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SCHWINGER-DYSON ANALYSIS OF 1+2 DIMENSIONAL QED

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The QED ground state is studied for a four component Dirac fermion with the topological Chern-Simons term in 1+2 dimension. The Lagrangian for a massless Dirac fermion has the parity and the chiral symmetries. Spontaneous breaking of these symmetries can be found by evaluating the vacuum expectation values for fermion and anti-fermion pairs. We introduce approximate vertex functions and calculate the expectation values by solving the Schwinger-Dyson equation. We observe a smaller gauge dependence for the Ball-Chiu vertex function.

Keywords: *1+2 dimensional QED, Schwinger-Dyson equation, Chern-Simons term.*

1 Introduction

Spontaneous symmetry breaking is one of the fundamental concepts in elementary particle physics. Non-perturbative effect is essential for spontaneous symmetry breaking caused by gauge interactions. The Schwinger-Dyson (SD) equation is often used to evaluate it. A solution of the SD equation for a propagator gives a full self-energy. However, a full-vertex function is necessary to solve the SD equation. We usually set some approximate vertex functions in order to define a closed form for the SD equation. In the ladder approximation we replace the full-vertex function by the tree level one.

The Ward-Takahashi (WT) identity is preserved in the ladder approximation with the Landau gauge for 1+3 dimensional QED. It should be noted that the physical observable in the solution of the SD equation depends on the gauge parameter because of the approximate vertex function. The ladder approximation with the Landau gauge can not preserve the WT identity in low dimensional systems or thermal environment. The validity of the ladder approximation can not be guaranteed in such systems. Much works have been done to find a better vertex function [1–5] and a gauge parameter [6–8].

The 1+2 dimensional QED with the Chern-Simons (CS) term is one of the simple models to develop the procedure to analyze the SD equation beyond the ladder approximation. In this paper we employ some vertex functions and solve the SD equation and show the gauge dependence of the result. In Sec. 2 we introduce the 1+2 dimensional QED with the CS term and review the chiral symmetry for a four component Dirac fermion. In Sec. 3 we give an explicit form for the SD equation and adopt some vertex functions. Evaluating the SD equation numerically, we show the gauge dependence for the vacuum expectation values for fermion and anti-fermion pairs.

2 Chiral symmetry in 1+2 dimensional QED

The QED Lagrangian is uniquely fixed by the gauge symmetry, the Lorentz covariance and the renormalizability. We start form the Lagrangian density with a massless Dirac fermion,

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu A^\mu)^2 + \frac{\mu}{2}\varepsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho \\ &+\bar{\psi}i\gamma^\mu D_\mu\psi. \end{aligned} \tag{1}$$

The first term in the right hand side is ordinary QED Lagrangian density written by the field strength, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. The second term is the gauge fixing part in the Lorentz gauge with the gauge parameter, α. The third term is called the CS term. This term is topologically determined and can be written as a total divergence. The coefficient μ introduces a topological mass for the gauge field, A^μ. $D_\mu = \partial_\mu + ieA_\mu$ represents the covariant derivative for the Dirac fermion, ψ. γ^μ denotes the 4×4 Dirac matrices. Since we consider the 1+2 dimensional space-time, the indices, μ, ν run from 0 to 2.

The Clifford algebra in 1+2 dimensions is defined by the anti-commutator for the Dirac matrices,

$$\{\gamma^\mu, \gamma^\nu\} = g^{\mu\nu}, \quad \mu, \nu \in \{0, 1, 2\}. \tag{2}$$

There are two Casimir operators, γ^3 and γ^5,

$$\{\gamma^3, \gamma^\mu\} = 0, \quad \{\gamma^5, \gamma^\mu\} = 0. \tag{3}$$

Thus we can define two-kinds of the chiral transformations,

$$\psi \to e^{i\gamma^3\theta^3}\psi, \quad \psi \to e^{i\gamma^5\theta^5}\psi, \tag{4}$$

where θ^3 and θ^5 are real parameters.

The Lagrangian (1) is invariant under the chiral transformations (4). If the fermion acquire a non-vanishing mass, the mass term, $m\bar{\psi}\psi$, breaks the chiral symmetries. We can introduce a parity odd mass term,

$$m_\tau \bar{\psi}\tau\psi, \quad \tau = \frac{1}{2}\{\gamma^3, \gamma^5\}. \tag{5}$$

This term is invariant under the chiral transformations. Therefore the spontaneous breaking for the chiral and the parity symmetries is found by observing the vacuum expectation values for the composite operators, $\langle\bar{\psi}\psi\rangle$ and $\langle\bar{\psi}\tau\psi\rangle$, respectively [6] .

3 SD euqation in 1+2 dimensional QED

The vacuum expectation values, $\langle\bar{\psi}\psi\rangle$ and $\langle\bar{\psi}\tau\psi\rangle$, are calculated by taking the trace of the fermion propagator. These values vanish for the massless fermion without considering the non-perturbative contribution of the gauge interaction. Here we introduce the non-perturbative effect by solving the SD equation. The SD equation for the fermion propagator, $S(p)$, is given by

$$S(p) = \frac{i}{/p - \Sigma(p) + i\varepsilon}, \tag{6}$$

with the fermion self-energy,

$$i\Sigma(p) = e^2 \int \frac{d^3k}{(2\pi)^3} \gamma^\mu D_{\mu\nu}(p-k) S(k) \Gamma^\nu(p,k), \tag{7}$$

where $D_{\mu\nu}(p-k)$ and $\Gamma^\mu(p,k)$ are the full gauge boson propagator and the full vertex function, respectively. The self-energy $\Sigma(p)$ has an 4×4 matrix form and can contain a parity odd and a chirality odd mass terms.

If we insert some functional forms for $D_{\mu\nu}(p-k)$ and $\Gamma^\mu(p,k)$, we can solve the SD equation (6) and find the fermion self-energy, $\Sigma(p)$. In the ladder approximation we employ a tree-level gauge boson propagator and a vertex function,

$$\begin{aligned} D_{\mu\nu}(p) &= \frac{-i}{p^2-\mu^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \\ &\quad -\mu \frac{1}{p^2-\mu^2}\frac{1}{p^2}\varepsilon_{\mu\nu\rho}p^\rho - i\alpha\frac{p_\mu p_\nu}{p^4}, \end{aligned} \tag{8}$$

and

$$\Gamma^\mu(p,k) = \gamma^\mu. \tag{9}$$

It is known that the ladder approximation is not consistent with the WT identity,

$$i(p-k)_\mu \Gamma^\mu(p,k) = S^{-1}(k) - S^{-1}(p). \tag{10}$$

Here we introduce the parity projection operators,

$$\chi_\pm \equiv \frac{1\pm\tau}{2} \tag{11}$$

and assume a decomposed form for the fermion propagator,

$$S(p) = \frac{i}{A_+ /p - B_+}\chi_+ + \frac{i}{A_- /p - B_-}\chi_-, \tag{12}$$

where $A_\pm$ and $B_\pm$ are functions of p. Then the WT identity (10) fix the longitudinal part of the vertex function [9],

$$\Gamma^\mu_L(p,k) = \Gamma^{BC\mu}_{L+}\chi_+ + \Gamma^{BC\mu}_{L-}\chi_-, \tag{13}$$

with

$$\begin{aligned} \Gamma^{BC\mu}_{L\pm} &= \frac{1}{2}[A_\pm(p) + A_\pm(k)]\gamma^\mu \\ &\quad + \frac{1}{2}[A_\pm(p) - A_\pm(k)]\frac{p^\mu + k^\mu}{p^2-k^2}(/p + /k) \\ &\quad - [B_\pm(p) - B_\pm(k)]\frac{p^\mu + k^\mu}{p^2-k^2}. \end{aligned} \tag{14}$$

The transverse part of the vertex function can not be fixed by the WT identity. We simply drop the transfers part below.

The Ball-Chiu (BC) vertex function (13) satisfies the WT identity by definition and may be a better assumption for the vertex function on the SD equation for the 1+2 dimensional QED with the CS term. Employing the Ball-Chiu vertex function and a tree-level gauge boson propagator, we numerically solve the SD equation and calculate the vacuum expectation values for fermion and anti-fermion pairs. It is found that the obtained expectation value $\langle\bar{\psi}\tau\psi\rangle$ depends on the gauge parameter, α. Between $\alpha = 0$ and 1 we fit the expectation value as a linear function of the gauge parameter, α,

$$\langle\bar{\psi}\tau\psi\rangle - \langle\bar{\psi}\tau\psi\rangle|_{\alpha=0} = a\alpha + O(\alpha^2), \tag{15}$$

and find $a = -(0.495 \pm 0.005)\times 10^{-3}$. In the ladder approximation we obtain $a = (1.79 \pm 0.06)\times 10^{-3}$. A smaller gauge dependence is observed for the BC vertex function compared with the ladder approximation. It should be noted that both results approach near the Landau gauge, $\alpha = 0$. Thus the ladder approximation seems to be a good assumption with the Landau gauge. Details of the numerical algorithm and the results are given in Ref. [10].

4 Concluding remarks

We have performed the SD analysis of 1+2 dimensional QED with the CS term. The often used ladder approximation is extended to avoid a large gauge dependence. For the BC vertex function a non-trivial vertex correction is introduced through the WT identity. The solution of the SD equation shows a smaller gauge dependence for the BC vertex function. We have also pointed out that the ladder approximation is not so bad with the Landau gauge.

Here we have applied some assumptions to solve the SD equation. The tree level propagator is adopted for the gauge boson. It is interesting to introduce the SD equation for the gauge boson propagator and simultaneously solve it with the SD equation (6). We have simply droped the transverse part of the vertex function. It is generated even at the one-loop level [11]. It is also interesting to study the gauge dependence for some possible assumptions of the transverse part of the vertex function. We hope to report any progress for these problems in future.

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Т. Инагаки

ФОРМАЛИЗМ ШВИНГЕРА-ДАЙСОНА В 1+2 РАЗМЕРНОЙ КЭД

Исследуется основное состояние четырехкомпонентного фермиона Дирака в КЭД с топологическим членом Черна–Саймона в 1+2 размерностях. Лагранжиан безмассового дираковского фермиона обладает киральной и P-симметриями. Спонтанное нарушение этих симметрий может быть найдено с помощью расчета значения вакуумного ожидания для фермион–антифермионных пар. Мы вводим приближенную вершинную функцию и вычисляем значения вакуумного ожидания решая уравнение Швингера–Дайсона. Мы наблюдаем слабую калибровочную зависимость вершинной функции Болла–Чью.

Ключевые слова: *1+2 размерная КЭД, уравнение Швингера-Дайсона, член Черна – Саймона.*

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