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SHOCK WAVES IN LIFSHITZ-LIKE BACKGROUNDS AND HOLOGRAPHIC DESCRIPTION OF QGP

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A holographic duality between gravitation and field theories with a Lifshitz scaling symmetry has recently aroused much attention. We consider Lifshitz-like backgrounds in the context of their applications to the anisotropic quark-gluon plasma created in heavy-ion collisions. We construct shock waves geometries for Lifshitz-like metrics[1]. For the geometry of two colliding Lifshitz domain walls the areas of trapped surfaces are calculated. According to the holographic approach, the multiplicity of particles produced in heavy-ion collisions can be estimated by the area of the trapped surface. We show that in the 5-dimensional case the dependence of multiplicity on the energy of colliding ions can fit the experimental data observed at RHIC and LHC.

Keywords: *Lifshitz-like metrics, shock waves, trapped surface, holography, quark-gluon plasma.*

1 Introduction

Applications of AdS/CFT correspondence to studies of the strongly coupled quark-gluon plasma have generated many important results. However, the properties of the QGP at the early stage of the heavy-ion collisions have not well understood yet. The elementary dual models considered in [2] require modification [3, 4] to describe the experimental data.

In [5] a holographic duality between gravity in anisotropic backgrounds and field theories with scale transformations

$$t \to \lambda^{\nu} t, \quad \vec{x} \to \lambda \vec{x}, \quad r \to \frac{1}{\lambda} r, \tag{1}$$

where ν is the so-called dynamical Lifshitz exponent.

In [6] Lifshitz geometries were generalized to anisotropic Lifshitz-like ones with the metric

$$ds^2 = L^2 \left(r^{2\nu} \left(-dt^2 + \sum_{i=1}^{p} dx_i^2 \right) + r^2 \sum_{j=1}^{q} dy_j^2 + \frac{dr^2}{r^2} \right), \tag{2}$$

where $p > 0$, $q > 0$ and $p + q + 2 = D \equiv d + 2$. We will refer to these metrics as Lifshitz-like metrics **(p,q)**-type ($Lif_{(p,q)}$). The line element (2) is invariant under a generalized scaling

$$(t, x_i, y_i, r) \to \left(\lambda^{\nu} t, \lambda^{\nu} x_i, \lambda y_i, \frac{r}{\lambda} \right). \tag{3}$$

We note that the scaling (3) differs from (1) by the extension of the anisotropy for space coordinates x_i. This allows to consider Lifshitz-like backgrounds (2) with the scaling (3) in the context of applications to the quark-gluon plasma [7], which is known to be spatially anisotropic at the very early stages of heavy-ion collisions.

[1] This note is based on the collaboration with I.Ya. Aref'eva [1].

Following to the holographic approach, the quark-gluon plasma formation can be examined through the analysis of shock waves. In this note, we study shock waves in the Lifshitz-like spacetimes (2) with an arbitrary dynamical exponent ν. Our main goal here is to construct shock wave geometries and show that estimations of multiplicities via areas of trapped surfaces are rather close to the experimental data.

2 Shock waves in D-dimensional Lifshitz-like spacetimes

Here we build shock waves in Lifshitz-like geometries of arbitrary dimensions (2). One can represent the metric (2) in terms of the coordinate $\rho = r^{1/\nu}$ as follows

$$ds^2 = L^2 \Big(\rho^2 (-dt^2 + \sum_{i=1}^{p} dx_i^2) + \rho^{\frac{2}{\nu}} \sum_{j=1}^{q} dy_j^2 + \frac{d\rho^2}{\rho^2} \Big). \tag{4}$$

To construct shock waves here and what follows we use the approach from the work [10]. There it is shown that for the d-dimensional metric

$$ds^2 = A(u,v) du\, dv + g(u,v) h_{ij}(x) dx^i dx^j, \tag{5}$$

with $i, j, = 1, 2, \ldots, d-2$, sourced some matter fields and cosmological constant, one can represent a shock wave solution as a metric of a light-like particle located at $u = 0$ and moving with the speed of light in the v-direction in the background (5)

$$ds^2 = 2A du\, dv - 2Af\delta(u) du^2 + g h_{ij} dx^i dx^j, \tag{6}$$

with the only non-zero component T_{uu} of the stress-energy tensor

$$T_{uu} = -4pA^2 \delta^{d-2}(x) \delta(u), \tag{7}$$

where δ is the Dirac delta-function and p is the momentum of the particle.

To write down the shock wave metric we introduce the light-cone coordinates $u = t - x_p$, $v = t + x_p$ and define the holographic coordinate $z = 1/r$. Now the metric ansatz for the shock wave in the geometry (4) takes the form

$$ds^2 = L^2\Big(z^{-2}(\phi(\vec{x}_{p-1}, \vec{y}_q, z)du^2 - dudv + \sum_{i=1}^{p-1} dx_i^2) + z^{-2/\nu}\sum_{j=1}^{q} dy_j^2 + \frac{dz^2}{z^2}\Big). \quad (8)$$

One can show that the profile function obeys the equation

$$\left[\Box_{Lif_{(p+q)}} - \frac{1}{L^2}\left(p + \frac{q}{\nu}\right)\right]\frac{\phi(x_i, y_j, z)}{z} = -2zJ_{uu}, \quad (9)$$

where $i = 1, \ldots p-1, j = 1, \ldots q$, J_{uu} is the density related with stress-energy tensor, and $\Box_{Lif_{(p+q)}}$ is the Laplace-Beltrami operator defined on the $(p+q)$-dimensional Lifshitz-like spacetime

$$ds^2_{Lif_{(p+q)}} = \frac{1}{z^2}\sum_{i=1}^{p-1} dx_i^2 + \frac{1}{z^{2/\nu}}\sum_{j=1}^{q} dy_j^2 + \frac{dz^2}{z^2}. \quad (10)$$

The shape of the shock wave is defined by the solution to (9), which looks to be rather complicated. In works [3, 4] a simpler form of shock waves called domain walls was suggested. To derive the equation for the profile, one should consider the mass of a point-like source averaged over the domain-wall. The profile of the domain wall has the dependence only on the holographic coordinate z and obeys the equation

$$\left[\Box_{Lif_{(p+q)}} - \tfrac{1}{L^2}\left(p + \tfrac{q}{\nu}\right)\right]\tfrac{\phi(z)}{z} = -16\pi G_D E z J_{uu}. \quad (11)$$

Eq. (11) is reduced to the following form

$$\frac{\partial^2\phi(z)}{\partial z^2} - \left(p + \frac{q}{\nu}\right)\frac{1}{z}\frac{\partial\phi(z)}{\partial z} = -16\pi G_D J_{uu} \quad (12)$$

with the source chosen as

$$J_{uu} = E\left(\frac{z_*}{L}\right)^{p+\frac{q}{\nu}}\delta(z - z_*). \quad (13)$$

The solution to eq. (12) with (13) can be represented as

$$\phi = \phi_a\Theta(z_* - z) + \phi_b\Theta(z - z_*), \quad (14)$$

with the profile functions given by

$$\phi_a(z) = C_0 z_a z_b\left(\frac{z_*^{p+1+\frac{q}{\nu}}}{z_b^{p+1+\frac{q}{\nu}}} - 1\right)\left(\frac{z^{p+1+\frac{q}{\nu}}}{z_a^{p+1+\frac{q}{\nu}}} - 1\right), \quad (15)$$

$$\phi_b(z) = C_0 z_a z_b\left(\frac{z_*^{p+1+\frac{q}{\nu}}}{z_a^{p+1+\frac{q}{\nu}}} - 1\right)\left(\frac{z^{p+1+\frac{q}{\nu}}}{z_b^{p+1+\frac{q}{\nu}}} - 1\right), \quad (16)$$

$$C_0 = -\frac{16\pi G_D E\nu z_a^{p+\frac{q}{\nu}} z_b^{p+\frac{q}{\nu}}}{(p\nu + q + \nu)L^{p+\frac{q}{\nu}+2}(z_b^{p+\frac{q}{\nu}+1} - z_a^{p+\frac{q}{\nu}+1})}. \quad (17)$$

Now let's turn to the consideration of the collision of two shock domain walls, as a model of heavy ion collisions. According to the proposal of [2], a collision of two nuclei in the bulk can be interpreted as a line element for two identical shock waves propagating towards one another in the gravity dual background. Here following [3, 4] we consider the collision of two shock domain walls with the metric before the collision given by

$$ds^2 = L^2\Big(-\tfrac{dudv}{z^2} + \tfrac{\phi_1(y,z)}{z^2}\delta(u)du^2 + \tfrac{\phi_2(y,z)}{z^2}\delta(v)dv^2 + \textstyle\sum_{i=1}^{p-1} dx_i^2) + z^{-2/\nu}\sum_{j=1}^{q} dy_j^2 + \tfrac{dz^2}{z^2}\Big). \quad (18)$$

The trapped surface formed in the wall-on-wall collision obeys the equation (12) and the following conditions must be satisfied for its formation

$$(\partial_z\phi)|_{z=z_a} = 2, \quad (\partial_z\phi)|_{z=z_b} = -2, \quad (19)$$

with the assumption for the points $z_a < z_* < z_b$.

The conditions (19) take the following form

$$\frac{8\pi G_{(D)}Ez_a^{p+\frac{q}{\nu}}\left(1 - z_b^{p+\frac{q}{\nu}+1}/z_*^{p+\frac{1}{\nu}+1}\right)}{L^{p+\frac{q}{\nu}+2}\left(z_b^{p+\frac{q}{\nu}+1}/z_*^{p+\frac{q}{\nu}+1} - z_a^{p+\frac{q}{\nu}+1}/z_*^{p+\frac{q}{\nu}+1}\right)} = -1,$$
$$\frac{8\pi G_{(D)}Ez_b^{p+\frac{q}{\nu}}\left(1 - z_b^{p+\frac{q}{\nu}+1}/z_*^{p+\frac{q}{\nu}+1}\right)}{L^{p+\frac{q}{\nu}+2}\left(z_b^{p+\frac{q}{\nu}+1}/z_*^{p+\frac{q}{\nu}+1} - z_a^{p+\frac{q}{\nu}+1}/z_*^{p+\frac{q}{\nu}+1}\right)} = 1. \quad (20)$$

The relations between the collision and boundary points read

$$z_a = \left(\frac{z_b^{p+\frac{q}{\nu}}}{-1 + z_b^{p+\frac{q}{\nu}}C}\right)^{\frac{\nu}{\nu p+q}},$$
$$z_* = \left(\frac{z_a^{p+\frac{q}{\nu}} z_b^{p+\frac{q}{\nu}}}{z_a^{p+\frac{q}{\nu}} + z_b^{p+\frac{q}{\nu}}}\right)^{\frac{\nu}{p\nu+\nu+q}}, \quad (21)$$

where $C = \dfrac{8\pi G_{(D)}E}{L^{p+\frac{q}{\nu}+2}}$.

One can calculate the area of the trapped surface using the relation

$$S = \frac{1}{2G_D}\int_C \sqrt{\det|g_{Lif_{(p+q)}}|} \times dzdx_1\dots dx_{p-1}dy_1\dots dy_q, \quad (22)$$

with the determinant $\det|g_{Lif_{(p+q)}}|$ of the $(p+q)$-dimensional Lifshitz space (10)

$$\det|g_{Lif_{(p+q)}}| = z^{-(p+\frac{q}{\nu})}. \quad (23)$$

Thus, the relative entropy is given by

$$s = \frac{1}{2G_5}\frac{\nu}{\nu p+q-\nu}\left(\frac{z_a}{z^{p+\frac{q}{\nu}}{}_a} - \frac{z_b}{z_b^{p+\frac{q}{\nu}}}\right). \quad (24)$$

3 Shock waves in 5d Lifshitz-like spacetimes

Consider a five-dimensional Lifshitz-like metric given by (2) with $p=1$, $q=2$ written in terms of the coordinate $z=r^{-\nu}$

$$ds^2 = L^2\Big[\frac{(-dt^2+dx^2)}{z^2} + \frac{(dy_1^2+dy_2^2)}{z^{2/\nu}} + \frac{dz^2}{z^2}\Big]. \quad (25)$$

The spacetime (25) for $\nu=1$ represents the Poincare patch of a 5-dimensional anti-de Sitter space.

The shock wave moving in the v-direction is given by

$$ds^2 = L^2\Big[\frac{\phi(y_1,y_2,z)\delta(u)}{z^2}du^2 - \frac{dudv}{z^2} + \frac{(dy_1^2+dy_2^2)}{z^{2/\nu}} + \frac{dz^2}{z^2}\Big] \quad (26)$$

with the profile function $\phi(y_1,y_2,z)$ satisfying the following equation

$$\left[\Box_{Lif_3} - \frac{1}{L^2}\left(1+\frac{2}{\nu}\right)\right]\frac{\phi(y_1,y_2,z)}{z} = -2zt_{uu}. \quad (27)$$

Eq. (27) can be obtained from the uu-component of the Einstein equations using expressions for R_{ij}, R and the cosmological constant. The Laplace-Beltrami operator $\Box_{Lif_3}$ has the following form

$$\Box_{Lif_3} = \frac{1}{\nu L^2}\Big(z^2\nu\frac{\partial^2}{\partial z^2} + \nu z\frac{\partial}{\partial z} - 2z\frac{\partial}{\partial z} + z^{2/\nu}\nu\frac{\partial^2}{\partial y_1^2} + \nu z^{2/\nu}\frac{\partial^2}{\partial y_2^2}\Big) \quad (28)$$

and is defined on the three-dimensional Lifshitz space with the metric

$$ds^2 = L^2\left[\frac{(dy_1^2+dy_2^2)}{z^{2/\nu}} + \frac{dz^2}{z^2}\right]. \quad (29)$$

Putting the dynamical exponent $\nu=1$, eq. (27) comes to the well-known equation for the 5d AdS-shock wave

$$\left[\Box_{H_3} - \frac{3}{L^2}\right]\frac{\phi(y_1,y_2,z)}{z} = -2zt_{uu}. \quad (30)$$

The equation for the domain-wall profile in the five-dimensional Lifshitz-like space is

$$\left[\Box_{Lif_3} - \frac{1}{L^2}\left(1+\frac{2}{\nu}\right)\right]\frac{\phi(z)}{z} = -16\pi G_5 zJ_{uu}, \quad (31)$$

which can be represented in the form

$$\frac{\partial^2\phi(z)}{\partial z^2} - \left(1+\frac{2}{\nu}\right)\frac{1}{z}\frac{\partial\phi(z)}{\partial z} = -16\pi G_5 J_{uu}, \quad (32)$$

with the source given by

$$J_{uu} = E\left(\frac{z}{L}\right)^{1+2/\nu}\delta(z-z_*). \quad (33)$$

The solution to the domain wave profile reads

$$\phi = \phi_a\Theta(z_*-z) + \phi_b\Theta(z-z_*), \quad (34)$$

where the profile functions are

$$\phi_a(z) = C_0 z_a z_b\left(\frac{z_*^{2(\nu+1)/\nu}}{z_b^{2(\nu+1)/\nu}} - 1\right) \times\left(\frac{z^{2(\nu+1)/\nu}}{z_a^{2(\nu+1)/\nu}} - 1\right), \quad (35)$$

$$\phi_b(z) = C_0 z_a z_b\left(\frac{z_*^{2(\nu+1)/\nu}}{z_a^{2(\nu+1)/\nu}} - 1\right) \times\left(\frac{z^{2(\nu+1)/\nu}}{z_b^{2(\nu+1)/\nu}} - 1\right), \quad (36)$$

$$C_0 = -\frac{8\nu\pi G_5 E z_a^{1+2/\nu} z_b^{1+2/\nu}}{(\nu+1)L^{3+\frac{2}{\nu}}(z_b^{2(\nu+1)/\nu} - z_a^{2(\nu+1)/\nu})}. \quad (37)$$

The line element for colliding shocks in the 5d Lifshitz-like background can be represented as:

$$ds^2 = L^2\Big[-\frac{1}{z^2}dudv + \frac{1}{z^2}\phi_1(y_1,y_2,z)\delta(u)du^2 + \frac{1}{z^2}\phi_2(y_1,y_2,z)\delta(v)dv^2 + \frac{1}{z^{2/\nu}}\left(dy_1^2+dy_2^2\right) + \frac{dz^2}{z^2}\Big]. \quad (38)$$

The trapped surface obeys the following conditions for the formation

$$\frac{8\pi G_5 E z_a^{1+\frac{2}{\nu}}\left(1 - z_b^{\frac{2}{\nu}+2}/z_*^{\frac{2}{\nu}+2}\right)}{L^{3+\frac{2}{\nu}}\left(z_b^{\frac{2}{\nu}+2}/z_*^{\frac{2}{\nu}+2} - z_a^{\frac{2}{\nu}+2}/z_*^{\frac{2}{\nu}+2}\right)} = -1,$$

$$\frac{8\pi G_5 E z_b^{1+\frac{2}{\nu}}\left(1 - z_b^{\frac{2}{\nu}+2}/z_*^{\frac{2}{\nu}+2}\right)}{L^{3+\frac{2}{\nu}}\left(z_b^{\frac{2}{\nu}+2}/z_*^{\frac{2}{\nu}+2} - z_a^{\frac{2}{\nu}+2}/z_*^{\frac{2}{\nu}+2}\right)} = 1. \quad (39)$$

Owing to (39) we obtain the relations between the collision and boundary points

$$z_a = \left(\frac{z_b^{1+\frac{2}{\nu}}}{-1+z_b^{1+\frac{2}{\nu}}C}\right)^{\frac{\nu}{\nu+2}},$$

$$z_* = \left(\frac{z_a^{1+\frac{2}{\nu}}z_b^{1+\frac{2}{\nu}}(z_a+z_b)}{z_a^{1+\frac{2}{\nu}}+z_b^{1+\frac{2}{\nu}}}\right)^{\frac{\nu}{2\nu+2}}, \quad (40)$$

where $C = \dfrac{8\pi G_5 E}{L^{\frac{2}{\nu}+3}}$.

The determinant $\det|g_{Lif_3}|$ of the metric (29) is

$$\det|g_{Lif_3}| = z^{-(1+\frac{2}{\nu})}. \quad (41)$$

Thus, we have for the relative area of the trapped surface

$$s = \frac{\nu}{4G_5}\left(\frac{1}{(z_a)^{2/\nu}} - \frac{1}{(z_b)^{2/\nu}}\right). \quad (42)$$

For large values of $z_b \to \infty$ we get

$$s(C, z_b) = \left(\tfrac{C}{2}\right)^{\frac{2}{2+\nu}} - \left(\tfrac{1}{z_b}\right)^{\frac{2}{\nu}} - \tfrac{2}{(\nu+2)}\left(\tfrac{2}{C}\right)^{\frac{2}{2+\nu}}\left(\tfrac{1}{z_b}\right)^{\frac{2+\nu}{\nu}} + \quad \ldots \quad . \quad (43)$$

The maximum value of the relative area s at infinite z_b takes the form

$$s|_{z_b\to\infty} = \frac{\nu}{4G_5}(8\pi G_5)^{2/(\nu+2)}E^{2/(\nu+2)}. \quad (44)$$

We can see that relations (35)–(37) and (44) are similar to those for the 5-dimensional AdS background deformed by the power-law factor $b = \dfrac{L}{z^a}$ from [9] with the profile equation

$$\frac{\partial^2\phi(z)}{\partial z^2} - \frac{3a}{z}\frac{\partial\phi(z)}{\partial z} = -16\pi G_5\left(\frac{z}{L}\right)^{3a}E^*\delta(z-z_*). \quad (45)$$

One can conclude that the following relation between the constant a and the Lifshitz exponent ν takes place

$$1 + \frac{2}{\nu} = 3a. \quad (46)$$

In [9] it is shown that the b-factor with $a = 1/2$ gives rise to the value of multiplicity, which is the most compatible to the experimental data. Thus, one should consider wall-on-wall-collisions in the Lifshitz-like spacetime with $\nu = 4$.

4 Shock waves on intersecting D3-D7 branes

In this section we consider constructing shock waves on a configuration of supergravity type IIB that describes intersecting $D3-D7$ branes. This solution represents the IR part of the zero-temperature solution from [7,8]. The intersecting branes are defined on a ten-dimension manifold $\mathcal{M}$ factorized as $\mathcal{M} = M_5 \times X_5$, where X_5 is an Einstein manifold and M_5 is a manifold with anisotropy. The metric of the $D3-D7$-intersection is given by

$$ds_E^2 = \tilde{R}^2\left[r^3(-dt^2 + dx^2 + dy^2) + r^2dw^2 + \tfrac{dr^2}{r^2}\right] + R^2ds^2_{X_5}. \quad (47)$$

The anisotropic part of the spacetime (47) coincides with (2) for $p = 2$, $q = 3$ and the critical exponent $\nu = 3/2$. In other words it realises the second possible Lifshitz-like geometry in $D = 5$ – type **(2,1)**.

Introducing light-cone coordinates $du = dt - dx$ and $dv = dt + dx$ and rewriting (47) in terms of the coordinate $z = 1/\rho^{2/3}$, one comes

$$ds^2 = \tilde{R}^2\left[-\tfrac{dudv}{z^2} + \tfrac{dy^2}{z^2} + \tfrac{dw^2}{z^{4/3}} + \tfrac{dz^2}{z^2}\right] + R^2ds^2_{X_5}. \quad (48)$$

We consider a shock wave propagating on the common brane worldvolume. The profile function $\Phi = \dfrac{\phi}{z}$ depends on coordinates of the overall transverse space. For simplicity, we restrict ourselves to the case $\phi = \phi(z)$ that yields the domain wall geometry. Thus, the metric of the shock is given by

$$ds^2 = \tilde{R}^2\left[\tfrac{\phi(z)}{z^2}\delta(u)du^2 - \tfrac{dudv}{z^2} + \tfrac{dy^2}{z^2} + \tfrac{dw^2}{z^{4/3}} + \tfrac{dz^2}{z^2}\right]. \quad (49)$$

The equation for the profile can be represented in the form

$$\frac{\partial^2\phi(z)}{\partial z^2} - \frac{8}{3z}\frac{\partial\phi(z)}{\partial z} = -16\pi G_5E^*z^{8/3}\delta(z-z_*), \quad (50)$$

Eq. (50) has the following solution for the profile

$$\phi(z) = \phi_a\Theta(z_* - z) + \phi_b\Theta(z - z_*), \quad (51)$$

where

$$\phi_a = C_0z_az_b\left(\left(\frac{z_*}{z_b}\right)^{11/3} - 1\right)\left(\left(\frac{z}{z_a}\right)^{11/3} - 1\right),$$

$$\phi_b = C_0z_az_b\left(\left(\frac{z_*}{z_a}\right)^{11/3} - 1\right)\left(\left(\frac{z}{z_b}\right)^{11/3} - 1\right), \quad (52)$$

with

$$C_0 = -\frac{48\pi G_5E}{11\tilde{R}^{14/3}}\frac{z_a^{8/3}z_b^{8/3}}{(z_b^{11/3} - z_a^{11/3})}. \quad (53)$$

Now consider colliding domain walls with the metric before the collision

$$ds^2 = -\frac{1}{z^2}dudv + \frac{1}{z^2}\phi_1(z)\delta(u)du^2 + \frac{1}{z^2}\phi_2(z)\delta(v)dv^2 + \frac{dy^2}{z^2} + \frac{dw^2}{z^{4/3}} + \frac{dz^2}{z^2}. \quad (54)$$

The trapped surface can be formed if the following boundary conditions are satisfied

$$\frac{8\pi G_5Ez_a^{8/3}\left(1 - \frac{z_b^{11/3}}{z_*^{11/3}}\right)}{\tilde{R}^{14/3}\left(\frac{z_b^{11/3}}{z_*^{11/3}} - \frac{z_a^{11/3}}{z_*^{11/3}}\right)} = -1,$$

$$\frac{8\pi G_5Ez_b^{8/3}\left(1 - \frac{z_a^{11/3}}{z_*^{11/3}}\right)}{\tilde{R}^{14/3}\left(\frac{z_b^{11/3}}{z_*^{11/3}} - \frac{z_a^{11/3}}{z_*^{11/3}}\right)} = 1. \quad (55)$$

From (55) we derive the relations

$$z_a = \frac{z_b}{\left(-1 + z_b^{8/3} C\right)^{3/8}},$$

$$z_* = z_a^{8/11} z_b^{8/11} \left(\frac{z_a + z_b}{z_a^{8/3} + z_b^{8/3}}\right)^{3/11}, \qquad (56)$$

with $C = 8\pi G_5 E/\tilde{R}^{14/3}$. To calculate the trapped surface area one should consider the $3d$ Lifshitz metrics

$$ds^2 = \frac{1}{\tilde{R}^2}\left(\frac{dy^2}{z^2} + \frac{dw^2}{z^{4/3}} + \frac{dz^2}{z^2}\right), \qquad (57)$$

with the determinant

$$\det|g_{Lif_3}| = z^{-\frac{16}{3}}. \qquad (58)$$

Now we can calculate the relative entropy

$$s = \frac{3}{10 G_5}\left(\frac{\tilde{R}}{z_a^{5/3}} - \frac{\tilde{R}}{z_b^{5/3}}\right). \qquad (59)$$

For large z_b one obtains

$$s(C, z_b) = \left(\frac{C}{2}\right)^{\frac{5}{8}} - \left(\frac{1}{z_b}\right)^{\frac{5}{3}} - \frac{5}{8}\left(\frac{2}{C}\right)^{\frac{3}{8}}\left(\frac{1}{z_b}\right)^{\frac{3}{8}} + \ldots. \qquad (60)$$

The relative area s of the trapped surface has the maximum value at infinite z_b

$$s|_{z_b\to\infty} = \frac{3\tilde{R}}{10 G_5}\left(\frac{8\pi G_5}{\tilde{R}^2}\right)^{5/8} E^{5/8}. \qquad (61)$$

5 Conclusion

Here we have constructed shock waves in D-dimensional Lifshitz-like geometries with the arbitrary dynamical exponent. We have found the solutions to the profile equations in the case of domain walls. We have considered wall-on-wall collisions and calculated areas of trapped surfaces.

As it is known, in the holographic language, the problem of the quark-gluon plasma formation is related to formation of bulk black holes, and one can estimate entropies of these black holes using areas of trapped surfaces. We have shown, that our estimate of the entropy for the five-dimensional Lifshitz spacetime with the critical exponent $\nu = 4$ can fit the experimental data for multiplicities. This result matches to that one calculated for the AdS background deformed by the power-law factor $b(z) = \left(\frac{L_{eff}}{z}\right)^{1/2}$ and supported by a phantom dilaton field [9].

An interesting extension of this work would be constructing solutions interpolating between AdS and Lifshitz-like backgrounds with non-zero temperature as it was done in [7] to track the process of isotropization.

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УДАРНЫЕ ВОЛНЫ В ЛИФШИЦЕВЫХ ПРОСТРАНСТВАХ И ГОЛОГРАФИЧЕСКОЕ ОПИСАНИЕ КВАРК-ГЛЮОННОЙ ПЛАЗМЫ

В последнее время голографическая дуальность между гравитацией и полевыми теориями, обладающими скейлингом Лифшица, привлекает много внимания. Мы рассматриваем лифшицевы пространства в контексте приложения к описанию анизотропной кварк-глюонной плазмы, возникающей при столкновении тяжелых ионов. Построены структуры типа ударных волн. Для случая столкновения доменных стенок в пространствах Лифшица вычислены площади ловушечных поверхностей. Согласно голографическому подходу, множественность частиц, рождающихся во время столкновения тяжелых ионов, можно оценить с помощью значения площади такой ловушечной поверхности. Показано, что для пятимерного случая зависимость множественности от энергии сталкивающихся ионов близка к экспериментальным данным, полученным на RHIC и LHC.

Ключевые слова: *лифшицевы метрики, ударные волны, ловушечная поверхность, голография, кварк-глюонная плазма.*

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