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Particle number production and time variation with non-equilibrium quantum field theory

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We study the particle number production and its time variation using non-equilibrium quantum field theory. We study the model proposed by Hotta et.al. [1] for particle number production with a heavy neutral scalar and a light complex scalar. The interaction Lagrangian contains CP violating phase and particle number violating interaction among the scalars. The particle number violating mass term is also introduced, which splits a complex scalar into two real scalars with small non-degenerate mass. Therefore, the term generates particle and anti-particle mixing. We study the long time behavior of the particle number production rate.

Keywords: *particle number production, non-equilibrium quantum field theory.*

1 Introduction

Study of the mechanism of the particle number production is a very important issue in baryogenesis and leptogensis. In experiments, there are many phenomena which violate the particle number. Such phenomena includes $B\bar{B}$ mixing and neutrino flavor oscillation. While they can be treated with time evolution of pure state, the baryogenesis and leptogenesis occur in the environment where the statistical treatment is suitable. This is because they occur when the definite particle number of the universe is unknown. In this study, We use the non-equilibrium field theory with the density matrix and study the time evolution of the particle number. This paper is organized as follows. In section II, we propose a particle number violating model which consists of a heavy neutral scalar and one complex scalar. In the next section, the current associated with the particle number is written in terms of a Green function of non-equilibrium field theory. In section III, the particle number production rate is computed and its property is discussed. The final section is devoted to conclusion.

2 Lagrangian for the scalar model and Particle Number Production

In the previous paper [1], we proposed the following model for particle number production with the interaction.

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu N\partial^\mu N - \frac{M_N^2}{2}N^2 + \\ &+ \partial_\mu\phi^\dagger\partial^\mu\phi - m_\phi^2\phi^\dagger\phi + A_\phi N\phi^\dagger\phi \\ &+ B^2\phi^2 + AN\phi^2 + h.c.,\end{aligned} \tag{1}$$

where N is a real scalar and ϕ is a complex scalar. There are two types of the interaction. One is particle number conserving interaction which coefficient is given by A_ϕ and the other is particle number violating interaction with the coefficient A. There are also two types of mass term. One of them with the coefficient B^2 violates the particle number and the other one is a particle number conserving one given by the mass term $m_\phi^2\phi^\dagger\phi$. One may take a phase convention that B^2 is real and A is complex. We denote the phase A as $\phi_A = \arg.A$ and it is a source of CP violation. The mass term B^2 breaks U(1) symmetry and it splits one complex scalar fields into the two mass eigenstates of real scalars. Introducing two real scalars as $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$, the Lagrangian is rewritten as,

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(\partial_\mu N\partial^\mu N - M_N^2N^2) \\ &+ \frac{1}{2}\sum_{i=1}^{2}(\partial_\mu\phi_i\partial^\mu\phi_i - m_i^2\phi_i^2) + \sum_{ij}\phi_i\mathcal{A}_{ij}\phi_j N,\end{aligned} \tag{2}$$

with $m_1^2 = m_\phi^2 - B^2, m_2^2 = m_\phi^2 + B^2$. $\mathcal{A}_{ij}$ is a two by two matrix and is given by,

$$\mathcal{A} = \begin{pmatrix} |A|\cos\phi_A + \frac{A_\phi}{2} & -|A|\sin\phi_A \\ -|A|\sin\phi_A & -|A|\cos\phi_A + \frac{A_\phi}{2}\end{pmatrix}. \tag{3}$$

3 Computing the current for U(1) charge; particle number

The particle number associated with the complex field ϕ is a U(1) current.

$$\begin{aligned}j_\mu &= i(\phi^\dagger\partial_\mu\phi - \partial_\mu\phi^\dagger\phi) \\ &= \phi_2\partial_\mu\phi_1 - \partial_\mu\phi_2\phi_1.\end{aligned} \tag{4}$$

We compute the divergence of the U(1) current with some initial condition specified with the density matrix.

$$\langle j_\mu(X)\rangle = \mathrm{Tr}\left(j_\mu(X)\rho(0)\right), \tag{5}$$

where $\rho(0)$ represents an initial quantum statistical state;

$$\rho(0) = \frac{\exp[-\beta(H_0-\mu L)]}{\mathrm{Tr}[\exp[-\beta(H_0-\mu L)]]}. \tag{6}$$

L is U(1) charge given by $L=\int d^3x j_0$ and β is $\frac{1}{T}$ with the temperature T. μ is chemical potential. We assume that at $t=0$ all the interaction terms including U(1) breaking mass term are zero and $t>0$, suddenly they are switched on. Therefore at t=0, Hamiltonian H_0 and particle number L commute as;

$$[H_0, L] = 0, \tag{7}$$

and at later time $t>0$, the U(1) breaking terms are switched on and the particle number is not conserved.

$$[H, L] \neq 0. \tag{8}$$

In $t=0$, Eq.(7), the density matrix can be written by the following product,

$$\rho(0) = \frac{\exp(-\mu L)\exp(-\beta H_0)}{\mathrm{Tr}[\exp(-\mu L)\exp(-\beta H_0)]}. \tag{9}$$

With the property Eq.(7), one can write the density matrix even for non-zero chemical potential case. Using the density matrix, the initial particle number at $t=0$ is given as,

$$\begin{aligned}\langle L(0)\rangle &\equiv \mathrm{Tr}L(0)\rho(0)\\ &= \int \frac{Vd^3k}{(2\pi)^3}\frac{\sinh\mu\beta}{\cosh\beta\omega_k-\cosh\beta\mu}.\end{aligned} \tag{10}$$

When chemical potential is zero, the initial particle number is zero. In the following, we use the density matrix of $\mu=0$ and focus on the particle number production with the interaction.

4 Green function and U(1) current

$U(1)$ current defined in Eq.(5) can be written in terms of the Green function of non-equilibrium field theory; [2], [3], [4],

$$\begin{aligned}G^{12}_{12}(x,y) &= <\phi_1(x)\rho(0)\phi_2(y)>\\ &= \mathrm{Tr}(\phi_2(y)\phi_1(x)\rho(0)).\end{aligned} \tag{11}$$

The lower indices distinguish the species of the light scalar fields; $\phi_i (i=1,2)$. The upper indices distinguish whether the operator is on the time ordered path 1 or on anti-time ordered path 2 in closed time path formulation [5]. Using the definition, the divergence of the averaged current with the density matrix is the production rate per unit time and unit volume. This can be related to the Green function as follows,

$$\frac{\partial}{\partial X^\mu}\langle J^\mu(X)\rangle = (\Box_x-\Box_y)G^{12}_{12}(x,y)\Big|_{x=y=X}. \tag{12}$$

If the space translation invariance holds, the current depends on only time. If this is the case, the divergence of the current is equal to time derivative of the particle number density,

$$\frac{\partial}{\partial X^\mu}\langle J^\mu(X)\rangle = \frac{\partial}{\partial X^0}\langle J^0(X^0)\rangle. \tag{13}$$

With the help of 2 PI (Two Particle Irreducible) formulation of the non-equilibrium quantum field theory [6], we derive Schwinger Dyson equation for the Green functions in [1]. The Schwinger Dyson equation can be solved perturbatively and the divergence of U(1) current obtained in one-loop level. We find,

$$\begin{aligned}&\frac{\partial}{\partial X^\mu}\langle J^\mu(X)\rangle \simeq -4|A|\sin\phi_A A_\phi\\ &\int\frac{d^3p}{2\omega_p(2\pi)^3}\int\frac{d^3k}{2\omega_k(2\pi)^3}\frac{1}{2\omega_N}\\ &\{(n_k+1)n_N(n_p+1)-n_k(n_N+1)n_p\}\\ &\{I(\omega_{2p}+\omega_{2k}-\omega_N, X^0)\\ &-I(\omega_{1p}+\omega_{1k}-\omega_N, X^0)\}.\end{aligned} \tag{14}$$

where we have ignored the terms which is proportional to B^2 and $\omega_{ik}=\sqrt{m_i^2+k^2}(i=1,2)$. The time dependent function I is given as,

$$I(\Omega, X^0) = \frac{\cos\Omega X^0-1}{\Omega}, \tag{15}$$

where n_N, $n_k(n_p)$ denote the thermal equilibrium distribution function for N and ϕ given as,

$$\begin{aligned}n_N &= \frac{1}{\exp(\beta\omega_N(k+p))-1},\\ n_k &= \frac{1}{\exp(\beta\omega_k)-1}.\end{aligned} \tag{16}$$

In the divergence of the current, first we study the factor related to distributions,

$$(n_k+1)n_N(n_p+1)-n_k(n_N+1)n_p. \tag{17}$$

The first term implies the decay N to two light scalars while the second term implies inverse decay. If the energy conservation in the following sense, holds,

$$\omega_k+\omega_p = \omega_N(p+k), \tag{18}$$

then,

$$(n_N+1)n_k n_p = n_N(n_k+1)(n_p+1). \tag{19}$$

The particle number production is cancelled between the decay process and inverse decay process. Therefore the net particle number can not be produced if the energy conservation of Eq.(18) is satisfied. Next we study the coupling constants in Eq.(14). The production rate is proportional to CP violating phase ϕ_A. In addition to CP violation, both types of the interactions, one is CP conserving and particle number conserving one A_ϕ and the other is CP violating and particle number violating one A are required. Finally we study time dependence. We are interested in the time length for which coherence of the two amplitudes in Eq.(14) corresponding to the $N \to \phi_i(k)\phi_i(p)$ $(i = 1, 2)$ is not lost yet. When the coherence remains, one can use the approximation,

$$I(\omega_{2p} + \omega_{2k} - \omega_N, X^0) - I(\omega_{1p} + \omega_{1k} - \omega_N, X^0) \simeq \frac{\sin \Omega_0 X^0}{\Omega_0} \sin\{(\frac{B^2}{2\omega_k} + \frac{B^2}{2\omega_p})X^0\}, \quad (20)$$

where $\Omega_0 = \omega_N - \omega_k - \omega_p$. We have used the approximation,

$$\omega_{2k} = \sqrt{k^2 + m_\phi^2 + B^2} = \omega_k + \frac{B^2}{2\omega_k},$$
$$\omega_{1k} = \sqrt{k^2 + m_\phi^2 - B^2} = \omega_k - \frac{B^2}{2\omega_k}.$$

We consider when X^0 is large and is proportional to $\frac{1}{B^2}$. In the limit of small B^2 with fixed B^2X^0, the time dependent factor becomes,

$$I(\omega_{2p} + \omega_{2k} - \omega_N, X^0) - I(\omega_{1p} + \omega_{1k} - \omega_N, X^0) \simeq \pi\delta(\Omega_0) \sin\{(\frac{B^2}{2\omega_k} + \frac{B^2}{2\omega_p})X^0\}. \quad (21)$$

Substituting the time dependent factor, the divergence of the current is,

$$\frac{\partial}{\partial X^\mu}\langle J^\mu(X)\rangle \simeq -4|A| \sin\phi_A A_\phi \int \frac{d^3p}{2\omega_p(2\pi)^3} \int \frac{d^3k}{2\omega_k(2\pi)^3} \frac{1}{2\omega_N} \pi\delta(\omega_N - \omega_k - \omega_p) \{(n_k + 1)n_N(n_p + 1) - n_k(n_N + 1)n_p\} \times \sin\{(\frac{B^2}{2\omega_k} + \frac{B^2}{2\omega_p})X^0\}. \quad (22)$$

The presence of the delta function implies the energy conservation $\omega_N = \omega_k + \omega_p$ holds. Therefore the decay contribution and inverse decay contribution are cancelled each other. Below, we consider only the decay contribution.

$$\frac{\partial}{\partial X^\mu}\langle J^\mu(X)\rangle\Big|_{\text{Decay}} \simeq -4\pi|A| \sin\phi_A A_\phi \int \frac{d^3p}{2\omega_p(2\pi)^3} \int \frac{d^3k}{2\omega_k(2\pi)^3} \frac{1}{2\omega_N} \delta(\omega_N - \omega_k - \omega_p) \frac{e^{\beta(\omega_k+\omega_p)}}{(e^{\beta\omega_k} - 1)(e^{\beta\omega_k} - 1)(e^{\beta\omega_N} - 1)} \times \sin\{(\frac{B^2}{2\omega_k} + \frac{B^2}{2\omega_p})X^0\}$$
$$\simeq -\frac{1}{64\pi^3}|A|A_\phi T^2 \sin\phi_A \int_{\beta m_N}^\infty \frac{dU}{\sinh\frac{U}{2}} \int_0^{V_{max}(U)} dV \frac{\sin\frac{2\beta B^2 X^0 U}{U^2 - V^2}}{\cosh\frac{U}{2} - \cosh\frac{V}{2}}, \quad (23)$$

where

$$U = \beta(\omega_k + \omega_p), \quad V = \beta(\omega_k - \omega_p), \quad (24)$$

and

$$V_{max} = \sqrt{(1 - \frac{4m_\phi^2}{m_N^2})(U^2 - \beta^2 m_N^2)}. \quad (25)$$

We introduce the following dimensionless quantities,

$$\hat\beta = \beta m_\phi, \hat X^0 = m_\phi X^0, \hat B = \frac{B}{m_\phi}, \hat m_N = \frac{m_N}{m_\phi}. \quad (26)$$

To study the long time behaviour of the production rate $\hat X^0 \sim \frac{1}{\hat B^2}$, we define the rescaled time t,

$$t = \frac{B^2 X^0}{m_\phi \pi} = \frac{\hat B^2 \hat X^0}{\pi}. \quad (27)$$

Using Eq.(27), one may write the time dependent part as,

$$F[t, \hat\beta] = \frac{1}{\hat\beta^2} \int_{\hat\beta \hat m_N}^\infty \frac{dU}{\sinh\frac{U}{2}} \times \int_0^{V_{max}(U)} dV \frac{\sin\frac{2\pi\hat\beta U t}{U^2 - V^2}}{\cosh\frac{U}{2} - \cosh\frac{V}{2}}. \quad (28)$$

In Fig.1, we show the time dependent factor Eq.(28) of the production rate as a function of the dimensionless rescaled time t for three inverse temperature $\hat\beta = \frac{1}{10}, \frac{1}{20}$,and $\frac{1}{30}$. We choose $\hat m_N = 20$. We can see the the production rate oscillates and the amplitude decreases. As temperature grows, the production rate becomes larger.

5 Conclusion

- We compute the time variation of the particle number with a scalar model.

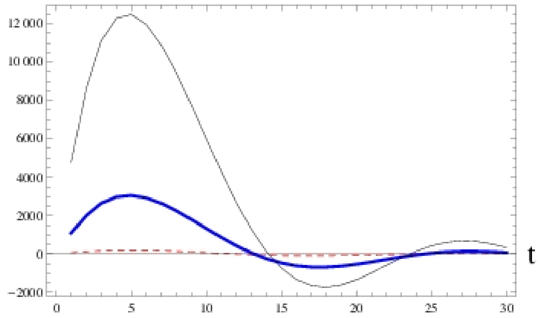

Figure 1: Time dependence of the production rate of the particle number density. The dashed line corresponds to $\hat{\beta} = \frac{1}{10}$, the thick solid line correspondnds to $\hat{\beta} = \frac{1}{20}$, and the thin solid line corresponds to $\hat{\beta} = \frac{1}{30}$.

- In the model, a neutral scalar and one complex scalar are included. U(1) charge related to the complex scalar is the particle number.
- The particle number and CP symmetry are violated due to the mass term and the interaction.
- Due to U(1) soft-breaking term, one complex scalar is not a mass eigenstate and mass eigenstates are two real scalars with non-degenerate mass.
- We have computed the time dependence of the production rate of the particle number density.
- The rate oscillates and is damping in their amplitude. The oscillation period is larger for the smaller non-degeneracy.
- Because of the damping, the particle number produced during the first half cycle of the oscillation, may remain after the integration of the rate with respect to time.
- We have not included the effect of the expanding universe, which effect leads to the decay and inverse decay contribution may not cancel.
- As an extension of our work, one can consider the case for $< L(0) > \neq 0$ and how the initial particle number is washed out. Also we may apply the method to flavored leptogenesis; $L_e(t), L_\mu(t), L_\tau(t)$.

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ОПЕРАТОР ЧИСЛА ЧАСТИЦ И ВРЕМЕННАЯ ЭВОЛЮЦИЯ В НЕРАВНОВЕСНОЙ КВАНТОВОЙ ТЕОРИИ ПОЛЯ

Мы изучаем временную эволюцию оператора числа частиц в рамках неравновесной квантовой теории поля. Используется модель, предложенная Хоттом и др. [Hotta R., Morozumi T. and Takata H. AIP Conf. Proc. **1467**, 239 (2012)] для оператора числа частиц с тяжелым нейтральным скаляром и легким комплексным скаляром.

Лагранжиан взаимодействия содержит CP нарушающие фазы и число частиц, нарушающие взаимодействие между скалярами. Вводится также число частиц, нарушающее массовый член, который расщепляет комплексные скаляры на два вещественных скаляра с малой невырожденной массой. Таким образом, возникает член в лагранжиане, порождающий смешивание частиц и античастиц. Мы изучаем поведение скорости рождения частиц на больших временах.

Ключевые слова: *particle number production, неравновесная квантовая теория поля.*

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