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Quantum Equivalence of Massive and Massless p-form Models in Curved D-Dimensional Space

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We consider massive and massless p-forms in arbitrary D-dimensional curved space-time. Quantization of these models and evaluation of effective actions have been performed. Results are presented in terms of d'Alembertians acting on p-forms. The massive theories of p-forms do not possess gauge invariance, in contrary of massless theories. The gauge invariance is restored with help of multi-step Stückelberg procedure. Comparising effective actions of classically equivalent theories (massless quantum theory of p-forms and theory of (D-p-2)-forms, massive quantum theory of p-forms and theory of (D-p-1)-forms), we demonstrate a quantum equivalence of corresponding models. A zeta-function excluding zero modes has been used.

Keywords: *quantum fields in curved space-time, p-forms, gauge field theories, effective action, quantum equivalence.*

1 Introduction

Totally antisymmetric tensor fields (ATFs, p-forms) have been investigated since the sixties ([1] etc.). ATFs have applications in string, supersymmetry and supergravity models, in quantum chromodynamics. ATGFs are considered in context of quantum equivalence of different field representations by [2], [3] and so on.

At present p-forms arouse interest for the most part in connexion with supergravity, M or string theory ([4] etc.). Some behavior of massive ATFs has been investigated by [5], [6] and others.

In this paper, we consider massive and massless p-form theories in arbitrary D-dimensional curved space-time. Sections 2 and 3 are devoted to describing and quantization of massive and massless p-form models. In section 4, we study the questions of quantum equivalence of classically equivalent massive and massless p-form theories with investigation of EAs structure in terms of a generalized zeta-functions associated with corresponding d'Alembertians. In conclusion, presented results are dicussed briefly.

2 Models

We consider p-form $\overset{(p)}{B}$, $p < D$, in arbitrary D-dimensional curved space:

$$\overset{(p)}{B} = \frac{1}{p!} \sum_{\mu_1 \dots \mu_p} B_{\mu_1 \dots \mu_p}(x)\, dx^{\mu_1} \wedge \dots \wedge dx^{\mu_p},$$

with an exterior derivative d,

$$d\overset{(p)}{B} = \frac{1}{(p+1)!} \sum_{\mu_1 \dots \mu_{p+1}} (dB)_{\mu_1 \dots \mu_{p+1}}(x)\, dx^{\mu_1} \wedge \dots \wedge dx^{\mu_{p+1}},$$

and a co-derivative δ, $(\delta B)_{\mu_1 \dots \mu_{p-1}} = -B^{;\nu}_{\nu\mu_1 \dots \mu_{p-1}}$.

Combination of d with δ yields de Rham Laplasian $\Delta = d\delta + \delta d = -\Box$, $\Box$ being the usual d'Alembertian.

Generalizing Maxwell action to p-forms, one has a classical action for ATGFs, massless p-forms, in arbitrary D-dimensional curved space:

$$S[\overset{(p)}{B}] = -\frac{1}{2}(d\overset{(p)}{B}, d\overset{(p)}{B}). \tag{1}$$

This action is invariant under the gauge transformations (GTs), because of $d^2 = 0$,

$$\overset{(k)}{B} \to \overset{(k)}{B'} = \overset{(k)}{B} - d\overset{(k-1)}{B}, 1 \le k \le p, \quad p \le D. \tag{2}$$

Adding to the action (1) a massive term, we get a generalized Proca action, which is not invariant under GTs (2) now:

$$S^m[\overset{(p)}{B}] = -\frac{1}{2}(d\overset{(p)}{B}, d\overset{(p)}{B}) + \frac{m^2}{2}(\overset{(p)}{B}, \overset{(p)}{B}).$$

To quantize this theory it is convenient to restore the gauge invariance with help of Stückelberg procedure [7]. Introducing an extra field $\overset{(p-1)}{C}$, we get new action

$$S^m[\overset{(p)}{B}, \overset{(p-1)}{C}] \equiv S^m[\overset{(p)}{B} + \frac{1}{m} d\overset{(p-1)}{C}], \tag{3}$$

which is invariant under the interrelated GTs of the p-forms $\overset{(p)}{B}$ and of the Stückelberg fields $\overset{(p-1)}{C}$:

$$\begin{aligned} &\overset{(k)}{B} \to \overset{(k)}{B'} = \overset{(k)}{B} - d\overset{(k-1)}{B}, \\ &\overset{(k-1)}{C} \to \overset{(k-1)}{C'} = \overset{(k-1)}{C} + m\overset{(k-1)}{B}, \ 1 \le k \le p. \end{aligned} \tag{4}$$

Now we proceed to quantization of models (1) and (3).

3 Quantization of massive and massless p-form models in curved D-dimensional space

To quantize theories (1) and (3) we use Faddeev-Popov multistep procedure in a form adapted for Abelian theories with linearly dependent generators [3, 8].

Formal path integrals over the p-forms $\overset{(p)}{B}$ and $\overset{(p-1)}{C}$, corresponding to the classical action (1),

$$I[\overset{(p)}{B}] \equiv I_p = \int D\overset{(p)}{B} \exp i(S[\overset{(p)}{B}]), \qquad (5)$$

and to the classical action (3),

$$I_m[\overset{(p)}{B}, \overset{(p-1)}{C}] = \int D\overset{(p)}{B}\, D\overset{(p-1)}{C} \exp(iS[\overset{(p)}{B}, \overset{(p-1)}{C}]), \qquad (6)$$

contain infinite gauge volumes, related to the GTs (2) and (4) correspondingly.

A gauge fixing function in massless case has usual form: $\overset{(p-1)}{K} = \delta\overset{(p)}{B}$. Separating a gauge group volume from path integral (5), one should input to this integral D-dimensional delta-function of $\overset{(p-1)}{K}$:

$$I_p = \Delta_{p-1} \int D\overset{(p)}{B}\ \tilde{\delta}[\overset{(p-1)}{K}] \exp i(S[\overset{(p)}{B}]),$$

here $\tilde{\delta}[\dots]$ being delta- function (tilde distinguishes it from co-derivative δ). However, it appears that $\tilde{\delta}[\overset{(p-1)}{K}]$ is ill defined, because of co-derivative obeys $\delta^2 = 0$.

We redefine D-dimensional delta-function of $\overset{(p-1)}{K}$, generalizing method presented in [3, 8]. Denoting new generalized delta- function as $\widehat{\delta}\ [...]$, we obtain:

$$\widehat{\delta}\ [\delta\overset{(k)}{V}] = \prod_{i=0}^{k-2} \Delta_i \prod_{s=0}^{(k-3+k_0)/2} \int D\overset{(2s+1-k_0)}{V} \times$$

$$\times \tilde{\delta}[\delta\overset{(2s+3-k_0)}{V} + d\overset{(2s+1-k_0)}{V}] \cdot (k_0 + (1-k_0) \cdot \tilde{\delta}[\delta\overset{(1)}{V}]),$$

where $\Delta_k = \prod_{s=0}^{k} (Det\,\Box_s)^{(k-s+1)\cdot(-1)^{(k-s)}}$, and

$$k_0(k) = (1 + (-1)^k)/2. \qquad (7)$$

Using this new generalized delta-function, we eliminate from path integral (5) infinite gauge volumes, tied with GTs (2) of all levels k, $1 \le k \le p$. Then we obtain an expression for p-order generating functional

$$Z_p = \prod_{k=0}^{p} (Det\,\Box_k)^{\frac{p+1-k}{2}} (-1)^{p+1-k}, \ p \le D,$$

and, in view of a relation $Z[\overset{(p)}{B}] = \exp(i\Gamma[\overset{(p)}{B}])$, an effective action

$$\Gamma_p = -\frac{i}{2} \sum_{k=0}^{p+1} (-1)^{p+1-k} (p+1-k) Tr\ \ln\Box_k. \qquad (8)$$

In massive theory (6), we choose the gauge fixing function as $\overset{(p-1)}{K_m} = \delta\overset{(p)}{B} + m(\overset{(p-1)}{C} + d\overset{(p-2)}{C})$, last term is absent if $(p-2) < 0$, at lower quantization level.

Constructing a “senior” Faddeev-Popov determinant

$$\Delta_{p-1}^{-1} = \int D\overset{(p-1)}{B}\ D\overset{(p-2)}{C}\ \tilde{\delta}[\overset{(p-1)}{K'}],$$

we find out that Δ_{p-1} contains infinite gauge volumes related to the lower GTs levels: $p-2$, $p-3$, $p-4$, ..., up to the $\Delta_0 = Det(\Box_0 + m^2)$ and $\overset{(0)}{K'} = \overset{(0)}{K} + (\Box_0 + m^2)\overset{(0)}{B}$. After extracting from (6) infinite gauge volumes, related to all levels GTs (4), we obtain p-order generating functional in massive theory (with $k_0(p)$ from (7)):

$$Z_p^m = \prod_{n=-1/2}^{+1/2} \prod_{s=0}^{(p-1)/2-n\cdot k_0(p)} Det^n(\Box_{p-2s-n-1/2} + m^2).$$

So, the effective action be

$$\Gamma_p^m = -i \sum_{n=-1/2}^{+1/2} \sum_{s=0}^{(p-1)/2-n\cdot k_0(p)} nTr\ \ln(\Box_{p-2s-n-1/2} + m^2). \qquad (9)$$

4 Quantum equivalence

It is known that massless p-form in D-dimensional space has $\binom{D-2}{p}$ physical degrees of freedom, and massive p-form has $\binom{D-1}{p}$ degrees. It occurs that quantum theories of p-forms reserve the properties of equivalence of classically equivalent theories.

Deriving a difference between EAs of massless p-forms and of (D-p-2)-forms, $\triangle\,\Gamma_p \equiv \Gamma_p - \Gamma_{D-p-2}$, we obtain with the help of (8):

$$\triangle\,\Gamma_p = \frac{i}{2} \sum_{k=0}^{D} (-1)^{p+k} (-k+p+1) Tr\ \ln\Box_k. \qquad (10)$$

Further we use in (10) a relation between operators $(-\Box_k)$ (in Euclidian formulation) and generalized zeta-functions, $Tr\ln(-\Box_k) = -(\zeta_k'(0) + \ln(\mu^2)\zeta_k(0))$, similar relation is hold for massive case. Zeta-function is defined by a formula, which excludes zero modes [9]:

$$\zeta_k(s) = \frac{1}{\Gamma(s)} = \int_0^\infty dt\ t^{s-1} Tr(e^{t\Box_k} - P_k),$$

P_k is a projector onto the space of the zero modes of the operator $\Box_k$.

Using some properties of zeta-functions [9], we can demonstrate that $\triangle\,\Gamma_p$ (10) is equal to zero, that implies quantum equivalence of massless theories of p-forms and of (D-p-2)-forms.

Under the usual definition of the zeta-function that includes zero modes this difference gives the Gauss-Bonnet topological invariant. Corresponding energy-momentum tensors coincide.

In massive case, a difference between EAs (9) of massive p-forms and of (D-p-1)-forms, $\triangle\,\Gamma^m_p \equiv \Gamma^m_p - \Gamma^m_{D-p-1}$, be

$$\triangle\,\Gamma^m_p = \frac{i}{2}(-1)^p \sum_{k=0}^{D}(-1)^k Tr\,\ln\left(\Box_k + m^2\right). \qquad (11)$$

Using an expansion of zeta-function at non-zero mass in power series in massless zeta-function, one gets a relation between operators $(-\Box_p + m^2)$ and generalized zeta-functions. Then we obtain for (11):

$$\triangle\,\Gamma^m_p = -\frac{i}{2}(-1)^p \sum_{k=0}^{D}(-1)^k[\zeta'_k(0,m) + \ln(\mu^2)\zeta_k(0,m)].$$

With relations for zeta-functions [9] we arrive to a conclusion that $\triangle\,\Gamma^m_p = 0$, consequently the EAs of massive p-form models and of (D-p-1)-form models are coincides, and these theories are quantum equivalent.

5 Conclusion

Quantization scheme of massive and massless p-form models in arbitrary D-dimensional curved space has been presented. Gauge invariance in massive theories is restored with the help of the Stückelberg multistep procedure. Effective actions are evaluated in terms of d'Alembertians acting on a p-forms.

We demonstrate a quantum equivalence of classically equivalent theories by means of comparison of EAs of corresponding models. A zeta-function excluding zero modes has been used. It was proven in D-dimensional curved space that massless quantum theory of p-forms is equivalent to theory of (D-p-2)-forms and massive quantum theory of p-forms is equivalent to theory of (D-p-1)-forms.

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КВАНТОВАЯ ЭКВИВАЛЕНТНОСТЬ МАССИВНЫХ И БЕЗМАССОВЫХ МОДЕЛЕЙ P-ФОРМ В ИСКРИВЛЕННОМ D-МЕРНОМ ПРОСТРАНСТВЕ

Рассматриваются массивные и безмассовые p-формы в произвольном D-мерном искривленном пространстве-времени. Выполнено квантование данных моделей, произведена оценка эффективного действия. Результаты представлены в терминах Даламбертианов, действующих на p-формы. Массивные теории p-форм не обладают калибровочной инвариантностью, в отличие от безмассовых теорий. Калибровочная инвариантность восстанавливается с помощью многоступенчатой процедуры Штюкельберга. Сравнение эффективного действия классически эквивалентных теорий (безмассовые квантовые теории p-форм и (D-p-2)-форм, массивные квантовые теории p-форм и (D-p-1)-форм) доказывает квантовую эквивалентность соответствующих моделей. Для доказательства используется дзета-функция с исключенными нулевыми модами.

Ключевые слова: *квантовые поля в искривленном пространстве-времени, p-формы, калибровочные полевые теории, эффективное действие, квантовая эквивалентность.*

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