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INTERACTION OF MASSIVE FERMIONIC HIGHER SPIN FIELDS WITH CONSTANT ELECTROMAGNETIC FIELD

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We develop a universal general gauge-invariant method of Lagrangian construction based on the BRST approach for half-integer higher spin fields interacting with constant electromagnetic field in Minkowski space of any dimension. No off-shell constraints for the fields and gauge parameters are imposed from the very beginning.

Keywords: *higher spin fields, BRST approach, fermionic fields.*

1 Introduction

Constructing of Lagrangians for interacting higher spin fields is a problem which has been attracting much attention for a long time due to the hope of finding new possibilities and approaches to the unification of the fundamental interactions and is one of the unsolved general problems of classical field theory (see, e.g., the reviews [1]).

In the present paper we investigate the interaction problem of fermionic fields. This problem has been studied much less than the problem of interacting bosonic fields. In particular it was studied construction of the cubic vertices in the light cone framework [2], and different problems of interaction with gravitational and elctromagnetic field (see e.g. [3–10]).

In this paper we develop a gauge-invariant approach based on the BRST construction to a Lagrangian construction for totally symmetric fermionic higher spin fields interacting with constant electromagnetic field in Minkowski space of any dimension and solve the problem in the linear in $F_{\mu\nu}$ approximation.

The paper is organized as follows. In section 2, we give brief reminder about BRST approach to Lagrangian construction for free massive fermionic field in Minkowski space. Then in section 3 we modify this procedure so as to construct interaction of the fermionic fields with constant electromagnetic field and realize it in the linear in $F_{\mu\nu}$ approximation.

2 Free massive fermionic fields in Minkowski space

Lagrangian construction for massive fermionic field in Minkowski space based on BRST approach was carried out in [11] (see also [12, 13]). In this section we briefly remind the result[1].

It is known that totally symmetric tensor-spinor field $\psi_{\mu_1\ldots\mu_n}$ (the Dirac index is suppressed) describing the irreducible massive spin $s = n+1/2$ representation of the Poincare group must satisfy the following conditions

$$(i\gamma^\nu\partial_\nu - m)\psi_{\mu_1\ldots\mu_n} = 0, \qquad \gamma^{\mu_1}\psi_{\mu_1\ldots\mu_n} = 0,$$
$$\partial^{\mu_1}\psi_{\mu_1\ldots\mu_n} = 0. \tag{1}$$

with $\{\gamma^\mu,\gamma^\nu\} = -2\eta^{\mu\nu}$.

In order to avoid explicit manipulation with the indices it is convenient to introduce an auxiliary Fock space generated by creation and annihilation operators a_μ^+, a_ν satisfying the commutation relations

$$[a_\mu^+, a_\nu] = \eta_{\mu\nu}. \tag{2}$$

Eqs (1) are realized in this Fock space as follows

$$(i\tilde\gamma^\nu\partial_\nu - \tilde\gamma m)|\psi_n\rangle = 0, \qquad \tilde\gamma^\mu a_\mu|\psi_n\rangle = 0,$$
$$ia^\mu\partial_\mu|\psi_n\rangle = 0, \tag{3}$$
$$|\psi_n\rangle = \psi_{\mu_1\ldots\mu_n} a^{+\mu_1}\ldots a^{+\mu_n}|0\rangle.$$

Here we introduce Grassmann odd "gamma-marix like objects" $\tilde\gamma^\mu$ and $\tilde\gamma$ which are connected with the usual Grassmann even gamma-matrices γ^μ by relation [11]

$$\gamma^\mu = \tilde\gamma^\mu\tilde\gamma, \tag{4}$$
$$\{\tilde\gamma^\mu,\tilde\gamma^\nu\} = -2\eta^{\mu\nu},\ \{\tilde\gamma^\mu,\tilde\gamma\} = 0,\ \{\tilde\gamma,\tilde\gamma\} = -2. \tag{5}$$

Next following the procedure described e.g. in [11–13] on the base of the operators (3) we construct all the operators involved in the BRST charge. These

[1] Unlike [11] the signature of the metrics in this paper is "mostly plus". There are also other differences in the notation.

operators are

$$t_0 = i\tilde{\gamma}^\mu\partial_\mu - \tilde{\gamma}m, \quad (6)$$
$$l_0 = \partial^2 - m^2, \quad (7)$$
$$l_1 = ia^\mu\partial_\mu + mb_1, \quad (8)$$
$$l_1^+ = ia^{+\mu}\partial_\mu + mb_1^+, \quad (9)$$
$$t_1 = \tilde{\gamma}^\mu a_\mu - \tilde{\gamma}b_1 + f^+b_2 - 2(b_2^+b_2 + h)f, \quad (10)$$
$$t_1^+ = a_\mu^+\tilde{\gamma}^\mu - \tilde{\gamma}b_1^+ + f^+ - 2b_2^+f, \quad (11)$$
$$l_2 = \tfrac{1}{2}a^\mu a_\mu + \tfrac{1}{2}b_1^2 + (b_2^+b_2 + f^+f + h)b_2, \quad (12)$$
$$l_2^+ = \tfrac{1}{2}a^{+\mu}a_\mu^+ + \tfrac{1}{2}b_1^{+2} + b_2^+, \quad (13)$$
$$g_0 = a_\mu^+a^\mu + b_1^+b_1 + 2b_2^+b_2 + f^+f + \tfrac{d+1}{2} + h. \quad (14)$$

To construct the operators above we enlarge the Fock space with two pairs of bosonic (b_1^+, b_1 and b_2^+, b_2) and one pair of fermionic (f^+, f) creation and annihilation operators with the standard commutation relations

$$[b_1, b_1^+] = 1, \qquad [b_2, b_2^+] = 1, \qquad \{f, f^+\} = 1$$

and h is an arbitrary parameter which, as will be shown below, is associated with spin of the field $h = 2 - s - \frac{d}{2}$. Note that the set of the operators is invariant under Hermitian conjugation if we change the scalar product in the (b_2, f)-sector of the Fock space as follows

$$\langle\Psi_1|\Psi_2\rangle_{\text{new}} = \langle\Psi_1|K|\Psi_2\rangle, \quad (15)$$
$$K_h = \sum_{n=0}^{\infty}\frac{1}{n!}\Big(|n\rangle\langle n|C(n,h) - 2f^+|n\rangle\langle n|f\,C(n+1,h)\Big), \quad (16)$$
$$C(n,h) = h(h+1)\cdots(h+n-1),$$
$$C(0,h) = 1, \qquad |n\rangle = (b_2^+)^n|0\rangle.$$

The BRST operator constructed on the base of operators (6)–(14) is

$$\begin{aligned}\tilde{Q} \;=\; & q_0t_0 + q_1^+t_1 + q_1t_1^+ + \eta_0l_0 + \eta_1^+l_1 + \eta_1l_1^+ \\ & + \eta_2^+l_2 + \eta_2l_2^+ + \eta_Gg_0 + 2q_0(q_1^+\mathcal{P}_1 + q_1\mathcal{P}_1^+) \\ & + (q_1^+\eta_1 - \eta_1^+q_1)ip_0 + (\eta_1^+\eta_1 - q_0^2)\mathcal{P}_0 \\ & + 2q_1^{+2}\mathcal{P}_2 + 2q_1^2\mathcal{P}_2^+ + q_1^+\eta_2ip_1^+ - \eta_2^+q_1ip_1 \\ & - \eta_2^+\eta_1\mathcal{P}_1 - \eta_1^+\eta_2\mathcal{P}_1^+ + (2q_1^+q_1 - \eta_2^+\eta_2)\mathcal{P}_G \\ & + \eta_G(q_1^+ip_1 - q_1ip_1^+ + \eta_1^+\mathcal{P}_1 - \eta_1\mathcal{P}_1^+ \\ & + 2\eta_2^+\mathcal{P}_2 - 2\eta_2\mathcal{P}_2^+).\end{aligned}$$

Here, q_0, q_1, q_1^+ and η_0, η_1^+, η_1, η_2^+, η_2, η_G are, respectively, the bosonic and fermionic ghost "coordinates" corresponding to their canonically conjugate ghost "momenta" p_0, p_1^+, p_1, $\mathcal{P}_0$, $\mathcal{P}_1$, $\mathcal{P}_1^+$, $\mathcal{P}_2$, $\mathcal{P}_2^+$, $\mathcal{P}_G$. They obey the (anti)commutation relations

$$\begin{aligned}\{\eta_1, \mathcal{P}_1^+\} =& \{\mathcal{P}_1, \eta_1^+\} = \{\eta_2, \mathcal{P}_2^+\} = \{\mathcal{P}_2, \eta_2^+\} \\ =& \{\eta_0, \mathcal{P}_0\} = \{\eta_G, \mathcal{P}_G\} = 1, \\ [q_0, p_0] =& [q_1, p_1^+] = [q_1^+, p_1] = i \end{aligned} \quad (17)$$

and possess the standard ghost number distribution, $gh(\mathcal{C}^i) = -gh(\mathcal{P}_i) = 1$, providing the property $gh(\tilde{Q}) = 1$.

Extracting from the BRST operator first dependence on ghosts η_G, $\mathcal{P}_G$ and then η_0, $\mathcal{P}_0$ and q_0, p_0 one obtains

$$\tilde{Q} = Q + \eta_G\big(N + \tfrac{d-3}{2} + h\big) + (2q_1^+q_1 - \eta_2^+\eta_2)\mathcal{P}_G,$$
$$\begin{aligned}N = & a_\mu^+a^\mu + b_1^+b_1 + 2b_2^+b_2 + f^+f + q_1^+ip_1 \\ & - ip_1^+q_1 + \eta_1^+\mathcal{P}_1 + \mathcal{P}_1^+\eta_1 + 2\eta_2^+\mathcal{P}_2 + 2\mathcal{P}_2^+\eta_2,\end{aligned}$$
$$\begin{aligned}Q = & q_0\tilde{t}_0 + \eta_0l_0 + \Delta Q \\ & + (q_1^+\eta_1 - \eta_1^+q_1)ip_0 + (\eta_1^+\eta_1 - q_0^2)\mathcal{P}_0,\end{aligned}$$
$$\begin{aligned}\Delta Q = & q_1^+t_1 + q_1t_1^+ + \eta_1^+l_1 + \eta_1l_1^+\eta_2^+l_2 + \eta_2l_2^+ \\ & + 2q_1^{+2}\mathcal{P}_2 + 2q_1^2\mathcal{P}_2^+ + q_1^+\eta_2ip_1^+ - \eta_2^+q_1ip_1 \\ & - \eta_2^+\eta_1\mathcal{P}_1 - \eta_1^+\eta_2\mathcal{P}_1^+,\end{aligned}$$
$$\tilde{t}_0 = t_0 + 2q_1^+\mathcal{P}_1 + 2q_1\mathcal{P}_1^+,$$

where Q is independent of η_G, $\mathcal{P}_G$ and ΔQ, $\tilde{t}_0$ are also independent of η_0, $\mathcal{P}_0$ and q_0, p_0.

Next following the procedure of [11] we choose the following representation of the Hilbert space

$$(p_0, q_1, p_1, \mathcal{P}_0, \mathcal{P}_G, \eta_1, \mathcal{P}_1, \eta_2, \mathcal{P}_2)\,|0\rangle = 0\,, \quad (18)$$

and suppose that the vectors and gauge parameters do not depend on η_G,

$$\begin{aligned}|\chi\rangle = & \sum_{k_i}(q_0)^{k_1}(q_1^+)^{k_2}(p_1^+)^{k_3}(\eta_0)^{k_4}(f^+)^{k_5}(\eta_1^+)^{k_6}\,\times \\ & \times(\mathcal{P}_1^+)^{k_7}(\eta_2^+)^{k_8}(\mathcal{P}_2^+)^{k_9}(b_1^+)^{k_{10}}(b_2^+)^{k_{11}}\,\times \\ & \times a^{+\mu_1}\cdots a^{+\mu_{k_0}}\chi^{k_1\cdots k_{11}}_{\mu_1\cdots\mu_{k_0}}(x)|0\rangle.\end{aligned} \quad (19)$$

The sum in (19) is taken over k_0, k_1, k_2, k_3, k_{10}, k_{11}, running from 0 to infinity, and over $k_4, k_5, k_6, k_7, k_8, k_9$, running from 0 to 1. Then, we derive from the equations that determine the physical vector, $\tilde{Q}|\chi\rangle = 0$, as well as from the reducible gauge transformations, $\delta|\chi\rangle = \tilde{Q}|\Lambda\rangle$, a sequence of relations

$$Q|\chi\rangle = 0, \qquad (\sigma + h)|\chi\rangle = 0, \quad (20)$$
$$\delta|\chi\rangle = Q|\Lambda\rangle, \qquad (\sigma + h)|\Lambda\rangle = 0, \quad (21)$$
$$\delta|\Lambda\rangle = Q|\Lambda^{(1)}\rangle, \qquad (\sigma + h)|\Lambda^{(1)}\rangle = 0, \quad (22)$$
$$\delta|\Lambda^{(i-1)}\rangle = Q|\Lambda^{(i)}\rangle, \qquad (\sigma + h)|\Lambda^{(i)}\rangle = 0, \quad (23)$$

The middle equation in (20) presents the equations for the possible values of h,

$$h = 2 - s - \frac{d}{2}. \quad (24)$$

By fixing the value of spin, we also fix the parameter h, according to (24). Having fixed a value of h, we must substitute it into each of the expressions (20)–(23). (See [11–13] for more details.)

As was shown in [11] the equation of motion (20) $Q|\chi\rangle = 0$ indeed reproduce equations (1) up to the reducible gauge transformations (21)–(23), but this equation of motion can't be obtained from a Lagrangian.

To extract from $Q|\chi\rangle = 0$ lagrangian set of equations of motion we decompose the state vector and gauge parameters in q_0, η_0 ghosts

$$|\chi\rangle = \sum_{k=0}^{\infty} q_0^k(|\chi_0^k\rangle + \eta_0|\chi_1^k\rangle),$$

$$|\Lambda\rangle = \sum_{k=0}^{\infty} q_0^k(|\Lambda_0^k\rangle + \eta_0|\Lambda_1^k\rangle).$$

Then following the procedure described in [12] we remove some of the fields with the help of a part of the gauge transformation and a part of the equations of motion and the remaining fields will be $|\chi_0^0\rangle$ and $|\chi_0^1\rangle$. Their equations of motion and reducible gauge transformations are

$$\Delta Q|\chi_0^0\rangle + \tfrac{1}{2}\{\tilde{t_0}, \eta_1^+\eta_1\}|\chi_0^1\rangle = 0, \qquad (25)$$

$$\tilde{t_0}|\chi_0^0\rangle + \Delta Q|\chi_0^1\rangle = 0, \qquad (26)$$

$$\delta|\chi_0^0\rangle = \Delta Q|\Lambda_0^0\rangle + \tfrac{1}{2}\{\tilde{t_0}, \eta_1^+\eta_1\}|\Lambda_0^1\rangle, \qquad (27)$$

$$\delta|\chi_0^1\rangle = \tilde{t_0}|\Lambda_0^0\rangle + \Delta Q|\Lambda_0^1\rangle, \qquad (28)$$

$$\delta|\Lambda^{(i)}{}_0^0\rangle = \Delta Q|\Lambda^{(i+1)}{}_0^0\rangle + \frac{1}{2}\{\tilde{T}_0, \eta_1^+\eta_1\}|\Lambda^{(i+1)}{}_0^1\rangle, \qquad (29)$$

$$\delta|\Lambda^{(i)}{}_0^1\rangle = \tilde{T}_0|\Lambda^{(i+1)}{}_0^0\rangle + \Delta Q|\Lambda^{(i+1)}{}_0^1\rangle, \qquad (30)$$

where $|\Lambda^{(0)}{}_0^0\rangle_n = |\Lambda_0^0\rangle$, and $|\Lambda^{(0)}{}_0^1\rangle_n = |\Lambda_0^1\rangle$.

Equations (26) can be obtained from Lagrangian

$$\mathcal{L} = \langle\tilde{\chi}_0^0|K_h\Big\{\tilde{t_0}|\chi_0^0\rangle + \Delta Q|\chi_0^1\rangle\Big\} + \langle\tilde{\chi}_0^1|K_h\Big\{\Delta Q|\chi_0^0\rangle + \tfrac{1}{2}\{\tilde{t_0}, \eta_1^+\eta_1\}|\chi_0^1\rangle\Big\}. \qquad (31)$$

Here $\{\tilde{t_0}, \eta_1^+\eta_1\} = \tilde{t_0}\eta_1^+\eta_1 + \eta_1^+\eta_1\tilde{t_0}$ and K_h is operator (16).

In the next section we generalize the above construction to the case of massive fermionic fields interacting with constant electromagnetic field.

3 Fermionic fields interacting with constant EM field in Minkowski space

Let us try to construct interaction of the fermionic fields with constant EM field $F_{\mu\nu} = const$ by the following way. First we replace all the partial derivatives by the covariant ones $D_\mu = \partial_\mu - ieA_\mu$ and enlarge the expressions of the operators (6)–(14) by terms vanishing at $F_{\mu\nu} \to 0$ limit (we will denote these enlarged expressions of the operators by the corresponding upper case letters) and demand that the new expressions for the operators form an algebra.

Before writing ansatz for the new expressions for the operators we give some comments. It is known that the trace of a field and its traceless part are independent each other and therefore we can shift the trace of the field so that the traceless condition remains unchanged. Thus we suppose that the operators related with the traceless condition t_1, t_1^+, l_2, l_2^+ and g_0 also remains unchanged. Moreover the (b_2, f)-sector of the Fock space was introduced to modify only these operators [see (10)–(14)] and therefore we also suppose that the creation and annihilation operators of this part of the Fock space will not take part in construction of enlarged expressions for the rest operators L_0, L_1, L_1^+.

Since we are going to consider only linear in $F_{\mu\nu}$ approximation we take the following ansatz for the operators

$$\begin{aligned} L_1 \;=\;& ia^\alpha D_\alpha + mb_1 + a^\alpha F_{\alpha\sigma}D^\sigma \sum_{k=0}^{\infty} f_{0k}\, b_1^{+k} b_1^k \\ &+ \tilde{\gamma}\tilde{\gamma}^\tau F_{\tau\sigma}D^\sigma \sum_{k=0}^{\infty} f_{2k}\, b_1^{+k} b_1^{k+1} \\ &+ a^{+\mu} F_{\mu\sigma}D^\sigma \sum_{k=0}^{\infty} f_{4k}\, b_1^{+k} b_1^{k+2} \\ &+ \tilde{\gamma}^{\mu\nu} F_{\mu\nu} \sum_{k=0}^{\infty} d_{0k}\, b_1^{+k} b_1^{k+1} \\ &+ \tilde{\gamma}\tilde{\gamma}^\sigma F_{\sigma\alpha} a^\alpha \sum_{k=0}^{\infty} d_{2k}\, b_1^{+k} b_1^k \\ &+ a^{+\mu} F_{\mu\alpha} a^\alpha \sum_{k=0}^{\infty} d_{8k}\, b_1^{+k} b_1^{k+1} \\ &+ \tilde{\gamma}\tilde{\gamma}^\sigma F_{\sigma\mu} a^{+\mu} \sum_{k=0}^{\infty} d_{4k}\, b_1^{+k} b_1^{k+2}, \end{aligned} \qquad (32)$$

$$\begin{aligned} T_0 \;=\;& i\tilde{\gamma}^\mu D_\mu - \tilde{\gamma}m + \tilde{\gamma}^\tau F_{\tau\sigma}D^\sigma \sum_{k=0}^{\infty} c_{0k}\, b_1^{+k} b_1^k \\ &+ \tilde{\gamma}a^\alpha F_{\alpha\sigma}D^\sigma \sum_{k=0}^{\infty} c_{4k}\, (b_1^+)^{k+1} b_1^k \\ &+ \tilde{\gamma}a^{+\mu} F_{\mu\sigma}D^\sigma \sum_{k=0}^{\infty} c_{5k}\, b_1^{+k} b_1^{k+1} \\ &+ \tilde{\gamma}\tilde{\gamma}^{\mu\nu} F_{\mu\nu} \sum_{k=0}^{\infty} a_{0k}\, b_1^{+k} b_1^k \\ &+ \tilde{\gamma}a^{+\mu} F_{\mu\alpha} a^\alpha \sum_{k=0}^{\infty} a_{4k}\, b_1^{+k} b_1^k \\ &+ \tilde{\gamma}^\sigma F_{\sigma\alpha} a^\alpha \sum_{k=0}^{\infty} a_{2k}\, (b_1^+)^{k+1} b_1^k \\ &+ \tilde{\gamma}^\sigma F_{\sigma\mu} a^{+\mu} \sum_{k=0}^{\infty} a_{3k}\, b_1^{+k} b_1^{k+1}, \end{aligned} \qquad (33)$$

$$L_1^+ \;=\; ia^{+\mu} D_\mu + mb_1^+ + a^{+\mu} F_{\mu\sigma}D^\sigma \sum_{k=0}^{\infty} f_{1k}\, b_1^{+k} b_1^k$$

$$+\tilde{\gamma}\tilde{\gamma}^{\tau}F_{\tau\sigma}D^{\sigma}\sum_{k=0}^{\infty}f_{3k}\,(b_1^+)^{k+1}b_1^k$$
$$+a^{\alpha}F_{\alpha\sigma}D^{\sigma}\sum_{k=0}^{\infty}f_{5k}\,(b_1^+)^{k+2}b_1^k$$
$$+\tilde{\gamma}^{\mu\nu}F_{\mu\nu}\sum_{k=0}^{\infty}d_{1k}\,(b_1^+)^{k+1}b_1^k$$
$$+\tilde{\gamma}\tilde{\gamma}^{\sigma}F_{\sigma\mu}a^{+\mu}\sum_{k=0}^{\infty}d_{3k}\,b_1^{+k}b_1^k$$
$$+a^{+\mu}F_{\mu\alpha}a^{\alpha}\sum_{k=0}^{\infty}d_{9k}\,(b_1^+)^{k+1}b_1^k$$
$$+\tilde{\gamma}\tilde{\gamma}^{\sigma}F_{\sigma\alpha}a^{\alpha}\sum_{k=0}^{\infty}d_{5k}\,(b_1^+)^{k+2}b_1^k, \qquad (34)$$

where a_{ik}, c_{ik}, d_{ik}, c_{ik} are arbitrary complex constants and by definition we suppose that $L_0 \equiv T_0^2$. The rest operators (10)–(14) are unchanged. One should also note that the ansatz for the operators L_1, T_0, L_1^+ (32)–(34) are not general.

Supposing that operator L_0 is hermitian and $(L_1)^+ = L_1^+$ with respect to the "new" scalar product[2] (15) the arbitrary coefficients in (32)–(34) are found from the condition that the new operators form an algebra in the linear approximation

$$a_{0(0)} = -\frac{ie}{8m} - \frac{ie}{2m}\zeta_0, \qquad a_{0(k)} = 0,$$
$$a_{2(0)} = \frac{ie}{2m} - \frac{e}{m}\xi_1, \qquad a_{2(k)} = -\frac{(-2)^k}{k!}\frac{e}{m}\xi_1,$$
$$a_{3(0)} = -\frac{ie}{2m} - \frac{e}{m}\xi_1, \qquad a_{3(k)} = -\frac{(-2)^k}{k!}\frac{e}{m}\xi_1,$$
$$a_{4(0)} = -\frac{ie}{2m} + \frac{2ie}{m}\zeta_0, \qquad a_{4(k)} = 0,$$
$$d_{0(0)} = -\frac{ie}{8m} + \frac{ie}{2m}(\zeta_0 + i\xi_1), \quad d_{0(k)} = \frac{(-2)^{k-1}}{k!}\frac{e}{m}\xi_1,$$
$$d_{1(0)} = -\frac{ie}{8m} + \frac{ie}{2m}(\zeta_0 - i\xi_1), \quad d_{1(k)} = -\frac{(-2)^{k-1}}{k!}\frac{e}{m}\xi_1,$$
$$d_{2(0)} = -\frac{ie}{4m} - \frac{e}{2m}\xi_1, \quad d_{2(k)} = \frac{(-2)^{k-1}}{k!}(k+1)\frac{e}{m}\xi_1,$$
$$d_{3(0)} = -\frac{ie}{4m} + \frac{e}{2m}\xi_1, \quad d_{3(k)} = -\frac{(-2)^{k-1}}{k!}(k+1)\frac{e}{m}\xi_1,$$
$$d_{4(k)} = \frac{(-2)^k}{k!}\frac{e}{m}\xi_1, \quad k \geq 0,$$
$$d_{5(k)} = -\frac{(-2)^k}{k!}\frac{e}{m}\xi_1, \quad k \geq 0,$$
$$d_{8(0)} = \frac{ie}{m} - \frac{ie}{m}(2\zeta_0 + i\xi_1), \qquad d_{8(k)} = \frac{(-2)^k}{k!}\frac{e}{m}\xi_1,$$
$$d_{9(0)} = \frac{ie}{m} - \frac{ie}{m}(2\zeta_0 - i\xi_1), \qquad d_{9(k)} = -\frac{(-2)^k}{k!}\frac{e}{m}\xi_1,$$

$$c_{0(0)} = \frac{ie}{2m^2}(2\zeta_1 + \xi_1), \qquad c_{0(k)} = \frac{(-2)^k}{k!}\frac{ie}{m^2}\xi_1,$$
$$c_{4(0)} = \frac{e}{m^2}(-2\zeta_0 + i\xi_1), \qquad c_{4(k)} = \frac{(-2)^k}{k!}\frac{ie}{m^2}\xi_1,$$
$$c_{5(0)} = \frac{e}{m^2}(2\zeta_0 + i\xi_1), \qquad c_{5(k)} = \frac{(-2)^k}{k!}\frac{ie}{m^2}\xi_1,$$
$$f_{0(0)} = \frac{e}{m^2}(\zeta_0 + i\zeta_1),$$
$$f_{0(1)} = \frac{e}{m^2}(2\zeta_0 - i\xi_1), \qquad f_{0(k)} = -\frac{(-2)^{k-1}}{k!}\frac{ie}{m^2}\xi_1,$$
$$f_{1(0)} = \frac{e}{m^2}(-\zeta_0 + i\zeta_1),$$
$$f_{1(1)} = -\frac{e}{m^2}(2\zeta_0 + i\xi_1), \qquad f_{1(k)} = -\frac{(-2)^{k-1}}{k!}\frac{ie}{m^2}\xi_1,$$
$$f_{2(0)} = \frac{e}{m^2}(\zeta_0 - \frac{i}{2}\xi_1), \qquad f_{2(k)} = \frac{(-2)^{k-1}}{k!}\frac{ie}{m^2}\xi_1,$$
$$f_{3(0)} = \frac{e}{m^2}(\zeta_0 + \frac{i}{2}\xi_1), \qquad f_{3(k)} = -\frac{(-2)^{k-1}}{k!}\frac{ie}{m^2}\xi_1,$$
$$f_{4(0)} = -\frac{2e}{m^2}\zeta_0, \qquad f_{4(k)} = 0 \quad k \geq 1,$$
$$f_{5(0)} = \frac{2e}{m^2}\zeta_0, \qquad f_{5(k)} = 0 \quad k \geq 1$$

Here ζ_0, ζ_1, ξ_1 are arbitrary real dimensionless constants. Note that similar problem was considered in [3], but we found two more arbitrary constant due to rejection of some condition on the coefficients in (32)–(34). The new operators form algebra which coincides with the algebra in the free case.

Further construction of Lagrangians go in the usual way [11,12].

4 Conclusion

Thus in the present paper we have developed a universal general gauge-invariant method of Lagrangian construction based on the BRST approach for half-integer higher spin fields interacting with constant electromagnetic field in Minkowski space of any dimension. In this procedure no off-shell constraints for the fields and gauge parameters are imposed from the very beginning.

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[2] In the (a_μ, b_1)-sector of the Fock space in which the operators L_0, L_1, L_1^+ are constructed scalar product (15) coincide with the usual scalar product in the Fock space.

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ВЗАИМОДЕЙСТВИЕ МАССИВНЫХ ФЕРМИОННЫХ ПОЛЕЙ ВЫСШИХ СПИНОВ С ПОСТОЯННЫМ ЭЛЕКТРОМАГНИТНЫМ ПОЛЕМ

Развивается БРСТ подход к построению лагранжианов для фермионных полей высших спинов, взаимодействующих с постоянным электромагнитным полем в пространстве Минковского произвольной размерности. В предлагаемой процедуре построения лагранжианов не предполагается никаких ограничений на поля и калибровочные параметры.

Ключевые слова: *поля высших спинов, БРСТ подход, фермионные поля.*

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