UDC 530.1; 539.1

MINIMAL MODELS OF INFLATION IN SUPERGRAVITY AND SUPERSTRINGS

S. V. Ketov [a,b,c]

[a] Department of Physics, Tokyo Metropolitan University,
Minami-ohsawa 1-1, Hachioji-shi, Tokyo 192-0397, Japan.
[b] Kavli Institute for the Physics and Mathematics of the Universe (IPMU),
The University of Tokyo, Chiba 277-8568, Japan.
[c] Institute of Physics and Technology, Tomsk Polytechnic University,
30 Lenin Ave., Tomsk 634050, Russia.

E-mail: ketov@tmu.ac.jp

A novel framework is proposed for embedding the natural inflation into the type IIA superstrings compactified on a Calabi-Yau three-fold. Inflaton is identified with axion of the universal hypermultiplet (UH). The other UH scalars (including dilaton) are stabilized by the CY fluxes whose impact can be described by gauging of the abelian isometry associated with the axion. The stabilizing scalar potential is controlled by the integrable three-dimensional Toda equation. The inflationary scalar potential of the UH axion is dynamically generated at a lower scale in the natural inflation via the non-perturbative quantum field effects such as gaugino condensation. The natural inflation has two scales that allow any values of the CMB observables (n_s, r).

Keywords: *inflation, supergravity, superstrings.*

1 Introduction

The most economical, simple and viable inflationary models are the *single-field* (inflaton) theories whose scalar potential is controlled by a one or two parameters. Amongst the most popular models of that type are (i) the *Starobinsky* inflation [1], the *Linde* inflation [2], the *Higgs* inflation [3] and the *natural* inflation [4].

The closed type II strings give the UV completion of quantized gravity, while the "closed string gravity" consist of the closed string zero modes including *metric, dilaton and B-field*, all being universally coupled to other fields. Their effective action (after integration of the string massive modes) gives rise to the (modified) Einstein gravity including the higher-order curvature terms. Those terms in the perturbative string effective action can be computed from either string amplitudes of the massless modes or their equations of motion given by the vanishing RG beta-functions of the *Non-Linear Sigma-model* describing string propagation in a background of the massless modes. However, the coefficients in front of all Ricci- and scalar- curvature dependent terms in the perturbative gravitational string effective action are *ambiguous*, because they are defined around the vacuum with the vanishing Ricci tensor. To resolve the ambiguity, one needs a non-perturbative setup for strings. It is usually unavailable, but there are some exceptions where the crucial role is played by *extended* local supersymmetry. Actually, the N=2 extended local supersymmetry in the critical dimension D=10 is required for consistency of closed (type II) strings, while their CY compactification gives rise to N=2 local supersymmetry in 4D spacetime. The corresponding low-energy string effective action is given by a matter-coupled N=2 supergravity, while its moduli space $\mathcal{M}$ is the direct product $\mathcal{M}_V \otimes \mathcal{M}_H$ of the moduli space $\mathcal{M}_V$ of $h_{1,1}$ N=2 vector multiplets and the moduli space $\mathcal{M}_H$ of $(1+h_{1,2})$ hypermultiplets, in terms of the CY Hodge numbers $h_{1,1}$ and $h_{1,2}$ (the UH is represented by 1 in the $(1+h_{1,2})$).

Inflaton can be interpreted as the *pseudo-Nambu-Goldstone boson* (pNGb) B associated with spontaneous breaking of the rigid scale invariance. When f is a scale of spontaneous breaking of the scale invariance, and Λ is a scale of inflation, a typical pNGb scalar potential takes the form [4]

$$V(B) = \Lambda^4 \left[1 - \cos\left(\frac{B}{f}\right)\right] \ . \tag{1}$$

In string theory, f is of the order of the $M_{\rm Pl}$, whereas Λ originates in particle physics *dynamically*, via gaugino condensation [5]. Our proposal is to identify the axion B of the natural inflation with the B-field of the UH in 4D.

2 UH moduli space

The hypermultiplet moduli space M_H of the CY-compactified 4D, type-IIA closed strings is known to be independent upon the CY complex structure but can receive non-trivial quantum corrections. The perturbative corrections are only possible at the 1-loop string level, being proportional to the CY Euler number [6]. The non-perturbative (instanton) corrections are due to the Euclidean D2-branes wrapped about the CY special (supersymmertic) 3-cycles and due to the solitonic (NS-type) Euclidean 5-branes wrapped about the entire CY space. The 4D instantons due to the wrapped D2-branes are called *D-instantons.*

In quantum 4D, N=2 closed string theory the non-perturbative UH moduli space is different from the classical UH space, as regards both its topology and its metric, because of the non-perturbative d.o.f. in 4D due to the wrapped branes, and because some UH scalars get the non-vanishing VEVs in quantum theory that break the classical symmetries. In addition, the CY flux quantizaton implies quantized brane charges that can be identified with the Noether charges of the *Peccei-Quinn* (PQ) symmetries. It is expected that the string duality symmetry, described by the *discrete* group $SL(2,\mathbb{Z})$, always survives.

The cosmological inflation can be associated with a special region of the quantum UH moduli space. We identify that region by demanding the smallness of the string coupling, where the NS5-brane instantons are suppressed and the axion isometry is preserved.

The quantum gravity corrections are encoded in the quaternionic-Kähler structure of the quantum UH moduli space. When assuming a single isometry survival, the appropriate framework is given by a reformulation of the UH quaternionic-Kähler geometry as the *Einstein-Weyl* geometry with a *negative* scalar curvature, defined by [7]

$$W^{-}_{abcd}=0\ , \quad R_{ab}=\frac{3}{2}\Lambda g_{ab}\ , \quad \Lambda=const.<0\ , \qquad (2)$$

where the W^{-}_{abcd} is the anti-self-dual part of the Weyl tensor, and the R_{ab} is the Ricci tensor of the UH moduli space metric g_{ab}, with $a,b=1,2,3,4$. Given the abelian isometry of the UH metric described by a Killing vector K obeying the equations

$$K^{a;b}+K^{b;a}=0\ , \quad K^2=g_{ab}K^aK^b\geq 0, \qquad (3)$$

one can choose some adapted coordinates, in which all the metric components are independent upon one coordinate (t). Then the Przanowski-Tod theorem [8,9] states that any such metric with the Killing vector ∂_t can be brought into the form

$$\begin{aligned} ds^2_{\rm Tod} &= \frac{1}{\rho^2}\left\{\frac{1}{P}(dt+\hat{\Theta})^2\right. \\ &+ \left. P\left[e^u(d\mu^2+d\nu^2)+d\rho^2\right]\right\} \end{aligned} \qquad (4)$$

in terms of the two potentials, P and u, and the 1-form $\hat{\Theta}$, in local coordinates (t,ρ,μ,ν).

It follows from Eq. (2) that the potential $P(\rho,\mu,\nu)$ is fixed by the second potential u as [9]

$$P=\frac{1}{|\Lambda|}\left(1-\frac{1}{2}\rho\partial_\rho u\right)\ , \qquad (5)$$

whereas the potential $u(\rho,\mu,\nu)$ obeys the 3D *non-linear* equation

$$-(\partial^2_\mu+\partial^2_\nu)u+\partial^2_\rho e^{-u}=0 \qquad (6)$$

that is known as the (integrable) $SU(\infty)$ or 3D continuous *Toda system*. Finally, the 1-form $\hat{\Theta}$ satisfies the *linear* differential equation [9]

$$\begin{aligned} -d\wedge\hat{\Theta} &= (\partial_\nu P)d\mu\wedge d\rho \\ &+ (\partial_\mu P)d\rho\wedge d\nu+\partial_\rho(Pe^{-u})d\nu\wedge d\mu, \end{aligned} \qquad (7)$$

whose integrability condition is just given by Eq. (6). The classical UH metric in the parameterization (4) is obtained by taking

$$\begin{aligned} &P=\frac{3}{2\,|\Lambda|}=const.>0\ , \quad e^{-u}=\rho\ , \\ &d\wedge\hat{\Theta}=d\nu\wedge d\mu\ . \end{aligned} \qquad (8)$$

so that $u=2\phi$. The string coupling is given by the dilaton VEV as $g_{\rm string}=\langle e^{\phi}\rangle$. The classical region of the UH moduli space corresponds to the vanishing $g_{\rm string}$.

The quantum UH moduli space was investigated in Refs. [10–16]. As was found in Refs. [12, 15], a summation of the D-instanton contributions is possible when there is the extended $U(1)\times U(1)$ isometry. In this case the UH metric is governed by the Calderbank-Petersen potential $F(\rho,\eta)$ obeying the equation [17]

$$\rho^2\left(\partial^2_\rho+\partial^2_\eta\right)F=\frac{3}{4}F\ . \qquad (9)$$

Its unique $SL(2,\mathbb{Z})$ modular invariant solution is given by the Eisenstein series $E_{3/2}$. The asymptotical expansion of the Eisenstein series reveals a sum of the classical contribution proportional to $\rho^{-1/2}$, the perturbative string 1-loop contribution proportional to $\zeta(3)\rho^{3/2}$, and the infinite sum of the D-instanton terms indeed [12].

3 CY fluxes and gauging the UH isometry

So far no scalar potential was generated for the UH scalars. As is well known in string theory, the moduli stabilization can be achieved via adding non-trivial fluxes of the NS-NS and RR three-forms in CY [18], while it amounts to *gauging* isometries of the

UH moduli space in the effective 4D, N=2 supergravity [19]. As the abelian gauge field one can employ either gravi-photon of N=2 supergravity multiplet or a vector field of an N=2 matter (abelian) vector multiplet. As a result, the UH gets a non-trivial *scalar* potential whose critical points determine the vacua of the theory [19].

The scalar potential arising from the gauging procedure takes the form [20]

$$V = \frac{9}{2} g^{ab}\partial_a W \partial_b W - 6W^2 \tag{10}$$

in terms of the UH metric g_{ab} and the superpotential W defined by [20]

$$W^2 = \frac{1}{3} dK \wedge {}^*dK - \frac{1}{6} dK \wedge dK \ , \tag{11}$$

where we have introduced the Killing 1-form $K = k_a dq^a$ of the gauged isometry and the Hodge star $(*)$ in any local coordinates (q) on the UH moduli space.

In the parametrization of Eq. (4) we have the Killing vector $K^a = (1, 0, 0, 0)$ that yields the Killing 1-form

$$K = \frac{1}{w^2 P}(dt + \Theta) \ , \tag{12}$$

whose square is given by $K^2 = g_{ab} K^a K^b = g_{tt} = \frac{1}{\rho^2 P}$.

It is straightforward to compute the superpotential squared. We find

$$\begin{aligned} W^2 = \ & \frac{\Lambda^2}{\rho^2} + \frac{1}{12P}\left(3 + 2\left|\Lambda\right| P + \frac{3}{2P}\rho\partial_\rho P\right)^2 \\ & + \frac{3\rho^2 e^{-u}}{4P^3}\left[(\partial_\mu P)^2 + (\partial_\nu P)^2\right] \ . \end{aligned} \tag{13}$$

The first term in the scalar potential (10) is always positive, whereas the second term is always negative, which is similar to the scalar potential in a generic matter-coupled N=1 supergravity [21]. The Minkowski vacua are determined by the fixed points of the scalar potential, related to the poles of the function $P\rho^2$. The existence of *meta-stable de Sitter* vacua was explicitly demonstrated in Refs. [14, 15].

4 Conclusion

We proposed the inflationary scenario in the 4D quantum gravity given by the type IIA closed strings compactified on a Calabi-Yau three-fold. Inflaton was identified with the axion of the Universal Hypermultiplet.

The other (non-inflaton) scalars of the Universal Hypermultiplet (including dilaton) were stabilized by the CY fluxes whose impact was calculated via the gauging procedure of the UH moduli space axion isometry. The latter survives when the NS5-brane instantons are suppressed, i.e. at a small string coupling $g_{\rm string} \ll 1$.

After the stabilization by CY fluxes/gauging, the N=2 local supersymmetry in 4D is unbroken, while axion is still massless and has no scalar potential. However, at a lower scale the axion can get a scalar potential due to some non-perturbative quantum field theory phenomena such as gaugino condensation. The slow-roll natural inflation can, therefore, take place with the scalar potential (1) whose structure is essentially dictated by the pNGb nature of the axion.

It is worth noticing here that the scalar potential (1) of the natural inflation yields the scalar index n_s and the tensor-to-scalar ratio r of the CMB anisotropy as [4]

$$n_s \approx 1 - \frac{M^2_{\rm Pl}}{8\pi f^2} \ , \quad \Lambda \approx 2.2 \cdot 10^{16}\, GeV \left(\frac{r}{0.002}\right)^{1/4} \ . \tag{14}$$

Therefore, the CMB observables (n_s, r) are directly related to the scales (f, Λ) of the natural inflation, respectively.

Acknowledgements

The author thanks S. Alexandrov, E. Kiritsis, A. Sagnotti, A. A. Starobinsky and S. Vandoren for discussions. This work was supported by a Grant-in-Aid of the Japanese Society for Promotion of Science (JSPS) under No. 26400252, the World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan, and the Competitiveness Enhancement Program of the Tomsk Polytechnic University in Russia.

References

[1] Starobinsky A. A., Phys. Lett. **B91** (1980) 99.

[2] Linde A. D., Phys. Lett. **B129** (1983) 177.

[3] Bezrukov F. and Shaposhnikov M., Phys. Lett. **B659** (2008) 703.

[4] Freese K., Frieman J. A. and Olinto A. V., Phys. Rev. Lett. **65** (1990) 3233.

[5] Dine M., Rohm R., Seiberg N. and Witten E., Phys. Lett. **B156** (1985) 55.

[6] Antoniadis I., Minasian R., Theisen S. and Vanhove P., Class. Quant. Grav. **20** (2003) 5079.

[7] Bagger J. and Witten E., Nucl. Phys. **B222** (1983) 1.

[8] Przanowski M., J. Math. Phys. **32** (1991) 1004.

[9] Tod K. P., Lecture Notes in Pure and Appl. Math. **184** (1997) 307.

[10] Ketov S. V., *D-instantons and universal hypermultiplet*, Los Angeles preprint CITUSC-01-046 (unpublished); arXiv:hep-th/0112012.

[11] Ketov S. V., Nucl. Phys. B **604** (2001) 256; arXiv:hep-th/0102099.

[12] Ketov S. V., Nucl. Phys. B **649** (2003) 365; arXiv:hep-th/0209003.

[13] Ketov S. V., Phys. Lett. B **558** (2003) 119; arXiv:hep-th/0302001.

[14] Behrndt K. and Mahapatra S., J. High Energy Phys. **01** (2004) 068.

[15] Davidse M., Saueressig F., Theis U. and Vandoren S., JHEP **0509** (2005) 065.

[16] Alexandrov S., Saueressig F. and Vandoren S., JHEP **0609** (2006) 040.

[17] Calderbank D. M. J. and Pedersen H., *Self-dual Einstein metrics with torus symmetry*, math.DG/0105263.

[18] Douglas M. R. and Kachru S., Rev. Mod. Phys. **79** (2007) 733.

[19] Polchinski J. and Strominger A., Phys. Lett. **388B** (1996) 736.

[20] Behrndt K. and Dall'Agata D., Nucl. Phys **B627** (2002) 357.

[21] Cremmer E., Julia B., Scherk J., Ferrara S., Girardello L. and van Nieuwenhuizen P., Nucl. Phys. **B147** (1979) 105.

Received 12.10.2014

С. В. Кетов

МИНИМАЛЬНЫЕ ИНФЛЯЦИОННЫЕ МОДЕЛИ В ТЕОРИЯХ СУПЕРГРАВИТАЦИИ И СУПЕРСТРУН

Предложено вложение естественной инфляции в ранней Вселенной в теорию IIA суперструн, компактифицированных на многообразиях Калаби-Яу. Инфлатон является аксионом универсального гипермультиплета. Остальные скаляры стабилизированы в результате локализации аксионной симметрии. Метод согласуется с любыми параметрами микроволнового реликтового излучения.

Ключевые слова: *инфляция, супергравитация, теория струн.*

Кетов С. В., доктор физико-математических наук, профессор.
Токийский столичный университет.
Minami-ohsawa 1-1, Hachioji-shi, Tokyo 192-0397, Япония.
Токийский университет.
Chiba 277-8568, Япония.
Томский Политехнический Университет.
пр. Ленина 30, 634050 Томск, Россия.
E-mail: ketov@tmu.ac.jp