ for right derivative of X with respect to ϕ^i.

2 Supersymmetric invariant theories

Our starting point is a theory of fields $\phi = \{\phi^i\}$ with Grassmann parities $\varepsilon(\phi^i) = \varepsilon_i$. We assume a non-degenerate action $S(\phi)$ of the theory so that the generating functional of Green functions is given by the standard functional integral

$$Z(J) = \int \mathcal{D}\phi \, \exp\Big\{\frac{i}{\hbar}\big[S(\phi) + J\phi\big]\Big\} \, . \tag{1}$$

We suppose invariance of $S(\phi)$ under supersymmetric transformations

$$\begin{aligned} \phi^i \quad &\mapsto \quad \phi^i = \varphi^i(\phi^{'}) \, , \\ \varphi^i(\phi) \quad &= \quad \phi^i + R^i(\phi)\xi \, , \quad \xi^2 = 0 \, , \end{aligned} \tag{2}$$

so that

$$S_{,i}(\phi)R^i(\phi) = 0 \, . \tag{3}$$

In (2) ξ is an odd Grassmann parameter and $R^i(\phi)$ are generators of supersymmetric transformations having the Grassmann parities opposite to fields ϕ^i: $\varepsilon(R^i) = \varepsilon_i + 1$.

It is very useful to use the so-called extended action $S(\phi, \phi^*)$ instead of the action $S(\phi)$ by introducing antifields ϕ^*_i with Grassmann parities opposite to fields ϕ^i, $\varepsilon(\phi^i) = \varepsilon_i + 1$:

$$S(\phi, \phi^*) = S(\phi) + \phi^*_i R^i(\phi), \tag{4}$$

and the extended generating functional of Green functions has form

$$Z(J, \phi^*) = \int \mathcal{D}\phi \, \exp\Big\{\frac{i}{\hbar}\big[S(\phi, \phi^*) + J\phi\big]\Big\}. \tag{5}$$

Then the condition (3) of invariance of the action can be conveniently represented in the form of classical master-equation written in terms of the antibracket [18]

$$(S, S) = 0, \tag{6}$$

where for any functions F, G the antibracket is defined by rule

$$(F,G)=\frac{\partial F}{\partial\phi}\frac{\partial G}{\partial\phi^*}-\frac{\partial G}{\partial\phi}\frac{\partial F}{\partial\phi^*}(-1)^{(\epsilon(F)+1)(\epsilon(G)+1)} \qquad (7)$$

with Grassmann parity

$$\epsilon((F,G))=\epsilon(F)+\epsilon(G)+1. \qquad (8)$$

Here we will restrict ourselves to a special supersymmetric theory when the generators $R^i(\phi)$ are subjected to the restriction [12]

$$R^i_{,i}(\phi)=0\ . \qquad (9)$$

Taking into account (9), the Ward identity for the generating functional $Z(J,\phi^*)$ (5) has form

$$J_i\frac{\delta Z(J,\phi^*)}{\delta\phi^*_i}=0. \qquad (10)$$

Introducing the generating functional of connected Green functions $W(J,\phi^*)=-i\hbar\ln Z$, the identity (10) can be rewritten as

$$J_i\frac{\delta W}{\delta\phi^*_i}=0. \qquad (11)$$

The generating functional of the vertex functions $\Gamma=\Gamma(\phi,\phi^*)$ is introduced in a standard way, through the Legendre transformation of W,

$$\Gamma(\phi,\phi^*)=W(J,\phi^*)-J_i\phi^i,$$
$$\phi^i=\frac{\delta W}{\delta J_i},\ \frac{\delta\Gamma}{\delta\phi^i}=-J_i. \qquad (12)$$

The Ward identity for the generating functional of the vertex functions can be obtained directly from (11) and (12), in the form

$$(\Gamma,\,\Gamma)=0. \qquad (13)$$

The Ward identity (13) has universal form and plays a very important role in proof of gauge invariant renormalizability of general gauge theories [16].

3 Supersymmetric invariant renormalization

Let us consider functional integro-differential equation for the generating functionals of vertex Green's functions (effective action)

$$\exp\left\{\frac{i}{\hbar}\Gamma(\phi,\phi^*)\right\} \qquad (14)$$
$$=\int d\phi^{'}\exp\left\{\frac{i}{\hbar}\Big[S(\phi+\phi^{'},\phi^*)-\frac{\delta\Gamma(\phi,\phi^*)}{\delta\phi^i}\phi^{i'}\Big]\right\}.$$

Solutions of this equation are studied within perturbation theory in $\hbar$.

Our study of the renormalization of a given supersymmetric invariant theory is based on the procedure proposed in [16]. The main points of this approach are: a) the action satisfies the classical master-equation; b) the effective action satisfies the Ward identity; c) there exists regularization, which retains forms of the equation (6) and identity (13).

Let us consider the one - loop approximation for Γ

$$\Gamma=S+\hbar\big(\Gamma^{(1)}_{div}+\Gamma^{(1)}_{fin}\big)+\mathcal{O}(\hbar^2),$$

where $\Gamma^{(1)}_{div}$ and $\Gamma^{(1)}_{fin}$ denote the divergent and finite parts of the one-loop approximation for Γ.

The functional $\Gamma^{(1)}_{div}$ determines the counterterms of the one-loop renormalized action S_{1R}:

$$S_{1R}=S-\hbar\Gamma^{(1)}_{div}$$

and satisfies the equation

$$(S,\Gamma^{(1)}_{div})=0. \qquad (15)$$

Then we find that S_{1R} satisfies the basic equation

$$(S_{1R},\ S_{1R})=\hbar^2E_2$$

up to certain terms E_2

$$E_2=(\Gamma^{(1)}_{div},\ \Gamma^{(1)}_{div})$$

of the second order in $\hbar$.

Let us construct the effective action Γ_{1R} with the help of the action S_{1R}. This functional is finite in the one-loop approximation and satisfies the equation

$$(\Gamma_{1R},\ \Gamma_{1R})=\hbar^2E_2+O(\hbar^3).$$

Represent Γ_{1R} in the form

$$\Gamma_{1R}\ =\ S+\hbar\Gamma^{(1)}_{fin}++\hbar^2(\Gamma^{(2)}_{1,div}+\Gamma^{(2)}_{1,fin})+O(\hbar^3)\ .$$

The divergent part $\Gamma^{(2)}_{1,div}$ of the two - loop approximation for Γ_{1R} determines the two - loop renormalization for S_{2R}

$$S_{2R}=S_{1R}-\hbar^2\Gamma^{(2)}_{1,div}$$

and satisfies the equation

$$(S,\ \Gamma^{(2)}_{1,div})=E_2.$$

Let us now consider

$$(S_{2R},\ S_{2R})=\hbar^3E_3+O(\hbar^4).$$

We find that S_{2R} satisfies the master-equation up to terms E_3

$$E_3=2(\Gamma^{(1)}_{div},\ \Gamma^{(2)}_{1,div})$$

of the third order in $\hbar$. Then the corresponding effective action Γ_{2R} generated by S_{2R} is finite in the two - loop approximation

$$\Gamma_{2R}\ =\ S+\hbar\Gamma^{(1)}_{fin}+\hbar^2\Gamma^{(2)}_{1,fin}+\hbar^3(\Gamma^{(3)}_{2,div}$$
$$+\ \ \Gamma^{(3)}_{2,fin})+O(\hbar^4)$$

and satisfies the equation

$$(\Gamma_{2R},\ \Gamma_{2R}) = \hbar^3 E_3 + O(\hbar^4)$$

up to certain terms E_3 of the third order in $\hbar$.

Applying the induction method we establish that the totally renormalized action S_R

$$S_R = S - \sum_{n=1}^{\infty} \hbar^n \Gamma^{(n)}_{n-1,div} \qquad (16)$$

satisfies the basic equation exactly:

$$(S_R,\ S_R) = 0, \qquad (17)$$

while the renormalized effective action Γ_R is finite in each order of $\hbar$ powers:

$$\Gamma_R = S + \sum_{n=1}^{\infty} \hbar^n \Gamma^{(n)}_{n-1,fin}, \qquad (18)$$

and satisfies the identity

$$(\Gamma_R,\ \Gamma_R) = 0. \qquad (19)$$

Here, we have denoted by $\Gamma^{(n)}_{n-1,div}$ and $\Gamma^{(n)}_{n-1,fin}$ the divergent and finite parts, respectively, of the n - loop approximation for the effective action which is finite in (n-1)th approximation and is constructed from the action $S_{(n-1)R}$.

Thus, the identity (19) means that after renormalization the effective action has the same symmetry properties as non-renormalized one.

Acknowledgement

The work is supported by Ministry of Education and Science of Russian Federation, project No. 867.

References

[1] Faddeev L. D. and Popov V. N. *Phys. Lett.* B**25** 29.

[2] Becchi C., Rouet A. and Stora R. 1975 *Commun. Math. Phys.* **42** 127.

[3] Tyutin I. V. 1975 *Gauge invariance in field theory and statistical physics in operator formalism*, Lebedev Inst. preprint N 39 arXiv:0812.0580 [hep-th].

[4] Curci G. and Ferrari R. 1976 *Nuovo Cim.* A**32** 151.

[5] Catterall S. and Chadab S. 2004 *JHEP* **0405** 044.

[6] Slavnov A. A. *Gauge fields beyond perturbation theory* arXiv:1310.8164 [hep-th].

[7] Quagri A. and Slavnov A. A. 2010 *JHEP* **1007** 087.

[8] Gribov V. N., 1978 *Nucl. Phys.* B**139** 1.

[9] Zwanziger D. 1989 *Nucl. Phys.* B**321** 591.

[10] Zwanziger D. 1989 *Nucl. Phys.* B **323** 513.

[11] Jourjine A. 2013 *Phys. Lett.* B**727** 211.

[12] Esipova S. R., Lavrov P. M. and Radchenko O. V. 2014 *Int. J. Mod. Phys.* A**29** 1450065.

[13] Lavrov P. M. and Shapiro I. L. 2010 *Phys. Rew.* D**81** 044026.

[14] Lavrov P. M. 2011 *Nucl. Phys.* B**849** 503.

[15] Lavrov P. M. 2012 *TSPU Bulletin* **13** 98.

[16] Lavrov P. M., Tyutin I. V. and Voronov B. L. 1982 *Sov. J. Nucl. Phys.* **36** 292.

[17] DeWitt B. S. 1965 *Dynamical Theory of Groups and Fields* (Gordon and Breach, New York) 288 p.

[18] Batalin I. A. and Vilkovisky G. A. 1981 *Phys. Lett.* B**102** 27.

Received 14.11.2014

П. М. Лавров, О. В. Радченко

ПЕРЕНОРМИРОВКА ПОЛЕВЫХ МОДЕЛЕЙ С ОДНОПАРАМАТРИЧЕСКОЙ ФЕРМИОННОЙ СИММЕТРИЕЙ

Доказано, что теории, инвариантные относительно однопараметрической фермионной симметрии, после перенормировки сохраняют это свойство инвариантности. Показано, что тождество Уорда для эффективного действия после перенормировки имеют ту же форму, что и до нее.

Ключевые слова: *перенормировка, суперсимметричная инвариантность.*

Лавров П. М., доктор физико-математических наук, профессор.
Томский государственный педагогический университет.
Ул. Киевская, 60, 634061 Томск, Россия.
E-mail: lavrov@tspu.edu.ru

Радченко О.В., кандидат физико-математических наук.
Томский государственный педагогический университет.
Ул. Киевская, 60, 634061 Томск, Россия.
E-mail: radchenko@tspu.edu.ru