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Triple M-brane solutions and supersymmetry

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We study composite M-brane solutions in 11-dimensional supergravity. The supersymmetric solutions describing orthogonally intersecting M-branes are defined on the product of Ricci-flat manifolds M_i. The amount of preserved supersymmetries depends upon certain numbers of (chiral) parallel spinors on factor spaces M_i and brane sign factors. Three examples of triple M-brane configurations are considered and the numbers of unbroken SUSY are obtained.

Keywords: *M-branes, parallel spinors, supergravity, supersymmetry.*

1 Introduction

Solutions to supergravity theories preserving some amount of supersymmeries play an important role in studies in non-pertubative M-theory and correspondences between gravity and gauge theory.

We consider intersecting M-brane solutions defined on the manifold of the form

$$M = M_0 \times M_1 \times \ldots \times M_n, \tag{1}$$

where M_i are Ricci-flat manifolds. In what follows we denote $d_i = \dim M_i$. In [1] the classification of supersymmetric M-brane configurations on product of flat factor spaces $\mathbb{R}^{d_i}$ was presented and the relation for the amount of preserved supersymmetry was found:

$$\mathcal{N} = 2^{-k}, \quad \text{with} \quad k = 1,2,3,4,5. \tag{2}$$

It was shown in [2] that for the simplest $M5$-brane configuration to obey the supergravity equations of motions, the brane world volume should be Ricci-flat and admit parallel spinors for supersymmetry. The relation (2) is no more valid if composite M-brane configurations on the product of Ricci-flat manifolds (1) are taken into consideration. In this case $\mathcal{N}$ depends upon certain numbers of chiral parallel (i.e. covariantly constant) spinors on M_i and brane sign factors $c_s = \pm 1$.

In this note we study supersymmetric solutions defined on the manifold (1) for triple M-brane configurations in $D = 11$ supergravity. The case of two intersecting M-branes (as well as the case of the basic $M2$- and $M5$-brane solutions) defined on the product of Ricci-flat manifolds was considered earlier in [4]. (For solutions with one brane see also [3].)

2 Generalized Killing spinor equations

The bosonic action in 11-dimensional supergravity is given by

$$S = \int d^{11}z \sqrt{|g|} \left\{ R[g] - \frac{1}{2(4!)} F^2 \right\} - \frac{1}{6} \int A \wedge F \wedge F, \tag{3}$$

where $F = dA = \frac{1}{4!} F_{NPQR} dz^N \wedge dz^P \wedge dz^Q \wedge dz^R$ is 4-form. We consider pure bosonic configurations in $D = 11$ supergravity (with zero fermionic fields) that are solutions to the equations of motion corresponding to the action (3). The amount of preserved supersymmetries (SUSY) corresponding to the bosonic background (e^A_M, $A_{M_1 M_2 M_3}$) is defined by the dimension of the space of solutions to (a set of) linear first-order differential equations (generalized Killing spinor equations) for 32-component spinor field $\varepsilon = \varepsilon(z)$

$$(D_M + B_M)\varepsilon = 0. \tag{4}$$

Here

$$D_M = \partial_M + \frac{1}{4} w_{ABM} \hat{\Gamma}^A \hat{\Gamma}^B \tag{5}$$

is covariant spinorial derivative, the gamma matrices satisfy the Clifford algebra relation

$$\hat{\Gamma}^A \hat{\Gamma}^B + \hat{\Gamma}^B \hat{\Gamma}^A = 2\eta^{AB} \mathbf{1}_{32}, \tag{6}$$

B_M is an operator induced by the 4-form field strength F

$$B_M = \frac{1}{288} \left(\Gamma_M \Gamma^N \Gamma^P \Gamma^Q \Gamma^R - 12 \delta^N_M \Gamma^P \Gamma^R \Gamma^R \right) F_{NPQR} \tag{7}$$

and Γ_M are world Γ-matrices.

The number of preserved SUSY is $\mathcal{N} = N/32$, where N is the dimension of the linear space of solutions to differential equations (5).

3 Intersecting M-brane solution

The metric of intersecting M-brane solutions of $D = 11$ supergravity can be written in the following form

$$g = e^{2\gamma(x)} g^0 + \sum_{i=1}^{n} e^{2\phi^i(x)} g^i, \tag{8}$$

$$e^{2\gamma} = (\prod_{s \in S_e} H_s)^{1/3} (\prod_{s \in S_m} H_s)^{2/3}, \quad e^{2\phi^i} = e^{2\gamma} \prod_{s \in S} H_s^{-\delta^i_{I_s}}, \tag{9}$$

$i = 1, \dots, n$.

Here $g^0 = g^0_{\mu\nu}(x) dx^\mu \otimes dx^\nu$ is a Ricci-flat metric on the manifold M_0 and $g^i = g^i_{m_i n_i}(y_i) dy_i^{m_i} \otimes dy_i^{n_i}$ is a Ricci-flat metric on M_i, $1, \dots, n$, $\delta_{iI_s} = \sum_{j \in I_s} \delta_{ij}$ is the indicator of i belonging to I_s: $\delta_{iI_s} = 1$ for $i \in I_s$ and $\delta_{iI_s} = 0$ otherwise.

The 4-form field strength reads

$$F = \sum_{s \in S_e} c_s dH_s^{-1} \wedge \tau(I_s) + \sum_{s \in S_m} c_s (*_0 dH_s) \wedge \tau(\bar{I}_s), \tag{10}$$

where $c_s^2 = 1$, $*_0$ is the Hodge operator on (M_0, g^0), H_s is a harmonic function on (M_0, g^0) and $\bar{I}_s = \{1, 2, \dots, n\} \setminus I_s$ is a dual set.

The set of indices S_e describes electric branes and S_m describes magnetic branes.

We put

$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{A_1} \hat{\Gamma}^{A_2} \hat{\Gamma}^{A_3}, \quad \text{for} \quad s \in S_e, \tag{11}$$

where $(\hat{\Gamma}^{A_{i_0}})^2 = -\mathbf{1}_{32}$ for some $i_0 \in \{1, 2, 3\}$, $(\hat{\Gamma}^{A_i})$ for $i \neq i_0$, and

$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{B_1} \hat{\Gamma}^{B_2} \hat{\Gamma}^{B_3} \hat{\Gamma}^{B_4} \hat{\Gamma}^{B_5}, \quad \text{for} \quad s \in S_m, \tag{12}$$

where $(\hat{\Gamma}^B_i)^2 = \mathbf{1}_{32}$ for all i.

It follows from the relations (11)-(12)

$$(\hat{\Gamma}_{[s]})^2 = \mathbf{1}_{32}. \tag{13}$$

In the recent work [4] it was shown that the relation (4) is satisfied identically if the spinor field ε can be represented in the form

$$\varepsilon = \left(\prod_{s \in S_e} H_s \right)^{-1/6} \left(\prod_{s \in S_m} H_s \right)^{-1/12} \eta, \tag{14}$$

where

$$\bar{D}^{(l)}_{m_l} \eta = 0, \tag{15}$$

$l = 0, \dots, n$ and

$$\hat{\Gamma}_{[s]} \eta = c_s \eta, \quad \text{for all} \quad s \in S. \tag{16}$$

Here $\bar{D}^{(l)}_{m_l} = \partial_{m_l} + \frac{1}{4} \omega^{(l)}_{a_l b_l m_l} \hat{\Gamma}^{a_l} \hat{\Gamma}^{b_l}$ is the modified operator of covariant spinorial derivative corresponding to factor-space M_l with the spin-connection $\omega^{(l)}$.

4 Triple M-brane backgrounds

In this section we present three examples of solutions.

4.1 $M5 \cap M5 \cap M5$

The solution describing three intersecting magnetic $M5$-branes is defined on the manifold

$$M_0 \times M_1 \times M_2 \times M_3 \times M_4, \tag{17}$$

where $d_0 = 1$, $d_1 = d_3 = d_4 = 2$, $d_2 = 4$.

The solution reads

$$g = H_1^{2/3} H_2^{2/3} H_3^{2/3} \Big\{ \hat{g}^0 + H_2^{-1} \hat{g}^1 + H_1^{-1} H_2^{-1} H_3^{-1} \hat{g}^2 + H_1^{-1} \hat{g}^3 + H_3^{-1} \hat{g}^4 \Big\}, \tag{18}$$

$$F = c_1 (*_0 dH_1) \wedge \hat{\tau}_1 \wedge \hat{\tau}_4 + c_2 (*_0 dH_2) \wedge \hat{\tau}_3 \wedge \hat{\tau}_4 + c_3 (*_0 dH_3) \wedge \hat{\tau}_1 \wedge \hat{\tau}_3, \tag{19}$$

where $c_1^2 = c_2^2 = c_3^2 = 1$; H_1, H_2, H_3 are harmonic functions on (M_0, g^0). The metrics g^i $(i = 0, 1, 3, 4)$ have Euclidean signatures and the metric g^2 has the signature $(-, +, +, +)$. The branes sets are $I_1 = \{2, 3\}$, $I_2 = \{1, 2\}$, $I_3 = \{2, 4\}$.

Using the rules of decomposition from [5] one can write Γ-matrices in the following form

$$(\hat{\Gamma}^A) = \begin{pmatrix} 1 & \otimes & \hat{\Gamma}_{(1)} & \otimes & \hat{\Gamma}_{(2)} & \otimes & \hat{\Gamma}_{(3)} & \otimes & \hat{\Gamma}_{(4)} \\ 1 & \otimes & i\hat{\Gamma}^{a_1}_{(1)} & \otimes & \hat{\Gamma}_{(2)} & \otimes & \hat{\Gamma}_{(3)} & \otimes & \hat{\Gamma}_{(4)} \\ 1 & \otimes & \mathbf{1}_2 & \otimes & \hat{\Gamma}^{a_2}_{(2)} & \otimes & \hat{\Gamma}_{(3)} & \otimes & \hat{\Gamma}_{(4)} \\ 1 & \otimes & \mathbf{1}_2 & \otimes & \mathbf{1}_4 & \otimes & i\hat{\Gamma}^{a_3}_{(3)} & \otimes & \hat{\Gamma}_{(4)} \\ 1 & \otimes & \mathbf{1}_2 & \otimes & \mathbf{1}_4 & \otimes & \mathbf{1}_2 & \otimes & \hat{\Gamma}^{a_4}_{(4)} \end{pmatrix}$$

Here

$$\hat{\Gamma}_{(1)} = \hat{\Gamma}^{1_1}_{(1)} \hat{\Gamma}^{2_1}_{(1)}, \quad \hat{\Gamma}_{(2)} = \hat{\Gamma}^{1_2}_{(2)} \hat{\Gamma}^{2_2}_{(2)} \hat{\Gamma}^{3_2}_{(2)} \hat{\Gamma}^{4_2}_{(2)}, \quad \hat{\Gamma}_{(3)} = \hat{\Gamma}^{1_3}_{(3)} \hat{\Gamma}^{2_3}_{(3)}, \quad \hat{\Gamma}_{(4)} = \hat{\Gamma}^{1_4}_{(4)} \hat{\Gamma}^{2_4}_{(4)} \tag{20}$$

obey

$$(\hat{\Gamma}_{(2)})^2 = -\mathbf{1}_4, \quad (\hat{\Gamma}_{(i)})^2 = -\mathbf{1}_2 \tag{21}$$

with $i = 1, 3, 4$. $\hat{\Gamma}^{a_1}_{(1)}, \hat{\Gamma}^{a_3}_{(3)}, \hat{\Gamma}^{a_4}_{(4)}$ are 2×2 Γ-matrices correspond to M_1, M_3, M_4 respectively.

The covariant derivatives can be represented as

$$\bar{D}^{(1)}_{m_1} = \partial_{m_1} + \frac{1}{4} w^{(1)}_{a_1 b_1 m_1} \left(1 \otimes \hat{\Gamma}^{a_1}_{(1)} \hat{\Gamma}^{b_1}_{(1)} \otimes \mathbf{1}_4 \otimes \mathbf{1}_2 \otimes \mathbf{1}_2\right),$$
$$\bar{D}^{(2)}_{m_2} = \partial_{m_2} + \frac{1}{4} w^{(2)}_{a_2 b_2 m_2} \left(1 \otimes \mathbf{1}_2 \otimes \hat{\Gamma}^{a_2}_{(2)} \hat{\Gamma}^{b_2}_{(2)} \otimes \mathbf{1}_2 \otimes \mathbf{1}_2\right),$$
$$\bar{D}^{(3)}_{m_3} = \partial_{m_3} + \frac{1}{4} w^{(3)}_{a_3 b_3 m_3} \left(1 \otimes \mathbf{1}_2 \otimes \mathbf{1}_4 \otimes \hat{\Gamma}^{a_3}_{(3)} \hat{\Gamma}^{b_3}_{(3)} \otimes \mathbf{1}_2\right),$$
$$\bar{D}^{(4)}_{m_4} = \partial_{m_4} + \frac{1}{4} w^{(4)}_{a_4 b_4 m_4} \left(1 \otimes \mathbf{1}_2 \otimes \mathbf{1}_4 \otimes \mathbf{1}_2 \otimes \hat{\Gamma}^{a_4}_{(4)} \hat{\Gamma}^{b_4}_{(4)}\right),$$

$w^{(i)}_{a_i b_i c_i}$ is a spin connection corresponding to the manifold M_i, $i = 1,2,3,4$, $D^{(i)}_{m_i}$ are covariant derivatives corresponding to M_i, $i = 1,2,3,4,$, $\bar{D}^{(0)}_{m_0} = \partial_{m_0}$ and $D^{(0)}_{m_0} = \partial_{m_0}$.

Let $\eta = \eta_0 \otimes \eta_1(y_1) \otimes \eta_2(y_2) \otimes \eta_3(y_3) \otimes \eta_4(y_4)$, where η_0 is a 1-component spinor on M_0, $\eta_2 = \eta_2(y_2)$ is a 4-component spinor on M_2, $\eta_i = \eta_i(y_i)$ is a 2-component spinor on M_i, $i = 1,3,4$.

The following relations for modified covariant derivatives take place:

$$\bar{D}^{(0)}_{m_0}\eta = \left(D^{(0)}_{m_0}\eta_0\right) \otimes \eta_1 \otimes \eta_2 \otimes \eta_3 \otimes \eta_4,$$
$$\bar{D}^{(1)}_{m_1}\eta = \eta_0 \otimes \left(D^{(1)}_{m_1}\eta_1\right) \otimes \eta_2 \otimes \eta_3 \otimes \eta_4,$$
$$\bar{D}^{(2)}_{m_2}\eta = \eta_0 \otimes \eta_1 \otimes \left(D^{(2)}_{m_2}\eta_2\right) \otimes \eta_3 \otimes \eta_4,$$
$$\bar{D}^{(3)}_{m_3}\eta = \eta_0 \otimes \eta_1 \otimes \eta_2 \otimes \left(D^{(3)}_{m_3}\eta_3\right) \otimes \eta_4,$$
$$\bar{D}^{(4)}_{m_4}\eta = \eta_0 \otimes \eta_1 \otimes \eta_2 \otimes \eta_3 \otimes \left(D^{(4)}_{m_4}\eta_4\right). \qquad (22)$$

The operators (12) corresponding to $M5$-branes read

$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{1_0}\hat{\Gamma}^{1_1}\hat{\Gamma}^{2_1}\hat{\Gamma}^{1_4}\hat{\Gamma}^{2_4} = 1 \otimes \mathbf{1}_2 \otimes \hat{\Gamma}_{(2)} \otimes \hat{\Gamma}_{(3)} \otimes \mathbf{1}_2,$$

for $s = I_1$,

$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{1_0}\hat{\Gamma}^{1_3}\hat{\Gamma}^{2_3}\hat{\Gamma}^{1_4}\hat{\Gamma}^{2_4} = 1 \otimes \hat{\Gamma}_{(1)} \otimes \hat{\Gamma}_{(2)} \otimes \mathbf{1}_2 \otimes \mathbf{1}_2,$$

for $s = I_2$ and

$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{1_0}\hat{\Gamma}^{1_1}\hat{\Gamma}^{2_1}\hat{\Gamma}^{1_3}\hat{\Gamma}^{2_3} = 1 \otimes \mathbf{1}_2 \otimes \hat{\Gamma}_{(2)} \otimes \mathbf{1}_2 \otimes \hat{\Gamma}_{(4)},$$

for $s = I_3$.

Then the chirality restrictions (16) are satisfied if

$$\hat{\Gamma}_{(j)}\eta_j = c_{(j)}\eta_j, \qquad c^2_{(j)} = -1, \qquad (23)$$

$j = 1,2,3,4$ and

$$c_{(2)}c_{(3)} = c_1, \quad c_{(1)}c_{(2)} = c_2, \quad c_{(2)}c_{(4)} = c_3. \qquad (24)$$

The solution to the SUSY equations corresponding to the field configuration from (18), (19) can be represented in the following form

$$\varepsilon = \prod_{s=1}^{3} H_s^{-\frac{1}{12}} \eta_0 \otimes \eta_1(y_1) \otimes \eta_2(y_2) \otimes \eta_2(y_3) \otimes \eta_4(y_4). \quad (25)$$

Here η_i, $i = 1,2,3,4$ are parallel spinors defined on M_i, respectively $\left(D^{(i)}_{m_i}\eta_i = 0\right)$, obeying (23) and (24).

Eqs. (24) have the following solutions

$$c_{(1)} = -ic_2, \quad c_{(2)} = i, \quad c_{(3)} = -ic_1, \quad c_{(4)} = -ic_3,$$

or

$$c_{(1)} = ic_2, \quad c_{(2)} = -i, \quad c_{(3)} = ic_1, \quad c_{(4)} = ic_3.$$

The number of linear independent solutions given by (25), and reads

$$N = 32\mathcal{N} = n_1(-ic_2)n_2(i)n_3(-ic_1)n_4(-ic_3) + n_1(ic_2)n_2(-i)n_3(ic_1)n_4(ic_3), \qquad (26)$$

where $n_j(c_j)$ is the number of chiral parallel spinors on M_j, $j = 1,2,3,4$.

4.2 $M2 \cap M2 \cap M5$

Let us consider the intersection of two electric and one magnetic branes. $M2 \cap M2 \cap M5$-solution is defined on the manifold

$$M_0 \times M_1 \times M_2 \times M_3 \times M_4 \times M_5 \times M_6, \qquad (27)$$

where $d_0 = 3$, $d_1 = d_2 = d_3 = d_4 = d_5 = 1$ and $d_6 = 3$. The metric and the 4-form field strength of two intersecting $M2$-branes and one $M5$-brane can be represented in the following form

$$g = H_1^{1/3} H_2^{1/3} H_3^{2/3} \Big\{ \hat{g}^0 + H_1^{-1}\hat{g}^1 + H_2^{-1}\hat{g}^2 + H_1^{-1}H_2^{-1}H_3^{-1}\hat{g}^3 + H_1^{-1}H_3^{-1}\hat{g}^4 + H_2^{-1}H_3^{-1}\hat{g}^5 + H_3^{-1}\hat{g}^6 \Big\}. \qquad (28)$$

The corresponding field strength is

$$F = c_1 dH_1^{-1} \wedge \hat{\tau}_1 \wedge \hat{\tau}_3 \wedge \hat{\tau}_4 + c_2 dH_2^{-1} \wedge \hat{\tau}_2 \wedge \hat{\tau}_3 \wedge \hat{\tau}_5 + c_3 (*_0 dH_3) \wedge \hat{\tau}_1 \wedge \hat{\tau}_2, \qquad (29)$$

where $c_1^2 = c_2^2 = c_3^2 = 1$; H_1, H_2, H_3 are harmonic functions on (M_0, g^0). The metrics g^i ($i = 0,1,2,4,5,6$) have Euclidean signatures and we put the metric $g^3 = -dt \otimes dt$. The branes sets are $I_1 = \{1,3,4\}$, $I_2 = \{2,3,5\}$, $I_3 = \{3,4,5,6\}$. The gamma-matrices may be chosen in the following form

$$(\hat{\Gamma}^A) = (\hat{\Gamma}^{a_0}_{(0)} \otimes \sigma_3 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes \mathbf{1}_2,$$
$$\mathbf{1}_2 \otimes \sigma_1 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes \mathbf{1}_2,$$
$$\mathbf{1}_2 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes \mathbf{1}_2,$$
$$\mathbf{1}_2 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes \sigma_1 \otimes i \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes \mathbf{1}_2,$$
$$\mathbf{1}_2 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 \otimes \mathbf{1}_2,$$
$$\mathbf{1}_2 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes \sigma_1 \otimes 1 \otimes \mathbf{1}_2,$$
$$\mathbf{1}_2 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes 1 \otimes \hat{\Gamma}^{a_6}_{(6)}).$$

Here the operators

$$\hat{\Gamma}_{(0)} = \hat{\Gamma}^{1_0}_{(0)}\hat{\Gamma}^{2_0}_{(0)}\hat{\Gamma}^{3_0}_{(0)}, \quad \hat{\Gamma}_{(6)} = \hat{\Gamma}^{1_6}_{(6)}\hat{\Gamma}^{2_6}_{(6)}\hat{\Gamma}^{3_6}_{(6)} \tag{30}$$

obey

$$\hat{\Gamma}_{(0)} = \hat{\Gamma}_{(6)} = i\mathbf{1}_2, \quad (\hat{\Gamma}_{(0)})^2 = (\hat{\Gamma}_{(6)})^2 = -\mathbf{1}_2. \tag{31}$$

We put $(\hat{\Gamma}^{a_i}_{(i)}) = (\sigma_1, \sigma_2, \sigma_3)$, where $i = 0, 6$ and hence $\hat{\Gamma}_{(i)} = i\mathbf{1}_2$.

The spinor monomial reads $\eta = \eta_0(x) \otimes \chi_1 \otimes \eta_1(y_1) \otimes \eta_2(y_2) \otimes \chi_2 \otimes \eta_3(y_3) \otimes \eta_4(y_4) \otimes \chi_3 \otimes \eta_5(y_5) \otimes \eta_6(y_6)$, where $\eta_i = \eta_i(y_i)$ is a 1-component spinor on M_i, $i = 1, 2, 3, 4, 5$, $\eta_0 = \eta_0(x)$ is a 2-component spinor on M_0 and $\eta_6 = \eta_6(y_6)$ is a 2-component spinor on M_6, χ_1, χ_2, χ_3 are "phantom" spinors.

Here the following relations for modified covariant derivatives take place:

$$\bar{D}^{(0)}_{m_0} = \partial_{m_0} + \frac{1}{4} w^{(0)}_{a_0 b_0 m_0} (\hat{\Gamma}^{a_0}_{(0)} \hat{\Gamma}^{b_0}_{(0)} \otimes \mathbf{1}_2 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes \mathbf{1}_2),$$

$$\bar{D}^{(6)}_{m_6} = \partial_{m_6} + \frac{1}{4} w^{(6)}_{a_6 b_6 m_6} (\mathbf{1}_2 \otimes \mathbf{1}_2 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes \hat{\Gamma}^{a_6}_{(6)} \hat{\Gamma}^{b_6}_{(6)}), \tag{32}$$

$\bar{D}_{m_i} = \partial_{m_i}$ for $1, 2, 3, 4, 5$. $D^{(i)}_{m_i}$ are covariant derivatives corresponding to M_i, $i = 0, 6$ and $D^{(i)}_{m_i} = \partial_{m_i}$, $i = 1, 2, 3, 4, 5$.

Here the "factorization" condition

$$\bar{D}^{(i)}_{m_i} \eta = \ldots \otimes \eta_{i-1} \otimes \left(D^{(i)}_{m_i} \eta_i \right) \otimes \eta_{i+1} \otimes \ldots \tag{33}$$

is satisfied identically.

The operators (11) corresponding to the $M2$-branes read

$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{1_1}\hat{\Gamma}^{1_3}\hat{\Gamma}^{1_4} = -\mathbf{1}_2 \otimes \sigma_1 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 \otimes \mathbf{1}_2, \tag{34}$$

for $s = I_1$ and

$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{1_2}\hat{\Gamma}^{1_3}\hat{\Gamma}^{1_5} = -\mathbf{1}_2 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes 1 \otimes \sigma_1 \otimes 1 \otimes \mathbf{1}_2, \tag{35}$$

for $s = I_2$. The operator (12) for the $M5$-brane can be written in the form

$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{1_0}\hat{\Gamma}^{2_0}\hat{\Gamma}^{3_0}\hat{\Gamma}^{1_1}\hat{\Gamma}^{1_2} = -\mathbf{1}_2 \otimes \mathbf{1}_2 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes \mathbf{1}_2, \tag{36}$$

for $s = I_3$.

After a suitable diagonalization of tensor products of Pauli matrices in $\hat{\Gamma}_{[s]}$ we get the following number of unbroken SUSY

$$\mathcal{N} = n_0 n_6 / 32, \tag{37}$$

where n_0 is the number of parallel spinors on the 3-dimensional manifold M_0 and n_6 is the number of chiral parallel spinors on the 2-dimensional manifold M_6.

4.3 $M2 \cap M5 \cap M5$

Configuration $M2 \cap M5 \cap M5$ is defined on manifold

$$M_0 \times M_1 \times M_2 \times M_3 \times M_4 \times M_5, \tag{38}$$

where $d_0 = 2$, $d_1 = 1$, $d_2 = d_3 = d_4 = d_5 = 2$.
The solution which describes an electric $M2$-brane and two magnetic ones is given by

$$g = H_1^{1/3} H_2^{2/3} H_3^{2/3} \Big\{ \hat{g}^0 + H_1^{-1} \hat{g}^1 + H_2^{-1} \hat{g}^2 + H_1^{-1} H_2^{-1} H_3^{-1} \hat{g}^3 + H_2^{-1} H_3^{-1} \hat{g}^4 + H_3^{-1} \hat{g}^5 \Big\}, \tag{39}$$

$$F = c_1 dH_1^{-1} \wedge \hat{\tau}_1 \wedge \hat{\tau}_3 + c_2 (*_0 dH_2) \wedge \hat{\tau}_1 \wedge \hat{\tau}_5 + c_3 (*_0 dH_3) \wedge \hat{\tau}_1 \wedge \hat{\tau}_2, \tag{40}$$

where $c_1^2 = c_2^2 = c_3^2 = 1$; H_1, H_2, H_3 are harmonic functions defined on (M_0, g^0). The metrics g^i $(i = 0, 1, 2, 4, 5)$ have Euclidean signatures and the metric g^3 has the signature $(-, +)$. The branes sets are $I_1 = \{1, 3\}$, $I_2 = \{2, 3, 4\}$ and $I_3 = \{3, 4, 5\}$.

We introduce the following set of Γ-matrices

$$\begin{aligned}(\hat{\Gamma}^A) = (&\hat{\Gamma}^{a_0}_{(0)} \otimes 1 \otimes \mathbf{1}_2 \otimes \mathbf{1}_2 \otimes \mathbf{1}_2 \otimes \mathbf{1}_2, \\ &\hat{\Gamma}_{(0)} \otimes 1 \otimes \hat{\Gamma}_{(2)} \otimes \hat{\Gamma}_{(3)} \otimes \hat{\Gamma}_{(4)} \otimes \hat{\Gamma}_{(5)} \\ &i\hat{\Gamma}_{(0)} \otimes 1 \otimes \hat{\Gamma}^{a_2}_{(2)} \otimes \mathbf{1}_2 \otimes \mathbf{1}_2 \otimes \mathbf{1}_2 \\ &\hat{\Gamma}_{(0)} \otimes 1 \otimes \hat{\Gamma}_{(2)} \otimes \hat{\Gamma}^{a_3}_{(3)} \otimes \mathbf{1}_2 \otimes \mathbf{1}_2 \\ &\hat{\Gamma}_{(0)} \otimes 1 \otimes \hat{\Gamma}_{(2)} \otimes \hat{\Gamma}_{(3)} \otimes \hat{\Gamma}^{a_4}_{(4)} \otimes \mathbf{1}_2 \\ &i\hat{\Gamma}_{(0)} \otimes 1 \otimes \hat{\Gamma}_{(2)} \otimes \hat{\Gamma}_{(3)} \otimes \hat{\Gamma}_{(4)} \otimes \hat{\Gamma}^{a_5}_{(5)}),\end{aligned} \tag{41}$$

where $\hat{\Gamma}^{a_i}_{(i)}$ are 2×2 Γ-matrices, $a_i = 1_i, 2_i$, $(i = 0, 2, 3, 4, 5)$ corresponding to M_i, respectively, and the operators

$$\hat{\Gamma}_{(3)} = \hat{\Gamma}^{1_3}_{(3)} \hat{\Gamma}^{2_3}_{(3)}, \quad \hat{\Gamma}_{(i)} = \hat{\Gamma}^{1_i}_{(i)} \hat{\Gamma}^{2_i}_{(i)}, \tag{42}$$

obey

$$(\hat{\Gamma}_{(i)})^2 = -\mathbf{1}_2, \quad (\hat{\Gamma}_{(3)})^2 = \mathbf{1}_2, \; i = 0, 2, 4, 5. \tag{43}$$

Consider η in the form $\eta = \eta_0(x) \otimes \eta_1 \otimes \eta_2(y_2) \otimes \eta_3(y_3) \otimes \eta_4(y_4) \otimes \eta_5(y_5)$, where $\eta_i = \eta_i(y_i)$ is a 2-component spinor on M_i, $i = 0, 2, 3, 4, 5$, η_1 is a 1-component spinor on M_1. Due to (41) and (43) the operator $\bar{D^{(i)}}_{m_i}$ acts on η as

$$\bar{D}^{(i)}_{m_i} \eta = \ldots \otimes \eta_{i-1} \otimes \left(D^{(i)}_{m_i} \eta_i \right) \otimes \eta_{i+1} \otimes \ldots \tag{44}$$

where $D^{(i)}_{m_i}$ is the spinorial covariant derivative corresponding to M_i, $i = 0, 2, 3, 4, 5$ and $D^{(1)}_{m_1} = \partial_{m_1}$.

The operator (11) corresponding $M2$-brane is

$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{1_1}\hat{\Gamma}^{1_3}\hat{\Gamma}^{2_3} = \hat{\Gamma}_{(0)} \otimes 1 \otimes \hat{\Gamma}_{(2)} \otimes \mathbf{1}_2 \otimes \hat{\Gamma}_{(4)} \otimes \hat{\Gamma}_{(5)},$$

for $s = I_1$ and the operators (12) corresponding to $M5$-branes are

$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{1_0}\hat{\Gamma}^{2_0}\hat{\Gamma}^{1_1}\hat{\Gamma}^{1_5}\hat{\Gamma}^{2_5} =$$

$$\mathbf{1}_2 \otimes 1 \otimes \hat{\Gamma}_{(2)} \otimes \hat{\Gamma}_{(3)} \otimes \hat{\Gamma}_{(4)} \otimes \mathbf{1}_2,$$

for $s = I_2$ and

$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{1_0}\hat{\Gamma}^{2_0}\hat{\Gamma}^{1_1}\hat{\Gamma}^{1_2}\hat{\Gamma}^{2_2} =$$

$$\mathbf{1}_2 \otimes 1 \otimes \mathbf{1}_2 \otimes \hat{\Gamma}_{(3)} \otimes \hat{\Gamma}_{(4)} \otimes \hat{\Gamma}_{(5)},$$

for $s = I_3$.

The chirality restrictions (16) are satisfied if

$$\hat{\Gamma}_{(3)}\eta_3 = c_{(3)}\eta_3, \qquad c_{(3)}^2 = 1,$$
$$\hat{\Gamma}_{(j)}\eta_j = c_{(j)}\eta_j, \qquad c_{(j)}^2 = -1, \tag{45}$$

$j = 0, 2, 4, 5$ and

$$c_{(0)}c_{(2)}c_{(4)}c_{(5)} = c_1,$$
$$c_{(2)}c_{(3)}c_{(4)} = c_2, \quad c_{(3)}c_{(4)}c_{(5)} = c_3. \tag{46}$$

For the field configuration (39) and (40) we obtain the following solution to SUSY equations $\varepsilon = H_1^{-1/6}H_2^{-1/12}H_3^{-1/12}\eta_0(x) \otimes \eta_1 \otimes \eta_2(y_2) \otimes \eta_2(y_3) \otimes \eta_4(y_4) \otimes \eta_5(y_5)$, where η_i, $i = 0, 2, 3, 4, 5$ are chiral parallel spinors defined on M_i, $\left(D^{(i)}_{m_i}\eta_i = 0\right)$, obeying (45) and (46), η_1 is constant.

Equations (46) have the following solutions $c_{(0)} = i\varepsilon_4 c_1 c_2 c_3$, $c_{(2)} = i\varepsilon_5 c_2 c_3$, $c_{(3)} = -\varepsilon_4\varepsilon_5 c_3$, $c_{(4)} = i\varepsilon_4$, $c_{(5)} = i\varepsilon_4$, where $\varepsilon_4 = \pm 1$, $\varepsilon_5 = \pm 1$. Thus, the number of linear independent solutions is

$$N = 32\mathcal{N} = \sum_{\varepsilon_4 = \pm 1, \varepsilon_5 = \pm 1} n_0(i\varepsilon_4 c_1 c_2 c_3) n_2(i\varepsilon_5 c_2 c_3) n_3(-\varepsilon_4\varepsilon_5 c_3) \times n_4(i\varepsilon_4) n_5(i\varepsilon_5),$$

where $n_j(c_j)$ is the number of chiral parallel spinors on M_j, $j = 0, 2, 3, 4, 5$.

5 Conclusions

As was discussed above, the problem of finding the solutions to SUSY equations is reduced to the search of parallel spinors on factor-spaces M_i and to the (technical) task of finding suitable sets of Γ-matrices corresponding to the product manifold M.

Here we have presented solutions corresponding to various intersecting M-brane configurations of $D = 11$ supergravity. Using the approach of [3,4] we have found the numbers of preserved supersymmetries for three configurations with three intersecting M-branes. For flat factor spaces $M_i = \mathbb{R}^{d_i}$ we get $\mathcal{N} = 1/8$ for any triple configuration in agreement with the classification of [1].

The presented approach may be of interest from the point of view of possible applications to studies of supersymmetric solutions defined on product of Ricci-flat manifolds for IIA, IIB supergravities and to supersymmetric localized branes.

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РЕШЕНИЯ ДЛЯ ПЕРЕСЕЧЕНИЯ ТРЕХ БРАН И СУПЕРСИММЕТРИИ

Исследованы композитные M-бранные конфигурации в 11-мерной супергравитации. Суперсимметричные решения описывающие ортогонально пересекающиеся M-браны определены на произведении риччи-плоских пространств M_i. Число суперсимметрий зависит от чисел ковариантно-постоянных киральных спиноров на соответствующих фактор-пространствах и знаковых множителей для бран. Представлены точные суперсимметричные решения для трех M-бран на произведении риччи-плоских многообразий со спинорной структурой. Для каждой из конфигураций получено число суперсимметрий и приведены конкретные примеры решений с различными риччи-плоскими фактор-пространствами.

Ключевые слова: *M-браны, суперсимметрии, параллельные спиноры, супергравитация.*

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