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OFF-SHELL INVARIANT SUPER YANG-MILLS WITH GAUGED CENTRAL CHARGES FOR N=D=2 AND N=D=4: "DO WE NEED A CONSTRAINT?"

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We investigate to derive off-shell invariant twisted super Yang-Mills for N=2 in 2-dimensions and N=4 in 4-dimensions with a central charge by super connection ansatz formalism. We find off-shell invariant N=2 algebra with and without an extra constraint in 2-dimensions. On the other hand in 4-dimensions we find off-shell invariant N=4 twisted SUSY algebra including one central charge always with a constraint.

Keywords: *superspace, supersymmetry, Yang-Mills, quantization.*

1 Introduction

It has been a long-standing question: "Can we construct off-shell invariant N=D=4 super Yang-Mills formulation ?" The answer was claimed to be negative especially for the case of R-symmetry SU(4) case where only on-shell invariance was realized [2]. It was, however, claimed later that off-shell invariance was realized for R-symmetry USp(4) case with a central charge [3,4]. However there appeared a constraint in this formulation [5].

There were intensive investigations on N=2 and N=4 SUSY algebra with central charge with a hope that N=D=4 super Yang-Mills can be formulated by superspace formalism [6]. There were also trials by harmonic superspace approach on this question [7, 8].

In the analyses of extended SUSY algebra it has been recognized that the quantization of gauge theory leads to a twisted version of SUSY algebra. It was especially shown that N=2 super Yang-Mills in 4-dimensions can be derived by quantizing topological Yang-Mills with instanton gauge fixing [9]. It has been intensively investigated to find a procedure of extending this formulation into N=4 super Yang-Mills formulation [10].

In dealing with N=D=4 supersymmetry algebra we proposed twisted superspace formulation by Dirac-Kaehler twisting procedure [11–13]. This Dirac-Kaehler twist is equivalent to Marcus twist of N=4 in 4-dimensions [14] among other twisting procedures [15, 16]. The Dirac-Kaehler twisting procedure, however, has nice generalization to other dimensions. Especially for 2-dimensional N=2 super Yang-Mills with central charge we found super connection ansatz where off-shell invariant super Yang-Mills can be formulated with and without a constraint [17].

It has already been formulated as a super connection ansatz where N=D=4 twisted super Yang-Mills with a central charge can be formulated at the off-shell level with a constraint [13]. It is an interesting question to ask if we can formulate N=4 super Yang-Mills in 4-dimensions without constraint by imposing the similar ansatz as 2-dimensional N=2 case leading to a super Yang-Mills formulation without constraint.

Throughout of this paper we use Euclidean formulation of SUSY algebra since we have in mind the application of the formulation into lattice SUSY [18–20].

2 Dirac-Kaehler twisted supersymmetry

We first show how N=D=2 twisted SUSY algebra naturally appears from a quantization of gauge theory. Let's first consider a very simple 2-dimensional abelian BF theory:

$$S = \int d^2x \phi \epsilon^{\mu\nu} \partial_\mu \omega_\nu, \tag{1}$$

which has the following gauge symmetry:

$$\phi = 0, \quad \delta\omega_\mu = \partial_\mu v. \tag{2}$$

After the Lorentz gauge fixing: $\partial^\mu \omega_\mu = 0$, a quantized action leads:

$$S = \int d^2x [\epsilon^{\mu\nu} \phi \partial_\mu \omega_\nu + b \partial^\mu \omega_\mu - i\bar{c} \partial^\mu \partial_\mu c], \tag{3}$$

which has BRST invariance with nilpotent BRST charge $s^2 = 0$. It is interesting to recognize that we can find family of BRST charges s_μ and $\tilde{s}$ which has the following fermionic symmetry at the on-shell level:

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ϕ^A	$s\phi^A$	$s_\mu\phi^A$	$\tilde{s}\phi^A$
ϕ	0	$-\epsilon_{\mu\nu}\partial^\nu\bar{c}$	0
ω_ν	$\partial_\nu c$	0	$-\epsilon_{\nu\rho}\partial^\rho c$
c	0	$-i\omega_\mu$	0
$\bar{c}$	$-ib$	0	$-i\phi$
b	0	$\partial_\mu\bar{c}$	0

On-shell N=D=2 twisted supersymmetry

In fact these family of femionic charges satisfy the following twisted N=D=2 supersymmetry algebra:

$$\{s, s_\mu\} = -i\partial_\mu, \quad \{\tilde{s}, s_\mu\} = i\epsilon_{\mu\nu}\partial^\nu, \tag{4}$$

$$s^2 = \{s, \tilde{s}\} = \tilde{s}^2 = \{s_\mu, s_\nu\} = 0. \tag{5}$$

What is surprising here is that we can find off-shell invariant N=D=2 supersymmetric action by introducing auxiliary fields λ and ρ:

$$S = \int d^2x[\epsilon^{\mu\nu}\phi\partial_\mu\omega_\nu + b\partial^\mu\omega_\mu - i\bar{c}\partial^\mu\partial_\mu c - i\lambda\rho] = \int d^2x s\tilde{s}\frac{1}{2}\epsilon^{\mu\nu}s_\mu s_\nu(-i\bar{c}c),$$

which has the s-exact form of the action with respect to the super charges. It has the following off-shell invariant twisted N=D=2 supersymmetry:

ϕ^A	$s\phi^A$	$s_\mu\phi^A$	$\tilde{s}\phi^A$
ϕ	$i\rho$	$-\epsilon_{\mu\nu}\partial^\nu\bar{c}$	0
ω_ν	$\partial_\nu c$	$-i\epsilon_{\mu\nu}\lambda$	$-\epsilon_{\nu\rho}\partial^\rho c$
c	0	$-i\omega_\mu$	0
$\bar{c}$	$-ib$	0	$-i\phi$
b	0	$\partial_\mu\bar{c}$	$-i\rho$
λ	$\epsilon^{\mu\nu}\partial_\mu\omega_\nu$	0	$-\partial_\mu\omega^\mu$
ρ	0	$-\partial_\mu\phi - \epsilon_{\mu\nu}\partial^\nu b$	0

Off-shell invariant N=D=2 supersymmetry

We call this fermionic symmetry algebra as twisted supersymmetry since the fermionic charges s_A are related with super charges of N=D=2 super symmetry algebra in the following way:

$$\{Q_{\alpha i}, Q_{\beta j}\} = 2\delta_{ij}\gamma^\mu{}_{\alpha\beta}P_\mu,$$
$$Q_{\alpha i} = \left(\mathbf{1}s + \gamma^\mu s_\mu - i\gamma^5\tilde{s}\right)_{\alpha i}, \tag{6}$$

where γ^μ and γ^5 are properly chosen γ-matrices in 2-dimensions [17]. We call this supersymmetry algebra as Dirac-Kaehler twisted supersymmetry algebra [11–13]. This Dirac-Kaehler twisting procedure can be extended into 4-dimensions for N=D=4 supersymmetry algebra [12,13].

We can extend the N=D=2 twisted supersymmetry algebra to include super charges [17]:

$$\{Q_{\alpha i}, Q_{\beta j}\} = 2\delta_{ij}\gamma^\mu_{\alpha\beta}P_\mu + 2\delta_{\alpha\beta}\delta_{ij}U_0 + 2\gamma^5_{\alpha\beta}\gamma^5_{ij}V_5, \tag{7}$$

$$[Q_{\alpha i}, R] = iS_{ij}Q_{\alpha j}, \quad [U_0, \text{any}] = [V_5, \text{any}] = 0, \tag{8}$$

where R is the R-symmetry generator and U_0 and V_5 are considered as central charges. Then twisted supersymmetry algebra with central charges is given by

$$\begin{aligned} &\{s, s_\mu\} = P_\mu\,, \ \{\tilde{s}, s_\mu\} = -\epsilon_{\mu\nu}P_\nu\,, \ \{s, \tilde{s}\} = 0, \\ &s^2 = \tilde{s}^2 = \frac{1}{2}(U_0 - V_5)\,, \ \{s_\mu, s_\nu\} = \delta_{\mu\nu}(U_0 + V_5). \end{aligned} \tag{9}$$

3 Twisted superspace and super connection

We introduce a twisted super field which is expanded by super coordinates θ_A corresponding to super charge s_A:

$$\Phi(x_\mu, \theta_A, z) = \phi(x_\mu, z) + \theta_A\phi_A(x_\mu, z) + \frac{1}{2}\theta_A\theta_B\phi_{AB}(x_\mu, z) + \cdots, \tag{10}$$

where x_μ is the space time coordinate and z is a coordinate (parameter) corresponding to a central charge. Twisted supersymmetry transformation is generated by super chages Q_A and we introduce super derivative $\mathcal{D}_J$ as:

$$\delta_\xi\Phi = \xi_A\mathcal{Q}_A\Phi, \quad \{\mathcal{Q}_I, \mathcal{D}_J\} = 0, \tag{11}$$

where ξ_A is super parameters. In order to consider gauge theory in this twisted superspace we introduce super covariant derivative

$$\nabla_I = \mathcal{D}_I - i\Gamma_I \quad (I = A), \tag{12}$$

where Γ_I can be identified as super connection. We introduce a notation to express the lowest order term with respect to the super coordinates; i.e. $\Phi|_{\theta_A=0} = \Phi| \equiv \phi$. If we generalize the notation of (12) for the gauge covariant derivative with $I = \underline{\mu}, z$ then $\nabla_{\underline{\mu}}| \equiv D_\mu = \partial_\mu - iA_\mu$.

Let us introduce a table notation of (anti-) commuting relations. For example the twisted supersymmetry algebra with central charge in (9) can be read as

	s	s_μ	$\tilde{s}$
s	$\frac{1}{2}(U_0 - V_5)$	P_μ	0
s_ν		$\delta_{\nu\mu}(U_0 + V_5)$	$-\epsilon_{\nu\rho}P^\rho$
$\tilde{s}$			$\frac{1}{2}(U_0 - V_5)$

where a low and a column crossing location of term represents the value of the corresponding anti-commutation relation of super charges.

4 Super connection ansatz and SUSY transformation

Once SUSY algebra is given, it is straightforward to examine the full closure of the SUSY algebra by super connection formalism [8, 10, 12, 21]. For an ansatz of given SUSY algebra all the possible combinations of Jacobi identities give a criteria for a consistency of the full algebra.

Let us consider the super connection Ansatz (A) (Table 1), where we included $\nabla_{\underline{\mu}}$ and ∇_z which are defined in $\{\nabla_A, \nabla_B\}$ in the table. For this ansatz N=D=2 twisted supersymmetry algebra with central charge can be read with an identification, $\nabla_A \to s_A,\quad W = 0,\ \nabla_z \to Z,\ -i\nabla_{\underline{\mu}} \to -i\partial_\mu = P_\mu$:

$$\{s, s_\mu\} = P_\mu\,,\ \{\tilde{s}, s_\mu\} = -\epsilon_{\mu\nu}P_\nu\,,\ \{s, \tilde{s}\} = 0\,,$$

$$s^2 = \tilde{s}^2 = \frac{1}{2}Z\,,\ \{s_\mu, s_\nu\} = \pm\delta_{\mu\nu}Z\,.$$

We next derive all possible non-trivial relations by using graded Jacobi identities until we don't get any new relation. In other words if we get inconsistent relations from the Jacobi identities we consider that the starting super connection ansatz is not taken correctly.

For example the following graded Jacobi identity is satisfied for boson W, fermion ψ and fermion χ:

$$[W, \{\psi, \chi\}] + \{\psi, [\chi, W]\} - \{\chi, [W, \psi]\} = 0. \tag{13}$$

As a concrete example of deriving a non-trivial relation is

$$[\nabla, \{\nabla, \tilde{\nabla}\}] + [\nabla, \{\tilde{\nabla}, \nabla\}] + [\tilde{\nabla}, \{\nabla, \nabla\}] = 0, \tag{14}$$

where $\{\nabla, \tilde{\nabla}\} = 0$. We then obtain the following relation:

$$[\tilde{\nabla}, -iW + \nabla_z] = -i\tilde{\nabla}W + i\tilde{G} = 0 \to \tilde{\nabla}W = \tilde{G}. \tag{15}$$

Similarly we obtain the following relations:

$$\nabla_\mu\nabla W = \epsilon_{\mu\nu}\nabla_\nu\tilde{\nabla}W\,,$$

$$F_{\underline{\mu}} = -i\nabla_\mu W\,, \quad \tilde{F}_{\underline{\mu}} = -i\epsilon_{\mu\nu}\nabla_\nu W\,,$$

$$F_{\mu\underline{\nu}} = \pm i\delta_{\mu\nu}\nabla W \mp i\epsilon_{\mu\nu}\tilde{\nabla}W\,,$$

$$F_{\underline{\mu}\underline{\nu}} = \pm\epsilon_{\mu\nu}\tilde{\nabla}\nabla W + \frac{1}{2}\epsilon_{\mu\nu}\epsilon_{\rho\sigma}\nabla_\rho\nabla_\sigma W\,,$$

$$G = \nabla W\,, \quad \tilde{G} = \tilde{\nabla}W\,, \quad G_\mu = -\nabla_\mu W\,,$$

$$G_{\underline{\mu}} = 2i\nabla_\mu\nabla W - \nabla_{\underline{\mu}}W.$$

We identify component fields of super multiplets as:

$$W| = \phi\,, \quad \nabla W| = \rho\,, \quad \tilde{\nabla}W| = \tilde{\rho}\,, \tag{16}$$

$$\nabla_\mu W| = \lambda_\mu\,, \quad \nabla_z W| = D\,, \quad G_{\underline{\mu}}| = g_\mu.$$

SUSY transformations of these component fields can be obtained by taking (anti-)commutator with the field and identifying the original Jacobi identities. For example the following SUSY transformation can be identified as

$$s\phi = s(W|) \equiv \mathcal{Q}W| = \mathcal{D}W| = \mathcal{D}W| - i[\Gamma, W]| \\ = \nabla W| = \rho, \tag{17}$$

where Wess-Zumino gauge is chosen here: $\Gamma = 0$. In a similar way we can derive all the N=D=2 SUSY transformations for these component fields (Table 2).

We can find off-shell invariant action under this SUSY transformation as:

$$S = \int d^2x\mathrm{Tr}\Big(\pm\frac{1}{2}(D_\mu\phi)^2 - \frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}D^2 \pm \frac{1}{2}g_\mu^2 \\ \mp 2i\lambda_\mu(D_\mu\rho - \epsilon_{\mu\nu}D_\nu\tilde{\rho}) - i\phi\{\rho, \rho\} \\ - i\phi\{\tilde{\rho}, \tilde{\rho}\} \pm i\phi\{\lambda_\mu, \lambda_\mu\}\Big). \tag{18}$$

In order to confirm off-shell closure of the algebra we need the following non-trivial constraint:

$$iD_\mu g_\mu \mp [\phi, D] - \{\lambda_\mu, \lambda_\mu\} \mp \{\rho, \rho\} \mp \{\tilde{\rho}, \tilde{\rho}\} = 0, \tag{19}$$

which cannot be obtained as one of Jacobi identities. For Abelian case the constraint becomes simple as: $\partial_\mu g_\mu = 0$, and can be solved as $g_\mu = \epsilon_{\mu\nu}\partial_\nu B$. We can then obtain off-shell SUSY invariant action without constraint:

$$S = \int d^2x\mathrm{Tr}\Big(\pm\frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}D^2 \pm \frac{1}{2}(\partial_\mu B)^2 \\ \mp 2i\lambda_\mu(\partial_\mu\rho - \epsilon_{\mu\nu}\partial_\nu\tilde{\rho}) \\ + e\Big(\frac{1}{2}\phi\epsilon_{\mu\nu}F_{\mu\nu} + 2\rho\tilde{\rho} + BD + \epsilon_{\mu\nu}\lambda_\mu\lambda_\nu\Big)\Big). \tag{20}$$

The constraint (19) cannot be solved for the non-Abelian case in a local way. This example is similar to the N=D=4 super Yang-Mills with R-symmetry USp(4) case where one constraint appears as a extra condition.

5 Off-shell N=D=2 SUSY invariant action without constraint

In order to find other ansatz which doesn't generate a constrained equation like (19), we impose another ansatz as in the following:

	∇	$\tilde{\nabla}$	∇_ν	$\nabla_{\underline{\nu}}$	∇_z
∇	0	0	$-i(\nabla_{\underline{\nu}} + F_\nu)$	$-iF_{\underline{\nu}}$	iG
$\tilde{\nabla}$		0	$i\epsilon_{\nu\rho}(\nabla_{\underline{\rho}} - F_\rho)$	$-i\tilde{F}_{\underline{\nu}}$	$i\tilde{G}$
∇_μ			$\delta_{\mu\nu}\nabla_z$	$-iF_{\mu\underline{\nu}}$	iG_μ
$\nabla_{\underline{\mu}}$				$-iF_{\underline{\mu}\underline{\nu}}$	$iG_{\underline{\mu}}$
∇_z					0

N=D=2 Ansatz (B)

From this ansatz we obtain the following relations by using graded Jacobi identities:

$$
\begin{aligned}
&\nabla F_\mu = \epsilon_{\mu\nu}\tilde\nabla F_\nu\,, \quad \nabla_\mu F_\nu + \nabla_\nu F_\mu = \delta_{\mu\nu}\nabla_\rho F_\rho\,,\\
&G_\mu = 0\,, \quad F_{\underline{\mu}} = -i\nabla F_\mu\,, \quad \tilde F_{\underline{\mu}} = i\tilde\nabla F_\mu\,,\\
&F_{\mu\underline{\nu}} = -\frac{i}{2}\delta_{\mu\nu}(\nabla_\rho F_\rho - G) + \frac{i}{2}\epsilon_{\mu\nu}(\epsilon_{\rho\sigma}\nabla_\rho F_\sigma - \tilde G)\,,\\
&F_{\underline{\mu}\underline{\nu}} = \nabla_\mu\nabla F_\nu - \nabla_\nu\nabla F_\mu + i[F_\mu, F_\nu] + \frac{1}{2}\epsilon_{\mu\nu}\nabla\tilde G\,,\\
&\nabla G = \tilde\nabla\tilde G = \nabla\tilde G + \tilde\nabla G = 0\,,\\
&\nabla_z F_\mu = \frac{1}{2}(\nabla_\mu G - \epsilon_{\mu\nu}\nabla_\nu\tilde G)\,,\\
&G_{\underline{\mu}} = \frac{i}{2}(\nabla_\mu G + \epsilon_{\mu\nu}\nabla_\nu\tilde G)\,. \qquad (21)
\end{aligned}
$$

In this ansatz we make the following identification of component fields of N=D=2 super multiplet:

$$
\begin{aligned}
&F_\mu| = \phi_\mu\,, \quad \nabla F_\mu| = \lambda_\mu\,, \quad \nabla_\mu F_\nu| = \frac{1}{2}(\delta_{\mu\nu}\rho + \epsilon_{\mu\nu}\tilde\rho)\,,\\
&\nabla_\mu\nabla F_\mu| = D. \qquad (22)
\end{aligned}
$$

SUSY transformation of these component fields can be obtained as in the previous example – see Table 3. In this ansatz the super charges have the following nilpotent nature:

$$
s^2 = \tilde s^2 = s_\pm^2 = 0, \quad (s_\pm \equiv s_1 \pm i s_2). \qquad (23)
$$

In order to derive off-shell SUSY invariant action it is convenient to find a s-exact form of an action. To find this type of action we recognized that among the relations in (21) the relations on G and $\tilde G$ are crucial to be solved. We actually found a solution of $\nabla\tilde G = \tilde\nabla G = 0$ as:

$$
G = a\epsilon_{\mu\nu}\nabla^-_{\underline{\mu}}\nabla F_\nu\,, \quad \tilde G = -a\nabla^+_{\underline{\mu}}\nabla F_\mu, \qquad (24)
$$

where a is a constant. We then found a off-shell SUSY invariant action:

$$
\begin{aligned}
S = \int d^2x \mathrm{Tr}\{&\frac{1}{2}(D_\mu\phi_\nu)^2 + \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}D^2 - i\rho D^+_\mu\lambda_\mu\\
&- i\tilde\rho\epsilon_{\mu\nu}D^-_\mu\lambda_\nu - \frac{1}{4}[\phi_\mu,\phi_\nu]^2 - ia^{-1}G|\tilde G|\}, \qquad (25)
\end{aligned}
$$

where $G|$ and $\tilde G|$ are fermionic fields. The action satisfies N=2 SUSY invariance at the off-shell level without constraint:

$$
sS = \tilde sS = s_\pm S = 0. \qquad (26)
$$

6 N=D=4 super Yang-Mills formulation with central charges

Most general N=4 supersymmetry algebra in 4-dimensions can be given by

$$
\begin{aligned}
\{Q_{\alpha i}, Q_{\beta j}\} &= 2C^{-1}_{ij}(\gamma^\mu C)_{\alpha\beta}P_\mu\\
&+ 2C_{\alpha\beta}(C^{-1}_{ij}U_0 + (C^{-1}\gamma_5)_{ij}U_5)\\
&+ 2(\gamma_5 C)_{\alpha\beta}(C^{-1}_{ij}V_0 + (C^{-1}\gamma_5)_{ij}V_5). \qquad (27)
\end{aligned}
$$

The N=D=4 super charges $Q_{\alpha i}$ can be decomposed into twisted super charges:

$$
Q_{\alpha i} = \frac{i}{\sqrt{2}}(1s + \gamma^\mu s_\mu + \frac{1}{2}\gamma^{\mu\nu}s_{\mu\nu} + \tilde\gamma^\mu\tilde s_\mu + \gamma^5\tilde s)_{\alpha i}. \qquad (28)
$$

In this Dirac-Kaehler twisting mechanism the spinor suffix α and the extended SUSY suffix i are rotated by angular momentum generators $J_{\mu\nu}$ and R-symmetry generators $R_{\mu\nu}$, respectively. In this way the fermionic super charges having spinor suffix change into the twisted super charges having scalar, vector, tensor. suffix which are now rotated by a new rotation generators $J'_{\mu\nu}$. They have the following relation [11–13]:

$$
J'_{\mu\nu} \equiv J_{\mu\nu} + R_{\mu\nu}. \qquad (29)
$$

The N=D=4 Dirac-Kaehler twisted SUSY algebra corresponding to (27) is given by

$$
\begin{aligned}
&\{s, s_\mu\} = \{\tilde s, \tilde s_\mu\} = P_\mu\,,\\
&\{s_\mu, s_{\rho\sigma}\} = -\delta_{\mu\nu\rho\sigma}P_\nu\,, \ \{\tilde s_\mu, s_{\rho\sigma}\} = \epsilon_{\mu\nu\rho\sigma}P_\nu\,,\\
&\{s, \tilde s_\mu\} = \{\tilde s, s_\mu\} = \{s, s_{\mu\nu}\} = \{\tilde s, s_{\mu\nu}\} = 0\,,\\
&2s^2 = 2\tilde s^2 = U_0 + V_5\,,\\
&\{s_\mu, s_\nu\} = \{\tilde s_\mu, \tilde s_\nu\} = \delta_{\mu\nu}(U_0 - V_5)\,,\\
&\{s, \tilde s\} = U_5 + V_0\,, \quad \{s_\mu, \tilde s_\nu\} = \delta_{\mu\nu}(U_5 - V_0)\,,\\
&\{s_{\mu\nu}, s_{\rho\sigma}\} = \delta_{\mu\nu\rho\sigma}(U_0 + V_5) - \epsilon_{\mu\nu\rho\sigma}(U_5 + V_0)\,,
\end{aligned}
$$

where $\delta_{\mu\nu\rho\sigma} \equiv \delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho}$ and $\epsilon_{\mu\nu\rho\sigma}$ is Euclidean ϵ-tensor.

We first investigate the case of ansatz where no central charge is inserted (Table 4). Graded Jacobi identities for this ansatz lead the following relations:

$$
\begin{aligned}
&\nabla W = \tilde\nabla W = \nabla_{\mu\nu}W = 0\,, \quad \nabla_\mu F = \tilde\nabla_\mu F = 0\,,\\
&-\delta_{\mu\nu}\nabla F = \tilde\nabla_\mu F_\nu + \tilde\nabla_\nu F_\mu\,, \quad \delta_{\mu\nu}\tilde\nabla F = \nabla_\mu F_\nu + \nabla_\nu F_\mu\,,\\
&\nabla_{[\mu}F_{\nu]} = \epsilon_{\mu\nu\rho\sigma}\tilde\nabla_\rho F_\sigma\,,\\
&\nabla_{\mu\nu}F_\rho = -\delta_{\mu\nu\rho\sigma}\nabla F_\sigma + \epsilon_{\mu\nu\rho\sigma}\tilde\nabla F_\sigma\,,\\
&F_{\underline{\mu}} = -i\nabla F_\mu\,, \quad \tilde F_{\underline{\mu}} = i\tilde\nabla F_\mu\,,\\
&\nabla_\mu W = -2\tilde\nabla F_\mu\,, \quad \tilde\nabla_\mu W = 2\nabla F_\mu\,,\\
&F_{\mu\underline{\nu}} = -\frac{i}{2}(\delta_{\mu\nu}\tilde\nabla F - \delta_{\mu\nu\rho\sigma}\nabla_\rho F_\sigma)\,,\\
&\tilde F_{\mu\underline{\nu}} = -\frac{i}{2}(\delta_{\mu\nu}\nabla F + \delta_{\mu\nu\rho\sigma}\tilde\nabla\rho F_\sigma)\,,\\
&F_{\mu\nu\underline{\rho}} = -i\delta_{\mu\nu\rho\sigma}\nabla F_\sigma - i\epsilon_{\mu\nu\rho\sigma}\tilde\nabla F_\sigma.
\end{aligned}
$$

We define component fields of N=4 super multiplets as

$$F_\mu| = \phi_\mu\,, \quad W| = A\,, \quad F| = B\,,$$
$$\nabla F_\mu| = \lambda_\mu\,, \quad \tilde{\nabla} F_\mu| = \tilde{\lambda}_\mu\,,$$
$$\nabla_\mu F_\nu| = \delta_{\mu\nu}\rho + \rho_{\mu\nu}\,, \quad \tilde{\nabla}_\mu F_\nu| = \delta_{\mu\nu}\tilde{\rho} + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\rho_{\rho\sigma}.$$

We can then obtain on-shell closed SUSY transformation of these fields as shown in Tables 5, 6. Then we obtain on-shell invariant N=4 super Yang-Mills action without central charge having SU(4) R-symmetry:

$$S = \int d^4x \mathrm{Tr}\{\frac{1}{4}(D_\mu\phi_\nu)^2 + \frac{1}{8}F_{\mu\nu}^2 + \frac{1}{8}D_\mu^+ A D_\mu^- B$$
$$+ \frac{1}{8}D_\mu^- A D_\mu^+ B - \frac{1}{8}[\phi_\mu, \phi_\nu]^2 + \frac{1}{16}[A, B]^2$$
$$- i\lambda_\mu(D_\mu^+\rho - D_\nu^-\rho_{\mu\nu} - [B, \tilde{\lambda}_\mu]) - i\tilde{\lambda}_\mu(D_\mu^-\tilde{\rho}$$
$$- \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}D_\nu^+\rho_{\rho\sigma}) + i\tilde{\rho}[A, \rho] - \frac{i}{8}\epsilon_{\mu\nu\rho\sigma}A\{\rho_{\mu\nu}, \rho_{\rho\sigma}\}\}.$$

We now investigate the super connection ansatz of N=D=4 with a central charge given by Table 7.

The corresponding twisted SUSY algebra with a central charge Z is given by

$$\{s, s_\mu\} = \{\tilde{s}, \tilde{s}_\mu\} = P_\mu\,, \quad \{s_\mu, s_{\rho\sigma}\} = -\delta_{\mu\nu\rho\sigma}P_\nu\,,$$
$$\{\tilde{s}_\mu, s_{\rho\sigma}\} = \epsilon_{\mu\nu\rho\sigma}P_\nu\,,$$
$$\{s, \tilde{s}_\mu\} = \{\tilde{s}, s_\mu\} = \{s, s_{\mu\nu}\} = \{\tilde{s}, s_{\mu\nu}\} = 0\,,$$
$$2s^2 = 2\tilde{s}^2 = 0\,, \ \{s_\mu, s_\nu\} = \{\tilde{s}_\mu, \tilde{s}_\nu\} = 0\,, \ \{s, \tilde{s}\} = Z\,,$$
$$\{s_\mu, \tilde{s}_\nu\} = \mp\delta_{\mu\nu}Z\,, \quad \{s_{\mu\nu}, s_{\rho\sigma}\} = -\epsilon_{\mu\nu\rho\sigma}Z,$$

where $Z = U_5$ for $+$ and $Z = V_0$. Graded Jacobi identities lead:

$$\nabla_{(\mu}F_{\nu)} = \delta_{\mu\nu}\tilde{\nabla}W\,, \qquad \tilde{\nabla}_{(\mu}F_{\nu)} = -\delta_{\mu\nu}\nabla W\,,$$
$$\nabla F_\mu = \pm\frac{1}{2}\tilde{\nabla}_\mu W\,, \qquad \tilde{\nabla}F_\mu = \mp\frac{1}{2}\nabla_\mu W\,,$$
$$\nabla_{\mu\nu}W = -\epsilon_{\mu\nu\rho\sigma}\nabla_\rho F_\sigma\,, \quad \nabla_{[\mu}F_{\nu]} = \epsilon_{\mu\nu\rho\sigma}\tilde{\nabla}_\rho F_\sigma\,,$$
$$\tilde{\nabla}_{[\mu}F_{\nu]} = \epsilon_{\mu\nu\rho\sigma}\nabla_\rho F_\sigma\,,$$
$$\nabla_{\mu\nu}F_\rho = -\delta_{\mu\nu\rho\sigma}\nabla F_\sigma + \epsilon_{\mu\nu\rho\sigma}\tilde{\nabla}F_\sigma\,,$$
$$F_{\underline{\mu}} = -i\nabla F_\mu\,, \quad \tilde{F}_{\underline{\mu}} = i\tilde{\nabla}F_\mu\,, \quad F_{\mu\underline{\nu}} = -i\nabla_\nu F_\mu\,,$$
$$\tilde{F}_{\mu\underline{\nu}} = i\tilde{\nabla}_\nu F_\mu\,, \quad F_{\mu\nu\underline{\rho}} = -i(\delta_{\mu\nu\rho\sigma}\nabla F_\sigma + \epsilon_{\mu\nu\rho\sigma}\tilde{\nabla}F_\sigma)\,,$$
$$F_{\underline{\mu}\underline{\nu}} = \nabla_{[\mu}\nabla F_{\nu]} + i[F_\mu, F_\nu]\,,$$
$$G = \tilde{G} = G_{\mu\nu} = 0\,, \quad G_\mu = 2\tilde{\nabla}F_\mu\,, \quad \tilde{G}_\mu = -2\nabla F_\mu\,,$$
$$G_{\underline{\mu}} = \frac{i}{2}(\nabla G_\mu + \tilde{\nabla}\tilde{G}_\mu)\,,$$
$$ZF_\mu = \frac{1}{2}(\nabla G_\mu - \tilde{\nabla}\tilde{G}_\mu)\,, \quad ZW = 2i\nabla_{\underline{\mu}}F_\mu + 2\nabla\tilde{\nabla}W$$

We define component fields of N=4 super multiplets:

$$F_\mu| = \phi_\mu\,, \quad W| = A\,, \quad \nabla F_\mu| = \lambda_\mu\,, \quad \tilde{\nabla}F_\mu| = \tilde{\lambda_\mu}\,,$$
$$\nabla_\mu F_\nu| = \delta_{\mu\nu}\rho + \rho_{\mu\nu}\,, \quad \tilde{\nabla_\mu}F_\nu| = \delta_{\mu\nu}\tilde{\rho} + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\rho_{\rho\sigma}\,,$$
$$\nabla\tilde{\nabla}W| = H\,, \quad G_{\underline{\mu}}| = g_\mu\,, \quad \nabla_z F_\mu| = H_\mu.$$

N=4 SUSY transformation of these component fields are given by Tables 8, 9. For the off-shell closure of the above SUSY algebra we need the following constraint:

$$iD_\mu g_\mu + [\phi_\mu, H_\mu] \mp 2\{\rho, \tilde{\rho}\} + 2\{\lambda_\mu, \tilde{\lambda}_\mu\}$$
$$\mp \frac{1}{4}\epsilon_{\mu\nu\rho\sigma}\{\rho_{\mu\nu}, \rho_{\rho\sigma}\} \pm \frac{i}{2}D_\mu^+ D_\mu^- A \pm \frac{1}{2}[A, H] = 0. \quad (30)$$

Off-shell twisted N=4 SUSY invariant action in this case is given by

$$S = \int d^4x \mathrm{Tr}\Big(\frac{1}{2}D_\mu\phi_\nu D_\mu\phi_\nu + \frac{1}{4}F_{\mu\nu}^2 \pm \frac{1}{2}(g_\mu^2 + H_\mu^2)$$
$$+ H(iD_\mu\phi_\mu + \frac{1}{2}H) - \frac{1}{2}(D_\mu\phi_\mu)^2$$
$$- 2i\rho D_\mu^+\lambda_\mu - 2i\tilde{\rho}D_\mu^-\tilde{\lambda}_\mu - 2i\rho_{\mu\nu}(D_\mu^-\lambda_\nu$$
$$+ \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}D_\rho^+\tilde{\lambda}_\sigma) - 2iA\{\lambda_\mu, \tilde{\lambda}_\mu\} - \frac{1}{4}[\phi_\mu, \phi_\nu]^2\Big),$$

where the constraint (30) is crucial to prove the off-shell invariance of SUSY for this action.

We now try another N=D=4 twisted SUSY ansatz which has similarity with the N=2 Ansatz (B) leading no constraint in 2-dimensions (Table 10). Graded Jacobi identities lead:

$$\nabla_{(\mu}F_{\nu)} = \delta_{\mu\nu}\tilde{\nabla}W\,, \qquad \tilde{\nabla}_{(\mu}F_{\nu)} = -\delta_{\mu\nu}\nabla W\,,$$
$$\nabla F_\mu = \pm\frac{1}{2}\tilde{\nabla}_\mu W\,, \qquad \tilde{\nabla}F_\mu = \mp\frac{1}{2}\nabla_\mu W\,,$$
$$\nabla_{\mu\nu}W = -\epsilon_{\mu\nu\rho\sigma}\nabla_\rho F_\sigma\,, \quad \nabla_{[\mu}F_{\nu]} = \epsilon_{\mu\nu\rho\sigma}\tilde{\nabla}_\rho F_\sigma\,,$$
$$\tilde{\nabla}_{[\mu}F_{\nu]} = \epsilon_{\mu\nu\rho\sigma}\nabla_\rho F_\sigma\,,$$
$$\nabla_{\mu\nu}F_\rho = -\delta_{\mu\nu\rho\sigma}\nabla F_\sigma + \epsilon_{\mu\nu\rho\sigma}\tilde{\nabla}F_\sigma\,,$$
$$F_{\underline{\mu}} = -i\nabla F_\mu\,, \quad \tilde{F}_{\underline{\mu}} = i\tilde{\nabla}F_\mu\,, \quad F_{\mu\underline{\nu}} = -i\nabla_\nu F_\mu\,,$$
$$\tilde{F}_{\mu\underline{\nu}} = i\tilde{\nabla}_\nu F_\mu\,, \quad F_{\mu\nu\underline{\rho}} = -i(\delta_{\mu\nu\rho\sigma}\nabla F_\sigma + \epsilon_{\mu\nu\rho\sigma}\tilde{\nabla}F_\sigma)\,,$$
$$F_{\underline{\mu}\underline{\nu}} = \nabla_{[\mu}\nabla F_{\nu]} + i[F_\mu, F_\nu]\,,$$
$$G = \tilde{G} = G_{\mu\nu} = 0\,, \quad G_\mu = 2\tilde{\nabla}F_\mu\,, \quad \tilde{G}_\mu = -2\nabla F_\mu\,,$$
$$G_{\underline{\mu}} = \frac{i}{2}(\nabla G_\mu + \tilde{\nabla}\tilde{G}_\mu)\,,$$
$$ZF_\mu = \frac{1}{2}(\nabla G_\mu - \tilde{\nabla}\tilde{G}_\mu)\,, \quad ZW = 2i\nabla_{\underline{\mu}}F_\mu + 2\nabla\tilde{\nabla}W$$

We define component fields of N=D=4 super multiplets as

$$F_\mu| = \phi_\mu\,, \quad W| = A\,, \quad \nabla F_\mu| = \lambda_\mu\,, \quad \tilde{\nabla}F_\mu| = \tilde{\lambda}_\mu\,,$$
$$\nabla_\mu F_\nu| = \delta_{\mu\nu}\rho + \rho_{\mu\nu}\,, \quad \tilde{\nabla}_\mu F_\nu| = \delta_{\mu\nu}\tilde{\rho} + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\rho_{\rho\sigma}\,,$$
$$\nabla\tilde{\nabla}W| = H\,, \quad G_{\underline{\mu}}| = g_\mu\,, \quad ZF_\mu| = H_\mu\,, \quad ZW| = K.$$

Twisted SUSY transformation of these component fields are given by Tables 11, 12, 13. For a closure of SUSY algebra for this ansatz we again need the following constraint:

$$
\begin{aligned}
& iD_\mu g_\mu - [\phi_\mu, H_\mu] - \frac{i}{2} D_\mu D_\mu A - \frac{i}{2}[\phi_\mu, [\phi_\mu, A]] \\
& \mp \frac{1}{4}[A, K] \mp \{\rho, \rho\} \mp \{\tilde{\rho}, \tilde{\rho}\} \mp \frac{1}{2}\{\rho_{\mu\nu}, \rho_{\mu\nu}\} \\
& - \{\lambda_\mu, \lambda_\mu\} - \{\tilde{\lambda}_\mu, \tilde{\lambda}_\mu\} = 0. \qquad (31)
\end{aligned}
$$

For the case of one central charge we have tried all possible super connection ansatz. We have found out two possible consistent Ansatz (B) and (C) but for both cases we need a constraint equation for the off-shell closure of N=D=4 twisted algebra with a central charge. Off-shell N=D=4 twisted SUSY invariant action for this Ansatz (C) can be given by

$$
\begin{aligned}
S = & \int d^4x \mathrm{Tr}\Big(\frac{1}{2} D_\mu \phi_\nu D_\mu \phi_\nu - \frac{1}{4} F_{\mu\nu}^2 \pm \frac{1}{2}(g_\mu - D_\mu A)^2 \\
& \mp \frac{1}{2}(H_\mu - i[A, \phi_\mu])^2 - \frac{1}{8}K^2 - 2i\rho(D_\mu \lambda_\mu \\
& - [\phi_\mu, \tilde{\lambda}_\mu]) - 2i\tilde{\rho}(D_\mu \tilde{\lambda}_\mu + [\phi_\mu, \lambda_\mu]) - 2i\rho_{\mu\nu}(D_\mu \lambda_\nu \\
& + [\phi_\mu, \tilde{\lambda}_\nu]) - i\epsilon_{\mu\nu\alpha\beta}\rho_{\mu\nu}(D_\alpha \tilde{\lambda}_\beta - [\phi_\alpha, \lambda_\beta]) \\
& \pm iA(\{\lambda_\mu, \lambda_\mu\} + \{\tilde{\lambda}_\mu, \tilde{\lambda}_\mu\}) + \frac{1}{4}[\phi_\mu, \phi_\nu]^2\Big). \qquad (32)
\end{aligned}
$$

7 Equivalence of the Ansatz (B) and Ansatz (C)

Consider the general Ansatz of N=D=4 (Tables 14, 15). Define the following new connections and curvatures:

$$
\begin{aligned}
& \nabla^{\mathrm{new}} = \frac{1}{\sqrt{2}}(-i\nabla + \tilde{\nabla}), \tilde{\nabla}^{\mathrm{new}} = \frac{1}{\sqrt{2}}(\nabla - i\tilde{\nabla}), \\
& \nabla_\mu^{\mathrm{new}} = \frac{1}{\sqrt{2}}(i\nabla_\mu + \tilde{\nabla}_\mu), \tilde{\nabla}_\mu^{\mathrm{new}} = \frac{1}{\sqrt{2}}(\nabla_\mu + i\tilde{\nabla}_\mu), \\
& \nabla_{\mu\nu}^{\mathrm{new}} = \frac{1}{\sqrt{2}}(-i\nabla_{\mu\nu} - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\nabla_{\rho\sigma}), \qquad (33)
\end{aligned}
$$

$$
\begin{aligned}
& X_0^{\mathrm{new}} = -iX'_0\,, \; X_\mu^{\mathrm{new}} = iX'_\mu\,, \; X_5^{\mathrm{new}} = -iX'_5 \qquad (34) \\
& X_0'^{\mathrm{new}} = -iX_0\,, \; X_\mu'^{\mathrm{new}} = iX_\mu\,, \; X_5'^{\mathrm{new}} = -iX_5
\end{aligned}
$$

It turns out that this Ansatz (D) and the ansatz given by the above new system has exactly the same form. Surprisingly the Ansatz (B) and Ansatz (C) have exactly the same relations as this new and old system. In other words these ansatz are essentially the same and thus both of cases naturally need the constraint relation for the N=4 SUSY closure.

8 Conclusion and Discussions

In 2-dimension we found two types of super connection ansatz which realize off-shell closure of N=2 twisted SUSY including central charge with and without constraint. Off-shell twisted SUSY invariant actions are found for each ansatz. On the other hand in 4-dimension we examined two possible ansatz of N=4 twisted SUSY algebra with a central charge and we found that both of ansatz having similarity with the 2-dimensional ansatz need a constraint equation for the off-shell closure of N=4 twisted SUSY algebra. In fact we found that these two ansatz are essentially equivalent to each other.

We have thus investigated a possibility of off-shell twisted N=D=4 invariant super Yang-Mills formulation without a constraint by super connection formalism. As far as N=4 twisted SUSY algebra with one super charge is concerned a constraint is inevitable for the off-shell closure of the algebra. We consider that this may be related to the fact that 10-dimensional N=1 super Yang-Mills theory can be formulated only at the on-shell level. N=D=4 super Yang-Mills can be dimensionally reduced from the 10-dimensional N=1 formulation and the on-shell nature could be kept invariant in the dimensionally reduced formulation and thus may lead to off-shell closure but with a constraint in 4-dimensions.

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	∇	$\tilde{\nabla}$	∇_ν	$\nabla_{\underline{\nu}}$	∇_z
∇	$-iW+\nabla_z$	0	$-i\nabla_{\underline{\nu}}$	$-iF_{\underline{\nu}}$	iG
$\tilde{\nabla}$		$\nabla_z - iW$	$i\epsilon_{\nu\rho}\nabla_{\underline{\rho}}$	$-i\tilde{F}_{\underline{\nu}}$	$i\tilde{G}$
∇_μ			$\pm\delta_{\mu\nu}(iW+\nabla_z)$	$-iF_{\mu\underline{\nu}}$	iG_μ
$\nabla_{\underline{\mu}}$				$-iF_{\underline{\mu\nu}}$	$iG_{\underline{\mu}}$
∇_z					0

Table 1. N=D=2 Ansatz (A)

	s	s_μ	$\tilde{s}$	Z
ϕ	ρ	λ_μ	$\tilde{\rho}$	D
A_ν	$-i\lambda_\nu$	$\pm i\delta_{\mu\nu}\rho \mp i\epsilon_{\mu\nu}\tilde{\rho}$	$-i\epsilon_{\nu\rho}\lambda_\rho$	g_ν
λ_ν	$\frac{i}{2}(g_\nu - D_\nu\phi)$	$\pm\frac{1}{2}\delta_{\mu\nu}D + \frac{1}{2}F_{\mu\nu}$	$-\frac{i}{2}\partial_{\nu\rho}(g_\rho - D_\rho\phi)$	$-iD_\nu\rho + i\epsilon_{\nu\rho}D_\rho\tilde{\rho} - i[\phi,\lambda_\nu]$
ρ	$\frac{1}{2}D$	$-\frac{i}{2}(g_\mu + D_\mu\phi)$	$\mp\frac{1}{4}\epsilon_{\mu\nu}F_{\mu\nu}$	$\mp iD_\mu\lambda_\mu + i[\phi,\rho]$
$\tilde{\rho}$	$\pm\frac{1}{4}\epsilon_{\mu\nu}F_{\mu\nu}$	$\frac{i}{2}\epsilon_{\mu\nu}(g_\nu + D_\nu\phi)$	$\frac{1}{2}D$	$\mp i\epsilon_{\mu\nu}D_\mu\lambda_\nu + i[\phi,\tilde{\rho}]$
D	$\mp iD_\mu\lambda_\mu$	$i\epsilon_{\mu\nu}D_\nu\tilde{\rho} - iD_\mu\rho$	$\mp i\epsilon_{\mu\nu}D_\mu\lambda_\nu$	$\pm D_\mu g_\mu \mp D_\mu D_\mu\phi$ $\pm 2i\{\lambda_\mu,\lambda_\mu\} + i[\phi, D]$
g_ν	$\epsilon_{\nu\rho}D_\rho\tilde{\rho} - [\phi,\lambda_\nu]$	$\epsilon_{\mu\sigma}\epsilon_{\nu\rho}D_\rho\lambda_\sigma$ $\mp\delta_{\mu\nu}[\phi,\rho] \pm \epsilon_{\mu\nu}[\phi,\tilde{\rho}]$	$-\epsilon_{\nu\rho}(D_\rho\rho + [\phi,\lambda_\rho])$	$\pm D_\rho F_{\nu\rho} - 2\epsilon_{\nu\rho}\{\lambda_\rho,\tilde{\rho}\}$ $-2\{\lambda_\nu,\rho\} + i[\phi, D_\nu\phi]$

Table 2. N=D=2 SUSY transformation for Ansatz (A)

	s	s_μ	$\tilde{s}$	Z
ϕ_ν	λ_ν	$\frac{1}{2}(\delta_{\mu\nu}\rho + \epsilon_{\mu\nu}\tilde{\rho})$	$-\epsilon_{\nu\rho}\lambda_\rho$	$\frac{1}{2}(\nabla_\nu G\vert - \epsilon_{\nu\rho}\nabla_\rho\tilde{G}\vert)$
A_ν	$-i\lambda_\nu$	$-\frac{i}{2}\delta_{\mu\nu}\rho + \frac{i}{2}\epsilon_{\mu\nu}\tilde{\rho}$ $+\frac{i}{2}\delta_{\mu\nu}G\vert - \frac{i}{2}\epsilon_{\mu\nu}\tilde{G}\vert$	$-i\epsilon_{\nu\rho}\lambda_\rho$	$\frac{i}{2}(\nabla_\nu G\vert + \epsilon_{\nu\rho}\nabla_\rho\tilde{G}\vert)$
λ_ν	0	$A_{\mu\nu}$	0	$-\frac{i}{2}(D^-_\nu G\vert - \epsilon_{\nu\rho}D^+_\rho\tilde{G}\vert)$
ρ	$\frac{i}{2}[D^+_\rho, D^-_\rho] - D$	$\frac{1}{2}(\nabla_\mu G\vert - \epsilon_{\mu\nu}\nabla_\nu\tilde{G}\vert)$	$\frac{i}{2}\epsilon_{\rho\sigma}[D^-_\rho, D^-_\sigma]$	$\frac{1}{2}(\nabla_z G\vert - \epsilon_{\rho\sigma}\nabla_\rho\nabla_\sigma\tilde{G}\vert)$
$\tilde{\rho}$	$-\frac{i}{2}\epsilon_{\rho\sigma}[D^+_\rho, D^+_\sigma]$	$\frac{1}{2}(\nabla_\mu\tilde{G}\vert + \epsilon_{\mu\nu}\nabla_\nu G\vert)$	$-\frac{i}{2}[D^+_\rho, D^-_\rho] - D$	$\frac{1}{2}(\nabla_z\tilde{G}\vert + \epsilon_{\rho\sigma}\nabla_\rho\nabla_\sigma G\vert)$
D	$-iD^+_\rho\lambda_\rho$	$\frac{i}{2}(D^+_\mu\rho - \epsilon_{\mu\nu}D^-_\nu\tilde{\rho})$ $-\frac{i}{2}(D_\mu G\vert - \epsilon_{\mu\nu}D_\nu\tilde{G}\vert)$	$-i\epsilon_{\rho\sigma}D^-_\rho\lambda_\sigma$	$-\frac{i}{2}(D^-_\rho\nabla_\rho G\vert + \epsilon_{\rho\sigma}D^+_{\underline{\rho}}\nabla_\sigma\tilde{G}\vert)$ $i\{\rho, G\vert\} + i\{\tilde{\rho}, \tilde{G}\vert\}$ $-\frac{i}{2}(\{G\vert, G\vert\} + \{\tilde{G}\vert, \tilde{G}\vert\})$

Table 3. N=D=2 SUSY transformation for Ansatz (B)

	∇	$\tilde{\nabla}$	∇_ρ	$\tilde{\nabla}_\rho$	$\nabla_{\rho\sigma}$	$\nabla_{\underline{\rho}}$
∇	0	$-iW$	$-i(\nabla_{\underline{\rho}} + F_\rho)$	0	0	$-iF_{\underline{\rho}}$
$\tilde{\nabla}$		0	0	$-i(\nabla_{\underline{\rho}} - F_\rho)$	0	$-i\tilde{F}_{\underline{\rho}}$
∇_μ			0	$-i\tilde{\delta}_{\mu\rho}F$	$i\delta_{\mu\nu\rho\sigma}(\nabla_{\underline{\nu}} - F_\nu)$	$-iF_{\mu\underline{\rho}}$
$\tilde{\nabla}_\mu$				0	$-i\epsilon_{\mu\nu\rho\sigma}(\nabla_{\underline{\nu}} + F_\nu)$	$-i\tilde{F}_{\mu\underline{\rho}}$
$\nabla_{\mu\nu}$					$i\epsilon_{\mu\nu\rho\sigma}W$	$-iF_{\mu\nu\underline{\rho}}$
$\nabla_{\underline{\mu}}$						$-iF_{\underline{\mu\rho}}$

Table 4. N=D=4 Ansatz without central charge (A)

	s	$\tilde{s}$	$s_{\mu\nu}$
ϕ_ρ	λ_ρ	$\tilde{\lambda}_\rho$	$-\delta_{\mu\nu\rho\sigma}\lambda_\sigma + \epsilon_{\mu\nu\rho\sigma}\tilde{\lambda}_\sigma$
A_ρ	$-i\lambda_\rho$	$i\tilde{\lambda}_\rho$	$-i\delta_{\mu\nu\rho\sigma}\lambda_\sigma - i\epsilon_{\mu\nu\rho\sigma}\tilde{\lambda}_\sigma$
A	0	0	0
B	$-2\tilde{\rho}$	2ρ	$-\epsilon_{\mu\nu\rho\sigma}\rho_{\rho\sigma}$
λ_ρ	0	$\frac{i}{2}D^-_\mu A$	$-\frac{i}{2}\epsilon_{\mu\nu\rho\sigma}D^+_\sigma A$
$\tilde{\lambda}_\rho$	$\frac{i}{2}D^+_\mu A$	0	$-\frac{i}{2}\delta_{\mu\nu\rho\sigma}D^-_\sigma A$
ρ	$-\frac{i}{2}(D_\alpha\phi_\alpha + \frac{1}{2}[A,B])$	0	$\frac{i}{2}[D^-_\mu, D^-_\nu]$
$\tilde{\rho}$	0	$-\frac{i}{2}(D_\alpha\phi_\alpha - \frac{1}{2}[A,B])$	$\frac{i}{4}\epsilon_{\mu\nu\rho\sigma}[D^+_\mu, D^+_\nu]$
$\rho_{\rho\sigma}$	$-\frac{i}{2}[D^+_\rho, D^+_\sigma]$	$\frac{i}{4}\epsilon_{\mu\nu\rho\sigma}[D^-_\mu, D^-_\nu]$	$\frac{i}{2}\delta_{\mu\nu\alpha\gamma}\delta_{\rho\sigma\beta\gamma}[D^-_\alpha, D^+_\beta] - \frac{i}{2}\delta_{\mu\nu\rho\sigma}(D_\alpha\phi_\alpha + \frac{1}{2}[A,B])$

Table 5.

	s_μ	$\tilde{s}_\mu$
ϕ_ρ	$\delta_{\mu\rho} + \rho_{\mu\rho}$	$\delta_{\mu\rho}\tilde{\rho} + \frac{1}{2}\epsilon_{\mu\rho\alpha\beta}\rho_{\alpha\beta}$
A_ρ	$-i\delta_{\mu\rho} + i\rho_{\mu\rho}$	$i\delta_{\mu\rho}\tilde{\rho} - \frac{i}{2}\epsilon_{\mu\rho\alpha\beta}\rho_{\alpha\beta}$
A	$-2\tilde{\lambda}_\mu$	$2\lambda_\mu$
B	0	0
λ_ρ	$\frac{i}{2}[D^+_\mu, D^-_\rho] + \frac{i}{2}\delta_{\mu\rho}(D_\alpha\phi_\alpha + \frac{1}{2}[A,B])$	$\frac{i}{4}\epsilon_{\mu\rho\alpha\beta}[D^+_\alpha, D^+_\beta]$
$\tilde{\lambda}$	$-\frac{i}{4}\epsilon_{\mu\rho\alpha\beta}[D^-_\alpha, D^-_\beta]$	$-\frac{i}{2}[D^-_\mu, D^+_\rho] + \frac{i}{2}\delta_{\mu\rho}(D_\alpha\phi_\alpha - \frac{1}{2}[A,B])$
ρ	0	$-\frac{i}{2}D^-_\mu B$
$\tilde{\rho}$	$\frac{i}{2}D^+_\mu B$	0
$\rho_{\rho\sigma}$	$-\frac{i}{2}\epsilon_{\mu\nu\rho\sigma}D^-_\nu B$	$\frac{i}{2}\delta_{\mu\nu\rho\sigma}D^+_\nu B$

Table 6.

	∇	$\tilde{\nabla}$	∇_ρ	$\tilde{\nabla}_\rho$	$\nabla_{\rho\sigma}$	$\nabla_{\underline{\rho}}$	∇_z
∇	0	∇_z	$-i(\nabla_{\underline{\rho}} + F_\rho)$	0	0	$-iF_{\underline{\rho}}$	iG
$\tilde{\nabla}$		0	0	$-i(\nabla_{\underline{\rho}} - F_\rho)$	0	$-i\tilde{F}_{\underline{\rho}}$	$i\tilde{G}$
∇_μ			0	$\delta_{\mu\rho}(\mp\nabla_z - iW)$	$i\delta_{\mu\nu\rho\sigma}(\nabla_{\underline{\nu}} - F_\nu)$	$-iF_{\mu\underline{\rho}}$	iG_μ
$\tilde{\nabla}_\mu$				0	$-i\epsilon_{\mu\nu\rho\sigma}(\nabla_{\underline{\nu}} + F_\nu)$	$-i\tilde{F}_{\mu\underline{\rho}}$	iG_μ
$\nabla_{\mu\nu}$					$-\epsilon_{\mu\nu\rho\sigma}\nabla_z$	$-iF_{\mu\nu\underline{\rho}}$	$iG_{\mu\nu}$
$\nabla_{\underline{\mu}}$						$-iF_{\underline{\mu\rho}}$	$iG_{\underline{\mu}}$
∇_z							0

Table 7. N=D=4 Ansatz with a central charge (B)

	s	$\tilde{s}$	s_μ	$\tilde{s}_\mu$
ϕ_ρ	λ_ρ	$\tilde{\lambda}_\rho$	$\delta_{\mu\rho}\rho + \rho_{\mu\rho}$	$\delta_{\mu\rho}\tilde{\rho} + \frac{1}{2}\epsilon_{\mu\rho\alpha\beta}\rho_{\alpha\beta}$
A_ρ	$-i\lambda_\rho$	$i\tilde{\lambda}_\rho$	$-i\delta_{\mu\rho}\rho + i\rho_{\mu\rho}$	$i\delta_{\mu\rho}\tilde{\rho} - \frac{i}{2}\epsilon_{\mu\rho\alpha\beta}\rho_{\alpha\beta}$
A'	$-\tilde{\rho}$	ρ	$\mp\tilde{\lambda}_\mu$	$\pm\lambda_{[}\mu]$
λ_ρ	0	g^+_ρ	$\frac{i}{2}[D^+_\mu, D^-_\rho] - \delta_{\mu\rho}H'$	$\frac{i}{4}\epsilon_{\mu\rho\alpha\beta}[D^+_\alpha, D^+_\beta]$
$\tilde{\lambda}_\rho$	$-g^-_\rho$	0	$-\frac{i}{4}\epsilon_{\mu\rho\alpha\beta}[D^-_\alpha, D^-_\beta]$	$-\frac{i}{2}[D^-_\mu, D^+_\rho] + \delta_{\mu\rho}(\frac{i}{2}[D^-_\sigma, D^+_\sigma] + H')$
ρ	H'	0	0	$\mp g^+_\mu - iD^-_\mu A'$
$\tilde{\rho}$	0	$-\frac{i}{2}[D^-_\mu, D^+_\mu] - H'$	$\pm g^-_\mu + iD^+_\mu A'$	0
$\rho_{\rho\sigma}$	$-\frac{i}{2}[D^+_\rho, D^+_\sigma]$	$\frac{i}{4}\epsilon_{\rho\sigma\alpha\beta}[D^-_\alpha, D^-_\beta]$	$\epsilon_{\mu\nu\rho\sigma}(\mp g^+_\nu - iD^-_\nu A')$	$\delta_{\mu\nu\rho\sigma}(\pm g^-_\nu + iD^+_\nu A')$
H'	0	$-iD^-_\mu\tilde{\lambda}_\mu$	$-iD^+_\mu\rho$	$-\frac{i}{2}\epsilon_{\mu\nu\rho\sigma}D^+_\nu\rho_{\rho\sigma}$
g^+_ρ	$\mp(\frac{i}{2}\epsilon_{\mu\nu\alpha\beta}D^+_\sigma\rho_{\alpha\beta}$ $-iD^-_\rho\tilde{\rho} + i[A, \lambda_\rho])$	0	$-i\delta_{\mu\rho}D^-_\nu\tilde{\lambda}_\nu + iD^-_\rho\tilde{\lambda}_\mu$	$-iD^-_\mu\lambda_\rho - i\epsilon_{\mu\rho\alpha\beta}D^+_\alpha\tilde{\lambda}_\beta$
g^-_ρ	0	$\pm(iD^-_\sigma\rho_{\rho\sigma}$ $-iD^+_\rho\rho + i[A, \tilde{\lambda}_\rho])$	$i\epsilon_{\mu\rho\alpha\beta}D^-_\alpha\lambda_\beta + iD^+_\mu\tilde{\lambda}_\rho$	$i\delta_{\mu\rho}D^+_\nu\lambda_\nu - iD^+_\rho\lambda_\mu$

Table 8. N=D=4 SUSY transformation with a central charge for Ansatz (B)

	$s_{\mu\nu}$	Z
ϕ_ρ	$-\delta_{\mu\nu\rho\sigma}\lambda_\sigma + \epsilon_{\mu\nu\rho\sigma}\tilde{\lambda}_\sigma$	H_ρ
A_ρ	$-i\delta_{\mu\nu\rho\sigma}\lambda_\sigma - i\epsilon_{\mu\nu\rho\sigma}\tilde{\lambda}_\sigma$	g_ρ
A'	$-\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\rho_{\rho\sigma}$	$\frac{i}{2}[D^-_\mu, D^+_\mu] + H$
λ_ρ	$\epsilon_{\mu\nu\rho\sigma}g^-_\sigma$	$\mp(-iD^-_\mu\tilde{\rho} + \frac{i}{2}\epsilon_{\mu\nu\rho\sigma}D^+\rho_{\rho\sigma} + i[A, \lambda_\mu])$
$\tilde{\lambda}_\rho$	$\delta_{\mu\nu\rho\sigma}g^+_\sigma$	$\mp(-iD^+_\mu\rho + iD^-_\nu\rho_{\mu\nu} + i[A, \tilde{\lambda}_\mu])$
ρ	$\frac{i}{2}[D^-_\mu, D^-_\nu]$	$-iD^-_\mu\tilde{\lambda}_\mu$
$\tilde{\rho}$	$\frac{i}{4}\epsilon_{\mu\nu\rho\sigma}[D^+_\rho, D^+_\sigma]$	$-iD^+_\mu\lambda_\mu$
$\rho_{\rho\sigma}$	$\frac{i}{2}\delta_{\mu\nu\alpha\gamma}\delta_{\rho\sigma\beta\gamma}[D^-_\alpha, D^+_\beta] + \delta_{\mu\nu\rho\sigma}H'$	$-i\epsilon_{\mu\nu\rho\sigma}D^-_\rho\lambda_\sigma - iD^+_{[\mu}\tilde{\lambda}_{\nu]}$
H'	$iD^-_{[\mu}\lambda_{\nu]}$	$iD^-_\mu g^-_\mu + 2i\{\lambda_\mu, \tilde{\lambda}_\mu\}$
g^+_ρ	$\mp\epsilon_{\mu\nu\rho\sigma}(-iD^+_\sigma\rho + iD^-_\alpha\rho_{\sigma\alpha} + i[A, \lambda_\sigma])$	$\mp i(\frac{i}{2}D^-_\nu D^+_\nu D^-_\mu + \epsilon_{\mu\nu\rho\sigma}\{\tilde{\lambda}_\nu, \rho_{\rho\sigma}\}+$ $2\{\rho, \lambda\} + D^-_\mu H' + 2[A', g^+_\mu])$
g^-_ρ	$\mp\delta_{\mu\nu\rho\sigma}(-iD^-_\sigma\tilde{\rho} + \frac{i}{2}\epsilon_{\sigma\gamma\alpha\beta}D^+_\gamma\rho_{\alpha\beta} + i[A, \lambda_\sigma])$	$\mp i(\frac{i}{2}D^-_\nu D^+_\mu D^+_\nu + 2\{\lambda_\nu, \rho_{\mu\nu}\}+$ $2\{\tilde{\rho}, \tilde{\lambda}_\mu\} + D^+_\mu H' + 2[A', g^-_\mu])$

Table 9. N=D=4 SUSY transformation with a central charge for Ansatz (B)

	∇	$\tilde{\nabla}$	∇_ρ	$\tilde{\nabla}_\rho$	$\nabla_{\rho\sigma}$	$\nabla_{\underline{\rho}}$	∇_z
∇	$\nabla_z - iW$	0	$-i\nabla_{\underline{\rho}}$	iF_ρ	0	$-iF_{\underline{\rho}}$	iG
$\tilde{\nabla}$		$\nabla_z - iW$	$-iF_\rho$	$-i\nabla_{\underline{\nu}}$	0	$-i\tilde{F}_{\underline{\rho}}$	$i\tilde{G}$
∇_μ			$\pm\delta_{\mu\nu}\nabla_z$	0	$i\delta_{\mu\nu\rho\sigma}\nabla_{\underline{\nu}} + i\epsilon_{\mu\nu\rho\sigma}F_\nu$	$-iF_{\mu\underline{\rho}}$	iG_μ
$\tilde{\nabla}_\mu$				$\pm\delta_{\mu\rho}\nabla_z$	$-i\epsilon_{\mu\nu\rho\sigma}\nabla_{\underline{\nu}} + i\delta_{\mu\nu\rho\sigma}F_\nu$	$-i\tilde{F}_{\mu\underline{\rho}}$	iG_μ
$\nabla_{\mu\nu}$					$\delta_{\mu\nu\rho\sigma}(\nabla_z - iW)$	$-iF_{\mu\nu\underline{\rho}}$	$iG_{\mu\nu}$
$\nabla_{\underline{\mu}}$						$-iF_{\underline{\mu\rho}}$	$iG_{\underline{\mu}}$
∇_z							0

Table 10. N=D=4 Ansatz with a central charge (C)

	s	$\tilde{s}$
ϕ_ρ	λ_ρ	$\tilde{\lambda}_\rho$
A_ρ	$i\tilde{\lambda}_\rho$	$-i\lambda_\rho$
A'	$\mp\tilde{\rho}$	$\pm\rho$
λ_ρ	H'_ρ	g'_ρ
$\tilde{\lambda}_\rho$	$-g'_\rho$	H'_ρ
ρ	$-\frac{i}{2}D_\mu\phi_\mu$	$\pm K'$
$\tilde{\rho}$	$\mp K'$	$-\frac{i}{2}D_\mu\phi_\mu$
$\rho_{\rho\sigma}$	$-\frac{1}{2}\epsilon_{\rho\sigma\alpha\beta}F^-_{\alpha\beta} - \frac{i}{2}D_{[\rho}\phi_{\sigma]}$	$F^-_{\rho\sigma} - \frac{i}{2}\epsilon_{\rho\sigma\alpha\beta}D_\alpha\phi_\beta$
K'	$\pm\frac{i}{2}(D_\mu\tilde{\lambda}_\mu + [\phi_\mu, \lambda_\mu])$	$\mp\frac{i}{2}(D_\mu\lambda_\mu - [\phi_\mu, \tilde{\lambda}_\mu])$
g'_ρ	$\pm\frac{i}{2}(D_\rho\tilde{\rho} - \frac{1}{2}\epsilon_{\rho\sigma\alpha\beta}D_\sigma\rho_{\alpha\beta}$	$\mp\frac{i}{2}(D_\rho\rho - D_\sigma\rho_{\rho\sigma} + [\phi_\rho, \tilde{\rho}]$
	$-[\phi_\rho, \rho] - [\phi_\sigma, \rho_{\rho\sigma}] \pm 2[A', \tilde{\lambda}_\rho])$	$+\frac{1}{2}\epsilon_{\rho\sigma\alpha\beta}[\phi_\sigma, \rho_{\alpha\beta}] \pm 2[A', \lambda_\rho])$
H'_ρ	$\mp\frac{i}{2}(D_\rho\rho - D_\sigma\rho_{\rho\sigma} + [\phi_\rho, \tilde{\rho}]$	$\mp\frac{i}{2}(D_\rho\tilde{\rho} - \frac{1}{2}\epsilon_{\rho\sigma\alpha\beta}D_\sigma\rho_{\alpha\beta}$
	$+\frac{1}{2}\epsilon_{\rho\sigma\alpha\beta}[\phi_\sigma, \rho_{\alpha\beta}] \pm 2[A', \lambda_\rho])$	$-[\phi_\rho, \rho] - [\phi_\sigma, \rho_{\rho\sigma}] \pm 2[A', \tilde{\lambda}_\rho])$

Table 11.

	s_μ	$\tilde{s}_\mu$
ϕ_ρ	$\delta_{\mu\rho}\rho + \rho_{\mu\rho}$	$\delta_{\mu\rho}\tilde{\rho} + \frac{1}{2}\epsilon_{\mu\rho\alpha\beta}\rho_{\alpha\beta}$
A_ρ	$-i\delta_{\mu\rho}\tilde{\rho} + \frac{i}{2}\epsilon_{\mu\rho\alpha\beta}\rho_{\alpha\beta}$	$i\delta_{\mu\rho}\rho - i\rho_{\mu\rho}$
A'	$-\tilde{\lambda}_\mu$	$\lambda_[\mu]$
λ_ρ	$\frac{1}{2}\epsilon_{\mu\rho\alpha\beta}F^-_{\alpha\beta} + \frac{i}{2}\delta_{\mu\rho}D_\nu\phi_\nu - \frac{i}{2}D_{(\mu}\phi_{\rho)}$	$F^+_{\mu\rho} \pm \delta_{\mu\rho}K' + \frac{i}{2}\epsilon_{\mu\rho\alpha\beta}D_\alpha\phi_\beta$
$\tilde{\lambda}_\rho$	$-F^+_{\mu\rho} \pm \delta_{\mu\rho}K' + \frac{i}{2}\epsilon_{\mu\rho\alpha\beta}D_\alpha\phi_\beta$	$-\frac{1}{2}\epsilon_{\mu\rho\alpha\beta}F^-_{\alpha\beta} + \delta_{\mu\rho}\frac{i}{2}D_\nu\phi_\nu - \frac{i}{2}D_{(\mu}\phi_{\rho)}$
ρ	$-\frac{i}{2}D_\mu\phi_\mu$	$\mp\frac{i}{2}g_\mu$
$\tilde{\rho}$	$\pm g_\mu$	$\frac{1}{2}H_\mu$
$\rho_{\rho\sigma}$	$\pm\frac{1}{2}\delta_{\mu\nu\rho\sigma}H_\nu \mp \frac{i}{2}\epsilon_{\mu\nu\rho\sigma}g_\nu$	$\pm\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}H_\nu \pm \frac{i}{2}\delta_{\mu\nu\rho\sigma}g_\nu$
K'	$\pm\frac{i}{2}(D_\mu\tilde{\rho} - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}D_\nu\rho_{\rho\sigma} - [\phi_\mu, \rho] - [\phi_\nu, \rho_{\mu\nu}])$	$\mp\frac{i}{2}(D_\mu\rho - D_\nu\rho_{\mu\nu} + [\phi_\mu, \tilde{\rho}] + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}[\phi_\nu, \rho_{\rho\sigma}])$
g'_ρ	$-\frac{i}{2}(\delta_{\mu\rho}(D_\nu\tilde{\lambda}_\nu + [\phi_\nu, \lambda_\nu]) - D_{(\mu}\tilde{\lambda}_{\rho)}$	$-\frac{i}{2}(-\delta_{\mu\rho}(D_\nu\lambda_\nu - [\phi_\nu, \tilde{\lambda}_\nu]) + D_{(\mu}\lambda_{\rho)}$
	$+[\phi_{[\mu}, \lambda_{\rho]}] - \epsilon_{\mu\rho\alpha\beta}(D_\alpha\lambda_\beta + [\phi_\alpha, \tilde{\lambda}_\beta]))$	$+[\phi_{[\mu}, \tilde{\lambda}_{\rho]}] + \epsilon_{\mu\rho\alpha\beta}(D_\alpha\tilde{\lambda}_\beta - [\phi_\alpha, \lambda_\beta]))$
H'_ρ	$-\frac{i}{2}(\delta_{\mu\rho}(D_\nu\lambda_\nu - [\phi_\nu, \tilde{\lambda}_\nu]) + D_{[\mu}\lambda_{\rho]}$	$-\frac{i}{2}(\delta_{\mu\rho}(D_\nu\tilde{\lambda}_\nu + [\phi_\nu, \lambda_\nu]) + D_{[\mu}\tilde{\lambda}_{\rho]}$
	$+[\phi_{(\mu}, \tilde{\lambda}_{\rho)}] + \epsilon_{\mu\rho\alpha\beta}(D_\alpha\tilde{\lambda}_\beta - [\phi_\alpha, \lambda_\beta]))$	$-[\phi_{(\mu}, \lambda_{\rho)}] + \epsilon_{\mu\rho\alpha\beta}(D_\alpha\lambda_\beta + [\phi_\alpha, \tilde{\lambda}_\beta]))$

Table 12.

	$s_{\mu\nu}$	Z
ϕ_ρ	$-\delta_{\mu\nu\rho\sigma}\lambda_\sigma + \epsilon_{\mu\nu\rho\sigma}\tilde{\lambda}_\sigma$	H_ρ
A_ρ	$i\delta_{\mu\nu\rho\sigma}\tilde{\lambda}_\sigma + i\epsilon_{\mu\nu\rho\sigma}\lambda_\sigma$	g_ρ
A'	$\mp\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\rho_{\rho\sigma}$	$2K'$
λ_ρ	$\delta_{\mu\nu\rho\sigma}H'_\sigma + \epsilon_{\mu\nu\rho\sigma}g'_\sigma$	$\mp i(D_\rho\rho - D_\sigma\rho_{\rho\sigma} + [\phi_\rho, \tilde{\rho}]$
		$+\frac{1}{2}\epsilon_{\rho\sigma\alpha\beta}[\phi_\sigma, \rho_{\alpha\beta}])$
$\tilde{\lambda}_\rho$	$\delta_{\mu\nu\rho\sigma}g'_\sigma - \epsilon_{\mu\nu\rho\sigma}H'_\sigma$	$\mp i(D_\rho\tilde{\rho} - \frac{1}{2}\epsilon_{\rho\sigma\alpha\beta}D_\sigma\rho_{\alpha\beta} - [\phi_\rho, \rho]$
		$-[\phi_\sigma, \rho_{\rho\sigma}])$
ρ	$\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^-_{\rho\sigma} - \frac{i}{2}D_{[\mu}\phi_{\nu]}$	$-i(D_\rho\lambda_\rho - [\phi_\rho, \tilde{\lambda}_\rho] - i[A, \rho])$
$\tilde{\rho}$	$F^-_{\mu\nu} + \frac{i}{2}\epsilon_{\mu\nu\rho\sigma}D_\rho\phi_\sigma$	$-i(D_\rho\tilde{\lambda}_\rho + [\phi_\rho, \lambda_\rho] - [A, \tilde{\rho}])$
$\rho_{\rho\sigma}$	$\epsilon_{\mu\nu\alpha\gamma}\delta_{\rho\sigma\beta\gamma}F^+_{\alpha\beta} + \frac{i}{2}\delta_{\mu\nu\alpha\gamma}\delta_{\rho\sigma\beta\gamma}D_{(\alpha}\phi_{\beta)}$	$-i(D_{[\mu}\lambda_{\nu]} + [\phi_{[\mu}, \tilde{\lambda}_{\nu]}]$
	$-\frac{i}{2}\delta_{\mu\nu\rho\sigma}D_\alpha\phi_\alpha \mp \epsilon_{\mu\nu\rho\sigma}K'$	$+\epsilon_{\mu\nu\rho\sigma}(D_\rho\tilde{\lambda}_\sigma - [\phi_\rho, \lambda_\sigma]) - [A, \rho_{\mu\nu}])$
K'	$\pm\frac{i}{2}(D_{[\mu}\tilde{\lambda}_{\nu]} - [\phi_{[\mu}, \lambda_{\nu]}] + \epsilon_{\mu\nu\rho\sigma}(D_\rho\lambda_\sigma$	$\pm i(\{\lambda, \lambda\} + \{\tilde{\lambda}, \tilde{\lambda}\} - D_\rho g'_\rho + [\phi_\rho, H'_\rho]$
		$\pm[A, K'])$
g'_ρ	$\pm\frac{i}{2}\delta_{\mu\nu\rho\sigma}(D_\sigma\tilde{\rho} - \frac{1}{2}\epsilon_{\sigma\gamma\alpha\beta}D_\gamma\rho_{\alpha\beta} - [\phi_\sigma, \rho]$	$\mp i(-\frac{1}{2}D_\sigma F_{\rho\sigma} \pm D_\rho K' - \frac{i}{2}[\phi_\sigma, D_\rho\phi_\sigma]$
	$+[\phi_{rho}, \tilde{\lambda}_\sigma])) - [\phi_\alpha, \rho_{\sigma\alpha}] \pm [A, \tilde{\lambda}_\sigma]) \pm \frac{i}{2}\epsilon_{\mu\nu\rho\sigma}(D_\sigma\rho$	$+\{\rho, \lambda_\rho\} + \{\tilde{\rho}, \tilde{\lambda}_\rho\} + \{\lambda_\sigma, \rho_{\rho\sigma}\}$
	$-D_\alpha\rho_{\sigma\alpha} + [\phi_\sigma, \tilde{\rho}] + \frac{1}{2}\epsilon_{\sigma\gamma\alpha\beta}[\phi_\gamma, \rho_{\alpha\beta}] \pm [A, \lambda_\sigma])$	$+\frac{1}{2}\epsilon_{\rho\sigma\alpha\beta}\{\tilde{\lambda}_\sigma, \rho_{\alpha\beta}\})$
H'_ρ	$\pm\frac{i}{2}\delta_{\mu\nu\rho\sigma}(D_\sigma\rho - D_\alpha\rho_{\sigma\alpha} + [\phi_\sigma, \tilde{\rho}]$	$\mp i(-\frac{i}{2}D_\sigma D_\sigma\phi_\rho + \frac{i}{2}[\phi_\sigma, [\phi_\rho, \phi_\sigma]]$
	$+\frac{1}{2}\epsilon_{\sigma\gamma\alpha\beta}[\phi_\gamma, \rho_{\alpha\beta}] \pm [A, \lambda_\sigma]) \mp \frac{i}{2}\epsilon_{\mu\nu\rho\sigma}(D_\sigma\tilde{\rho}$	$\mp[\phi_\rho, K'] + \{\rho, \tilde{\lambda}_\rho\} - \{\tilde{\rho}, \lambda_\rho\}$
	$-\frac{1}{2}\epsilon_{\sigma\gamma\alpha\beta}D_\gamma\rho_{\alpha\beta} - [\phi_\sigma, \rho] - [\phi_\alpha, \rho_{\sigma\alpha}] \pm [A, \tilde{\lambda}_\sigma])$	$-\{\tilde{\lambda}_\sigma, \rho_{\rho\sigma}\} + \frac{1}{2}\epsilon_{\rho\sigma\alpha\beta}\{\lambda_\sigma, \rho_{\alpha\beta}\})$

Table 13.

	∇	$\tilde{\nabla}$	∇_ρ	$\tilde{\nabla}_\rho$
∇	$X_0 + X'_5$	$X_5 + X'_0$	$-i(\nabla_{\underline{\rho}} + iX_\rho)$	$-X'_\rho$
$\tilde{\nabla}$		$X_0 + X'_5$	X'_ρ	$-i(\nabla_{\underline{\rho}} - iX_\rho)$
∇_μ			$\delta_{\mu\rho}(X_0 - X'_5)$	$\delta_{\mu\rho}(X_5 - X'_0)$
$\tilde{\nabla}_\mu$				$\delta_{\mu\rho}(X_0 - X'_5)$
$\nabla_{\mu\nu}$				
$\nabla_{\underline{\mu}}$				

Table 14. Ansatz (D)

	$\nabla_{\rho\sigma}$	$\nabla_{\underline{\rho}}$	∇_z
∇	0	$-iF_{\underline{\rho}}$	iG
$\tilde{\nabla}$	0	$-i\tilde{F}_{\underline{\rho}}$	$i\tilde{G}$
∇_μ	$i\delta_{\mu\nu\rho\sigma}(\nabla_{\underline{\nu}} - iX_\nu) - \epsilon_{\mu\nu\rho\sigma}X'_\nu$	$-iF_{\mu\underline{\rho}}$	iG_μ
$\tilde{\nabla}_\mu$	$-i\epsilon_{\mu\nu\rho\sigma}(\nabla_{\underline{\nu}} + iX_\nu) - \delta_{\mu\nu\rho\sigma}X'_\nu$	$-i\tilde{F}_{\mu\underline{\rho}}$	$i\tilde{G}_\mu$
$\nabla_{\mu\nu}$	$\delta_{\mu\nu\rho\sigma}(X_0 + X'_5) - \epsilon_{\mu\nu\rho\sigma}(X_5 + X'_0)$	$-iF_{\mu\nu\underline{\rho}}$	$iG_{\mu\nu}$
$\nabla_{\underline{\mu}}$		$-iF_{\underline{\mu\rho}}$	$iG_{\underline{\mu}}$
∇_z			0

Table 15. Ansatz (D)

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СУПЕРСИММЕТРИЧНАЯ ТЕОРИЯ ПОЛЯ ЯНГА-МИЛЛСА С КАЛИБРОВАННЫМ ЦЕНТРАЛЬНЫМ ЗАРЯДОМ ВНЕ МАССОВОЙ ОБОЛОЧКИ ДЛЯ N=D=2 И N=D=4: «НУЖНЫ ЛИ НАМ СВЯЗИ?»

Мы исследуем вывод твистованной суперсимметричной теории поля Янга-Миллса с центральным зарядом вне массовой оболочки для N=2 в двух измерениях и N=4 в четырех измерениях, используя формализм суперсвязностей. В двух измерениях мы находим N=2 инвариантную вне массовой оболочки супералгебру без дополнительной связи. Однако в четырех измерениях мы находим, что твистованная N=4 супералгебра с одним центральным зарядом всегда содержит связь.

Ключевые слова: *суперпространство, суперсимметрия, Янг-Миллс, квантование.*

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