UDC 530.1; 539.1

Regularization parameter independent approach in a four fermion interaction model

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In QCD a non-perturbative effect is essential in low energy phenomena. A usual perturbative procedure loses its validity. A four-fermion interaction model is used to evaluate the low energy phenomena for quarks and gluons. Since the four-fermion interaction is nonrenormalizable, the results depend on the regularization parameter in four spacetime dimensions. Here we show the possibility to eliminate the regularization parameter dependence for some of physical observables.

Keywords: *four-fermion interaction, dimensional regularization.*

1 Introduction

The quantum chromodynamics (QCD) is the fundamental theory for color charged particles. Because of the asymptotic freedom the coupling constant for the QCD interaction blows up below the QCD scale $\sim$ 1GeV. The QCD dynamics is essetially nonperturbative at low energy. We need to use the method free from the usual perturbative expansion in terms of the coupling constant.

One of the methods is developed under a phenomenological model of QCD. Nambu and Jona-Lasinio introduce a four-fermion interaction model as a low energy phenomenological model for hadrons [1, 2]. We can adopt the $1/N$ expansion method as a non-perturbative approach. Then the model spontaneously breaks the chiral symmetry and well describes the low energy phenomena [3–5].

The four-fermion interaction is described by an operator whose mass dimension is $2D-2$ in D space-time dimensions. The operator is irrelevant and the model is nonrenormalizable for $D > 2$ in a sense of the usual perturbation. The model is renormalizable in three dimensions in the 1/N expansion scheme [6–8]. The model is nonrenormalizable in four dimensions. The result depends on the regularization parameter, after we apply a normal renormalization procedure [10].

On the other hands, the four-fermion interaction is known to be trivial in four dimensions [9]. The four-fermion interaction model reduces to a free fermion theory. All the radiative corrections disappear at the naive four dimensional limit. At a glance the model seems to be renormalizable in four-dimensions. However, the four-fermion interaction vanishes and we lose all the dynamics.

In this paper a simple scalar type four-fermion interaction is considered. We evaluate the model in $4-2\varepsilon$ dimension. The regularization dependence can be eliminated by taking the limit, $\varepsilon \to 0$. We show that the four-fermion interaction vanishes at the limit but the results are not always reduces to the free fermion theory.

2 Four-fermion interaction model

Here we consider a simple model with scalar type four-fermion interactions.

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m_i\right)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi_j)^2\right], \tag{1}$$

where ψ describes a quark field. We suppose the order of the coupling constant G to be $GN_c \simeq O(1)$, where N_c is the number of colors.

2.1 Vacuum diagram

Here we calculate the chiral condensate $\langle\bar{\psi}\psi\rangle$. In the leading order of $1/N_c$ expansion, it is given by,

$$\langle\bar{\psi}\psi\rangle = M^{4-D}\int\frac{d^Dp}{i(2\pi)^D}\mathrm{tr}\frac{1}{p\!\!\!/ - m^*}, \tag{2}$$

where m^* represents the constituent quark mass, "tr" denotes the trace with respect to spinor and color indices. We rescale the chiral condensate by multiplying the mass scale M in order to have the mass dimension in four-dimension. Evaluating the integral in $D(\equiv 4-2\epsilon)$ dimensions [11–15], we obtain

$$\langle\bar{\psi}\psi\rangle = -\frac{N_c M^{4-D}}{(2\pi)^{D/2}}\Gamma\left(1-\frac{D}{2}\right)m^*(m^{*2})^{D/2-1}. \tag{3}$$

At the four dimensional limit Eq.(3) reduces to

$$\langle\bar{\psi}\psi\rangle \to \frac{N_c}{(2\pi)^2}\frac{1}{\varepsilon}m^{*3}. \tag{4}$$

Thus the chiral condensate is divergent at the limit, $\varepsilon \to 0$.

2.2 Diagram with two external lines

The constituent quark mass, m^*, is given by the non-trivial solution for the gap equation,

$$m^* = m + 2G\frac{N_c M^{4-D}}{(2\pi)^{D/2}}\Gamma\left(1-\frac{D}{2}\right)m^*(m^{*2})^{D/2-1}. \quad (5)$$

Taking the four dimensional limit, we obtain

$$m^* \to m - \frac{2N_c G}{(2\pi)^2}\frac{1}{\varepsilon}m^{*3}. \quad (6)$$

The parameter of the model can be fixed to reproduce some of low energy phenomena. It is know that the four-fermion coupling, G, behaves as $O(\varepsilon)$ in the dimensional regularization, see, for example, Refs. [16–18]. Thus the four-fermion coupling vanishes at the limit, $\varepsilon \to 0$, but a finite correction remains at the second term of the right-hand side in Eq.(6).

2.3 Diagram with four external lines

We consider the radiative correction of the four-fermion coupling for the scalar S channel. It is given by the summation of all bubble-type diagrams in the leading order of the 1=N expansion [14],

$$G_s(k^2) = \frac{4G^2}{2G - \Pi(k^2)}, \quad (7)$$

where $\Pi(p^2)$ is

$$\Pi(k^2) = -4G^2 M^{4-D}\int\frac{d^D p}{i(2\pi)^D}\mathrm{tr}\left(\frac{1}{\not p - m^*}\frac{1}{\not p - \not k - m^*}\right). \quad (8)$$

At the four dimensional limit Eq. (8) simplifies to

$$\Pi(k^2) \to G^2\frac{2N_c}{(2\pi)^2}\frac{1}{\varepsilon}(k^2 - 6m^{*2}). \quad (9)$$

Thus the four-point function for the scalar S channel is found to be

$$G_s(k^2) = \frac{2(2\pi)^2}{N_c}\varepsilon\frac{1}{-k^2 + 6m^{*2} + (4\pi)^2\varepsilon/(N_c G)}. \quad (10)$$

It vanishes at the four dimensional limit, but a scalar meson pole is found at

$$k^2 = 6m^{*2} + \frac{(4\pi)^2\varepsilon}{N_c G}. \quad (11)$$

Thus we can obtain a finite scalar meson mass.

3 Order counting of divergence

Finiteness of the radiative corrections can be diagrammatically understood by counting the ε dependence. We consider a diagram with $\#e$ external, $\#i$ internal lines and $\#v_4$ four-fermion interactions. There is a relationship

$$\#e + 2\#i = 4\#v_4. \quad (12)$$

A momentum integral is assigned on each internal lines and a delta function on each vertices. One of the delta functions shows the overall momentum conservation between the external lines. Thus the number of the loop integrals, $\#l$, is given by

$$\#l = \#i - \#v_4 + 1. \quad (13)$$

From Eqs. (12) and (13) we obtain

$$\#e = 2(\#v_4 - \#l + 1). \quad (14)$$

Each loop integrals behave as at most $O(1/\varepsilon)$ in the dimensional regularization. The four-fermion interaction behaves as $O(\varepsilon)$. Thus the order of a diagram with $\#e$ external lines is $O(\varepsilon^{(\#e-2)/2})$. Any odd number is not allowed for $\#e$ in the NJL model. Hence, only the vacuum diagram, $\#e = 0$, is divergent at the four dimensional limit. The diagram with two external lines is finite. If the number of the external lines is greater than two, $\#e > 2$, the diagram have to be vanish at the four dimensional limit. As is known, the NJL model is trivial in four dimensions. It should be noted that the chiral condensate and the constituent quark mass are represented by the vacuum diagram and the diagram with two external lines, respectively.

The order of the each diagram is fixed by only the number of the external lines. The order cannot be modified by any radiative corrections. It means that all the radiative corrections, δ_{rad}, have to be finite in four dimensions.

$$I = I_{tree}(1 + \delta_{rad}), \quad (15)$$

where I and I_{tree} represent the value for a diagram at the full and the tree level, respectively. Some of low energy phenomena can be evaluated from the finite radiative corrections, δ_{rad} [16–18]. The obtained properties are independent on the regularization parameter, ε.

4 Further cancellation

In Sec. 2 we assume that the mass scale, M, is constant and take the four-dimensional limit. The mass scale, M, is introduced to modifies the mass dimension by hand. It is one of parameters of the model to be fixed. If the scale, M, changes as a function of the dimension, D, the four dimensional limit may be modified. According to the procedure in Ref. [16, 18], we find that the mass scale behaves as $M^{4-D} \to O(\varepsilon)$ and $GM^{4-D} \to O(\varepsilon)$. Then we find

$$\langle\bar{\psi}\psi\rangle \to \frac{N_c M^{4-D}}{(2\pi)^2}\frac{1}{\varepsilon}m^{*3}, \tag{16}$$

$$m^* \to m - \frac{2N_c G M^{4-D}}{(2\pi)^2}\frac{1}{\varepsilon}m^{*3}, \tag{17}$$

and

$$G_s(k^2) = \frac{2(2\pi)^2}{N_c}\varepsilon\frac{1}{-k^2 + 6m^{*2} + (4\pi)^2\varepsilon/(N_c G M^{4-D})}. \tag{18}$$

Eqs.(16) and (17) are convergent and Eq.(18) vanishes at the four-dimensional limit.

In the dimensional regularization a loop integral is rescaled in order to have the correct mass dimension.

$$\int \frac{d^D p}{i(2\pi)^D} \to M^{4-D}\int \frac{d^D p}{i(2\pi)^D}. \tag{19}$$

Since the integral behaves as at most $O(1/\varepsilon)$ in the dimensional regularization, all the divergences can be cancelled by the vanishing factor, $M^{4-D} \to O(\varepsilon)$, at the limit, $\varepsilon \to 0$. Thus the vacuum diagram is also convergent at the four dimensional limit.

5 Conclusion

We have studied the regularization parameter dependences of a four-fermion interaction model. The four-fermion interaction vanishes at the four-dimensional limit. Thus the theory reduces to be free at the limit. All the divergence in n-point functions can be canceled not only at the leading order but also at an arbitrary order. If $n > 2$ the diagram vanishes at the four dimensional limit. Some meson properties can be evaluated at this free fermion limit.

In our approach we may drop some part of physics. Here we regard the model as a low energy effective model of QCD. We may drop some parts of physics in the four-fermion model. However, the procedure is useful, if it can describe some QCD phenomena. We hope that the bubble type diagrams, i.e. meson propagation processes, contribute to the results, and some quark propagation processes are dropped.

The renormalizability of the scalar type four-fermion model is not discussed in this paper. It is also interesting to apply a normal renormalization procedure to subtract the divergence. We will continue the work in this direction and hope to publish reports on this problem.

Acknowledgement

The main part of this paper is based on the work in collaboration with Kimura D., Kohyama H. and Kvinikhidze A.

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Received 01.10.2012

Т. Инагаки

НЕЗАВИСИМЫЙ ОТ ПАРАМЕТРА РЕГУЛЯРИЗАЦИИ ПОДХОД В ЧЕТЫРЕХФЕРМИОННОЙ МОДЕЛИ ВЗАИМОДЕЙСТВИЯ

В квантовой хромодинамике непертурбативный эффект является неотъемлемым явлением при низких энергиях. Обычная пертурбативность теряет силу. Четырехфермионная модель взаимодействия используется для оценки явления низких энергий для кварков и глюонов. Так как четырехфермионное взаимодействие неперенормируемо, результаты зависят от параметра регуляризации в четырехмерном пространстве-времени. Здесь мы покажем возможность устранения зависимости параметра регуляризации для некоторых физических наблюдений.

Ключевые слова: *четырехфермионное взаимодействие, размерная регуляризация.*

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