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A DYNAMICAL THEORY FOR FREE MASSIVE SUPERSPIN 3/2

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We present a new theory for free massive superspin $Y = 3/2$ irreducible representation of the $4D$, $\mathcal{N} = 1$ Super-Poincaré group, which has linearized non-minimal supergravity (superhelicity $Y = 3/2$) as its massless limit. The theory is being described in terms of a real bosonic superfield $H_{\alpha\dot{\alpha}}$ and two fermionic ones χ_α , u_α. The new results will illuminate the underlying structure of auxiliary superfields required for the description of higher massive superspin systems.

Keywords: *superspin, superspace.*

1 Introduction

After four decades of exploring the topic of supersymmetry (SUSY), the problem of writing a manifestly susy-invariant action that describes a free, off-shell massive arbitrary superspin irreducible representation of the Super-Poincaré group still possesses puzzles. Although the non-supersymmetric case of massive higher spin theory has been developed [1, 2] and is well understood, the off-shell supersymmetric case has yet to be understood with a comparable level of clarity. There has been progress for on-shell supersymmetry [3], but these results do not capture the rich off-shell structure of supersymmetric theories. There is a need for a manifestly susy invariant theory of massive integer and half-integer superspins which includes all the auxiliary superfields a theory of this nature is expected to possess.

Progress in this direction was made with the works presented in [4–6] where free massive irreducible representations of superspin 1 and 3/2 were constructed. These results provided a proof of concept that constructions like these are possible, but they do not shed light to the heart of the problem which is to determine the set of auxiliary superfields required to describe an arbitrary superspin system with a proper massless limit. Specifically in [4] the focus was on massive extension of linearized old-minimal supergravity and new-minimal supergravity. These theories do not generalize to the arbitrary spin case, therefore the results obtained do not provide clues about the underlying structure of the auxiliary superfields for the general case.

This is not the case with the work presented in [6] where a free massive extension of linearized non-minimal supergravity is derived. Linearized non-minimal supergravity supermultiplet is a member of a tower of irreducible representations that can be extended to the arbitrary super-helicity and that makes it a good starting point. However, their construction uses a lagrange multiplier technique in order to impose constraints that were not derived in a dynamical way.

We will show that there is an alternative formulation of the theory where all the constraints required, for the description of a free massive irreducible representation of Y=3/2, are dynamically generated from the equations of motion of a set of superfields $\{H_{\alpha\dot{\alpha}},\ \chi_\alpha,\ u_\alpha\}$. Superfields $H_{\alpha\dot{\alpha}}$, χ_α in the massless limit form the free linearized non-minimal theory (superhelicity $Y = 3/2$) and u_α is an auxiliary superfield that decouples when $m \to 0$.

Finally, the theory presented here is a free theory without interactions. The full interactions, non-linear problem is still an open and very hard problem and for sure it is one of the motivations for this kind of investigations. In a realistic approach we can not talk about interactions if we have not established the free theories first. The results presented here extend our understanding for the free massive theory of the non-minimal superspin 3/2 supermultiplet, which is the first non-trivial. Furthermore, we provide clues for some of the degrees of freedom that must be present in the non-linear, interacting theory. These are the superfields that have auxiliary status in the free linearized theory.

Our presentation is organized as follows: In section 2, we quickly review the representation theory of the $4D$, $\mathcal{N} = 1$ Super-Poincaré group for a free massive arbitrary superspin system. In section 3, we present the constraints imposed in the theory in order to have a proper massless limit. In the last section 4 we present the new massive theory for $Y = 3/2$.

2 Arbitrary superspin representation theory

The irreducible representations of the Super-Poincaré group are labeled by its two Casimir operators. The first one is the mass and the other one is a supersymmetric extension of the Poincaré Spin operator. For the massive case the Super-spin Casimir operator takes the form

$$C_2 = \frac{W^2}{m^2} + \left(\frac{3}{4} + \lambda\right) P_{(o)}, \tag{1}$$

where W^2 is the ordinary spin operator (the square of the Pauli-Lubanski vector), $P_{(o)}$ is the projection operator $P_{(o)} = -\frac{1}{m^2}\mathrm{D}^{\gamma}\bar{\mathrm{D}}^2\mathrm{D}_{\gamma}$ and the parameter λ satisfies the equation

$$\lambda^2 + \lambda = \frac{W^2}{m^2} \quad . \tag{2}$$

In order to diagonalize C_2 we want to diagonalize both $W^2, P_{(o)}$. The superfield $\Phi_{\alpha(n)\dot{\alpha}(m)}$ that does this

$$\begin{aligned} W^2\Phi_{\alpha(n)\dot{\alpha}(m)} &= j(j+1)m^2\Phi_{\alpha(n)\dot{\alpha}(m)} \ , \\ P_{(o)}\Phi_{\alpha(n)\dot{\alpha}(m)} &= \Phi_{\alpha(n)\dot{\alpha}(m)} \ , \ j = \frac{n+m}{2} \end{aligned} \tag{3}$$

and describes the highest possible representation (highest superspin)

$$\lambda = \frac{n+m}{2} \quad , \tag{4}$$

$$C_2\Phi_{\alpha(n)\dot{\alpha}(m)} = Y(Y+1)\Phi_{\alpha(n)\dot{\alpha}(m)}, \ Y = \frac{n+m+1}{2} \ ,$$

has to satisfy the following constraints:

$$\begin{aligned} &\mathrm{D}^2\Phi_{\alpha(n)\dot{\alpha}(m)} = 0 \ , && \bar{\mathrm{D}}^2\Phi_{\alpha(n)\dot{\alpha}(m)} = 0 \ , \\ &\mathrm{D}^{\gamma}\Phi_{\gamma\alpha(n-1)\dot{\alpha}(m)} = 0 \ , && \partial^{\gamma\dot{\gamma}}\Phi_{\gamma\alpha(n-1)\dot{\gamma}\dot{\alpha}(m-1)} = 0 \ , \\ &\Box\Phi_{\alpha(n)\dot{\alpha}(m)} = m^2\Phi_{\alpha(n)\dot{\alpha}(m)}, \end{aligned} \tag{5}$$

where all dotted and undotted indices are fully symmetrized and the spin content of this supermultiplet is $j = Y + 1/2, \ Y, \ Y, \ Y - 1/2$.

A superfield that describes a superspin Y system has index structure such that $n + m = 2Y - 1$, where n, m are integers. This Diophantine equation has a finite number of different solutions for (n, m) pairs but the corresponding superfields are all equivalent because we can use the $\partial_{\beta\dot{\beta}}$ operator to convert one kind of index to another. Therefore we can pick one of them to represent the entire class.

One last comment has to be made about the reality of the representation. The reality condition imposed on the superfield differs with the character of the superfield. The bosonic superfields, have even number of indices therefore describe half-integer superspin systems, $Y{=}s{+}1/2$. In this case we can pick to have $n{=}m{=}s$ ($H_{\alpha(s)\dot{\alpha}(s)}$) and the reality condition is $H_{\alpha(s)\dot{\alpha}(s)}{=}\bar{H}_{\alpha(s)\dot{\alpha}(s)}$. On the other hand the fermionic superfields have odd number of indices and describe integer superpsin systems, $Y{=}s{+}1$. For that case we can pick $n{=}s{+}1$, $m{=}s$ ($\Psi_{\alpha(s+1)\dot{\alpha}(s)}$) and the reality condition is the Dirac equation $i\partial_{\alpha_{s+1}}{}^{\dot{\alpha}_{s+1}}\bar{\Psi}_{\alpha(s)\dot{\alpha}(s+1)} + m\Psi_{\alpha(s+1)\dot{\alpha}(s)}{=}0$.

3 The massless limit

Representation theory tells us the type of superfields and constraints we need to consider in order to describe a specific irreducible representation. We would like to have a dynamical way to derive these constraints, through an action. Very quickly we realize that we need a set of auxiliary superfields to help us generate these constraints, as in the case of non-supersymmetric free massive arbitrary spins. The heart of the problem is to find the minimum number and type of these auxiliary superfields needed. A helpful clue in the process of constructing these massive representations is their massless limit. We demand the massless limit of our theory to give the corresponding massless irreducible representation.

The list of available massless highest superhelicity irreducible representations was presented in [7–10]. There is one infinite tower for theories of integer superhelicity and two different infinite towers for theories of half integer super-helicities. However there are a few theories that do not fit into this pattern, like the old minimal, new minimal and new-new minimal supermultiplets. These are special cases that can not be generalized to the arbitrary superhelicity. If our goal is towards the construction of an arbitrary massive superspin supermultiplet, then it is obvious that we should start with massless theories that are memebers of an infinite tower and not a special case.

The conclusion is that the construction of massive theories must start with the corresponding massless action and then add to it deformations proportional to m and m^2, so the massless theory decouples in the massless limit, along with extra auxiliary superfields if necessary.

4 New massive $Y{=}3/2$ theory

We will follow this suggested strategy to build a theory of superspin $\frac{3}{2}$. The starting point is the theory of superhelicity $\frac{3}{2}$, and specifically the one that is the linear limit of non-minimal supergravity (s =1 in [9]). Linear Non-minimal supergravity is formulated in terms of a real bosonic superfield $H_{\alpha\dot{\alpha}}$ and a fermionic superfield χ_{α}. We will add mass corrections to that action and check if 1) we can make χ_{α} vanish on-shell (auxiliary status) and 2) generate the constraints $\mathrm{D}^{\alpha}H_{\alpha\dot{\alpha}}{=}0$, $\Box H_{\alpha\dot{\alpha}}{=}m^2 H_{\alpha\dot{\alpha}}$ required by representation theory. The rest of the constraints can be generated out of this subset due to the reality of $H_{\alpha\dot{\alpha}}$ and the D-algebra. The starting action is:

$$S = \int d^8z \Big\{ H^{\alpha\dot{\alpha}} \mathrm{D}^\gamma \bar{\mathrm{D}}^2 \mathrm{D}_\gamma H_{\alpha\dot{\alpha}} + a_1 m H^{\alpha\dot{\alpha}} (\bar{\mathrm{D}}_{\dot{\alpha}} \chi_\alpha + cc) \\ -2H^{\alpha\dot{\alpha}} \bar{\mathrm{D}}_{\dot{\alpha}} \mathrm{D}^2 \chi_\alpha + a_2 m H^{\alpha\dot{\alpha}} \mathrm{D}^2 H_{\alpha\dot{\alpha}} + c.c. \\ -2\chi^\alpha \mathrm{D}^2 \chi_\alpha + a_3 m \chi^\alpha \chi_\alpha + c.c. \\ +2\chi^\alpha \mathrm{D}_\alpha \bar{\mathrm{D}}^{\dot{\alpha}} \bar{\chi}_{\dot{\alpha}} + a_4 m^2 H^{\alpha\dot{\alpha}} H_{\alpha\dot{\alpha}} \Big\}. \tag{6}$$

The equations of motion are:

$$\mathcal{E}^{(H)}_{\alpha\dot{\alpha}} = 2\mathrm{D}^\gamma \bar{\mathrm{D}}^2 \mathrm{D}_\gamma H_{\alpha\dot{\alpha}} + 2(\mathrm{D}_\alpha \bar{\mathrm{D}}^2 \bar{\chi}_{\dot{\alpha}} - \bar{\mathrm{D}}_{\dot{\alpha}} \mathrm{D}^2 \chi_\alpha) \\ +a_1 m(\bar{\mathrm{D}}_{\dot{\alpha}} \chi_\alpha - \mathrm{D}_\alpha \bar{\chi}_{\dot{\alpha}}) + 2a_2 m(\mathrm{D}^2 H_{\alpha\dot{\alpha}} + \bar{\mathrm{D}}^2 H_{\alpha\dot{\alpha}}) \\ +2a_4 m^2 H_{\alpha\dot{\alpha}}, \tag{7a}$$

$$\mathcal{E}^{(\chi)}_\alpha = -4\mathrm{D}^2 \chi_\alpha + 2\mathrm{D}_\alpha \bar{\mathrm{D}}^{\dot{\alpha}} \bar{\chi}_{\dot{\alpha}} - 2\mathrm{D}^2 \bar{\mathrm{D}}^{\dot{\alpha}} H_{\alpha\dot{\alpha}} \\ +a_1 m \bar{\mathrm{D}}^{\dot{\alpha}} H_{\alpha\dot{\alpha}} + 2a_3 m \chi_\alpha. \tag{7b}$$

Now we can use these equations and attempt to remove any $H_{\alpha\alpha}$-dependence to derive one equation that depends solely on χ_α. That will tell us if we can pick coefficients in a way that χ_α vanishes on-shell. Consider the following combination:

$$I_\alpha = A\mathrm{D}^2 \bar{\mathrm{D}}^{\dot{\alpha}} \mathcal{E}^{(H)}_{\alpha\dot{\alpha}} + B\mathrm{D}^2 \bar{\mathrm{D}}^2 \mathcal{E}^{(\chi)}_\alpha + m^2 \mathcal{E}^{(\chi)}_\alpha \\ -2(A+B)\Box \mathrm{D}^2 \bar{\mathrm{D}}^{\dot{\alpha}} H_{\alpha\dot{\alpha}} + 2(A+B)\mathrm{D}^2 \bar{\mathrm{D}}^2 \mathrm{D}_\alpha \bar{\mathrm{D}}^{\dot{\alpha}} \bar{\chi}_{\dot{\alpha}} \\ -Aa_1 m \mathrm{D}^2 \bar{\mathrm{D}}^{\dot{\alpha}} \mathrm{D}_\alpha \bar{\chi}_{\dot{\alpha}} + 2(Aa_4 - 1) m^2 \mathrm{D}^2 \bar{\mathrm{D}}^{\dot{\alpha}} H_{\alpha\dot{\alpha}} \\ -4(A+B)\Box \mathrm{D}^2 \chi_\alpha - 4m^2 \mathrm{D}^2 \chi_\alpha \\ +a_1 m^3 \bar{\mathrm{D}}^{\dot{\alpha}} H_{\alpha\dot{\alpha}} + 2(Aa_1 + Ba_3) m \mathrm{D}^2 \bar{\mathrm{D}}^2 \chi_\alpha \\ +2m^2 \mathrm{D}_\alpha \bar{\mathrm{D}}^{\dot{\alpha}} \bar{\chi}_{\dot{\alpha}} + 2a_3 m^3 \chi_\alpha. \tag{8}$$

From that it is obvious that there is a choice of coefficients that will remove any $H_{\alpha\dot{\alpha}}$ dependence:

$$A + B = 0, \quad Aa_4 - 1 = 0, \quad a_1 = 0 \tag{9}$$

and for that choice we get,

$$I_\alpha = -4m^2 \mathrm{D}^2 \chi_\alpha + 2Ba_3 m \mathrm{D}^2 \bar{\mathrm{D}}^2 \chi_\alpha \\ +2m^2 \mathrm{D}_\alpha \bar{\mathrm{D}}^{\dot{\alpha}} \bar{\chi}_{\dot{\alpha}} + 2a_3 m^3 \chi_\alpha \ . \tag{10}$$

It is clear that there is no freedom left in order to make χ_α vanish on-shell. Therefore we must introduce an auxiliary superfield. Its purpose will be to impose a constraint on χ_α when it vanishes. That constraint will be used to simplify the above expression for I_α and set χ_α to zero. But a more careful examination of I_α will convince us that there is no single differential constraint on χ_α that will make all dynamical terms vanish. The inescapable conclusion is that we have to treat χ_α=0 as the desired constraint. This suggests that we must introduce a spinorial superfield u_α that couples only with χ_α through a mass term $mu^\alpha\chi_\alpha$. Hence when u_α=0 then immediately we see χ_α=0.

We must update the action with the addition of the interaction term $m\ u^\alpha\chi_\alpha$, the kinetic energy terms for u_α (the most general quadratic action) and the mass term of u_α. The new action is

$$S = \int d^8z \Big\{ H^{\alpha\dot{\alpha}} \mathrm{D}^\gamma \bar{\mathrm{D}}^2 \mathrm{D}_\gamma H_{\alpha\dot{\alpha}} \\ -2H^{\alpha\dot{\alpha}} (\bar{\mathrm{D}}_{\dot{\alpha}} \mathrm{D}^2 \chi_\alpha - \mathrm{D}_\alpha \bar{\mathrm{D}}^2 \bar{\chi}_{\dot{\alpha}}) \\ -2\chi^\alpha \mathrm{D}^2 \chi_\alpha + a_2 m H^{\alpha\dot{\alpha}} \mathrm{D}^2 H_{\alpha\dot{\alpha}} + c.c. \\ +2\chi^\alpha \mathrm{D}_\alpha \bar{\mathrm{D}}^{\dot{\alpha}} \bar{\chi}_{\dot{\alpha}} + a_4 m^2 H^{\alpha\dot{\alpha}} H_{\alpha\dot{\alpha}} \\ +a_3 m \chi^\alpha \chi_\alpha + \gamma m u^\alpha \chi_\alpha + c.c. \\ +b_1 u^\alpha \mathrm{D}^2 u_\alpha + b_2 u^\alpha \bar{\mathrm{D}}^2 u_\alpha + c.c. \\ +b_3 u^\alpha \bar{\mathrm{D}}^{\dot{\alpha}} \mathrm{D}_\alpha \bar{u}_{\dot{\alpha}} + b_4 u^\alpha \mathrm{D}_\alpha \bar{\mathrm{D}}^{\dot{\alpha}} \bar{u}_{\dot{\alpha}} \\ +b_5 m u^\alpha u_\alpha + c.c. \Big\}. \tag{11}$$

and the updated equations of motion are

$$\mathcal{E}^{(H)}_{\alpha\dot{\alpha}} = 2\mathrm{D}^\gamma \bar{\mathrm{D}}^2 \mathrm{D}_\gamma H_{\alpha\dot{\alpha}} + 2(\mathrm{D}_\alpha \bar{\mathrm{D}}^2 \bar{\chi}_{\dot{\alpha}} - \bar{\mathrm{D}}_{\dot{\alpha}} \mathrm{D}^2 \chi_\alpha) \\ +2a_2 m(\mathrm{D}^2 H_{\alpha\dot{\alpha}} + \bar{\mathrm{D}}^2 H_{\alpha\dot{\alpha}}) + 2a_4 m^2 H_{\alpha\dot{\alpha}}, \tag{12a}$$

$$\mathcal{E}^{(\chi)}_\alpha = -4\mathrm{D}^2 \chi_\alpha + 2\mathrm{D}_\alpha \bar{\mathrm{D}}^{\dot{\alpha}} \bar{\chi}_{\dot{\alpha}} - 2\mathrm{D}^2 \bar{\mathrm{D}}^{\dot{\alpha}} H_{\alpha\dot{\alpha}} \\ +2a_3 m \chi_\alpha + \gamma m u_\alpha. \tag{12b}$$

$$\mathcal{E}^{(u)}_\alpha = 2b_1 \mathrm{D}^2 u_\alpha + 2b_2 \bar{\mathrm{D}}^2 u_\alpha + b_3 \bar{\mathrm{D}}^{\dot{\alpha}} \mathrm{D}_\alpha \bar{u}_{\dot{\alpha}} \\ +b_4 \mathrm{D}_\alpha \bar{\mathrm{D}}^{\dot{\alpha}} \bar{u}_{\dot{\alpha}} + 2b_5 m u_\alpha + \gamma m \chi_\alpha. \tag{12c}$$

We repeat the process of eliminating $H_{\alpha\dot{\alpha}}$, but since u_α does not couple to $H_{\alpha\dot{\alpha}}$ nothing will be changed regarding the $H_{\alpha\dot{\alpha}}$-dependent terms. The same choice of coefficients as in (9) must be made to remove $H_{\alpha\dot{\alpha}}$. The updated expression for I_α is

$$I'_\alpha = 2Ba_3 m \mathrm{D}^2 \bar{\mathrm{D}}^2 \chi_\alpha - 4m^2 \mathrm{D}^2 \chi_\alpha + 2m^2 \mathrm{D}_\alpha \bar{\mathrm{D}}^{\dot{\alpha}} \bar{\chi}_{\dot{\alpha}} \\ +B\gamma m \mathrm{D}^2 \bar{\mathrm{D}}^2 u_\alpha + \gamma m^3 u_\alpha + 2a_3 m^3 \chi_\alpha \ . \tag{13}$$

Now we want to use the equation of motion of u_α to remove any dependences on χ_α in order to derive an equation just for u_α. For that, consider the combination

$$J_\alpha = I'_\alpha + mK\mathrm{D}^2 \mathcal{E}^{(u)}_\alpha + m\Lambda \mathrm{D}_\alpha \bar{\mathrm{D}}^{\dot{\alpha}} \bar{\mathcal{E}}^{(u)}_{\dot{\alpha}} \\ = [2Ba_3]\mathrm{D}^2 \bar{\mathrm{D}}^2 \chi_\alpha + [B\gamma + 2Kb_2 + \Lambda b_3] m \mathrm{D}^2 \bar{\mathrm{D}}^{\dot{\alpha}} u_\alpha \\ -[4 - K\gamma] m^2 \mathrm{D}^2 \chi_\alpha + [Kb_3 + 2\Lambda b_2] m \mathrm{D}^2 \bar{\mathrm{D}}^{\dot{\alpha}} \mathrm{D}_\alpha \bar{u}_{\dot{\alpha}} \\ +[2 + \Lambda\gamma] m^2 \mathrm{D}_\alpha \bar{\mathrm{D}}^{\dot{\alpha}} \bar{\chi}_{\dot{\alpha}} + [\Lambda(2b_4 - b_3)] \mathrm{D}_\alpha \bar{\mathrm{D}}^2 \mathrm{D}^\beta u_\beta \\ +[Kb_5] m^2 \mathrm{D}^2 u_\alpha + [\Lambda b_5] m^2 \mathrm{D}_\alpha \bar{\mathrm{D}}^{\dot{\alpha}} \bar{u}_{\dot{\alpha}} \\ +[2a_3] m^3 \chi_\alpha + \gamma m^3 u_\alpha. \tag{14}$$

If we choose

$$a_3 = 0\ ,\ -4 + K\gamma = 0\ ,\ 2 + \Lambda\gamma = 0\ , \tag{15}$$

all the χ_α dependence disappears and we get:

$$J_\alpha = [B\gamma + 2Kb_2 + \Lambda b_3] m \mathrm{D}^2 \bar{\mathrm{D}}^{\dot{\alpha}} u_\alpha + [Kb_5] m^2 \mathrm{D}^2 u_\alpha \\ +[Kb_3 + 2\Lambda b_2] m \mathrm{D}^2 \bar{\mathrm{D}}^{\dot{\alpha}} \mathrm{D}_\alpha \bar{u}_{\dot{\alpha}} + [\Lambda b_5] m^2 \mathrm{D}_\alpha \bar{\mathrm{D}}^{\dot{\alpha}} \bar{u}_{\dot{\alpha}} \\ +[\Lambda(2b_4 - b_3)] \mathrm{D}_\alpha \bar{\mathrm{D}}^2 \mathrm{D}^\beta u_\beta + \gamma m^3 u_\alpha. \tag{16}$$

As a result, we have the freedom to further choose coefficients in order to cancel all the dynamical terms of u_α and force it to vanishe on-shell

$$\begin{aligned} &B\gamma + 2Kb_2 + \Lambda b_3 = 0\ ,\ Kb_3 + 2\Lambda b_2 = 0\ ,\\ &2b_4 - b_3 = 0\ ,\qquad b_5 = 0\ ,\\ &\gamma \neq 0 \qquad . \end{aligned} \tag{17}$$

Since u_α=0 on-shell, now we can reverse the arguments. The equation of motion of u_α (12c) will give χ_α=0 on-shell and that will impose constraints on $H_{\alpha\dot\alpha}$:

$$\mathcal{E}^{(\chi)}_{\alpha}|_{u_\alpha=\chi_\alpha=0} = -2\mathrm{D}^2\bar{\mathrm{D}}^{\dot\alpha}H_{\alpha\dot\alpha} \Rightarrow \mathrm{D}^2\bar{\mathrm{D}}^{\dot\alpha}H_{\alpha\dot\alpha} = 0, \tag{18a}$$

$$\mathcal{E}^{(H)}_{\alpha\dot\alpha}|_{u_\alpha=\chi_\alpha=0} = 2\mathrm{D}^\gamma\bar{\mathrm{D}}^2\mathrm{D}_\gamma H_{\alpha\dot\alpha} + 2a_4m^2H_{\alpha\dot\alpha} + 2a_2m(\mathrm{D}^2H_{\alpha\dot\alpha} + \bar{\mathrm{D}}^2H_{\alpha\dot\alpha}). \tag{18b}$$

therefore due to (18a) we get that

$$\mathrm{D}^\alpha\mathcal{E}^{(H)}_{\alpha\dot\alpha} = 2a_2m\mathrm{D}^\alpha\bar{\mathrm{D}}^2H_{\alpha\dot\alpha} + 2a_4m^2\mathrm{D}^\alpha H_{\alpha\dot\alpha}. \tag{19}$$

For $a_2 = 0$, $a_4 \neq 0$ this gives the desired condition $\mathrm{D}^\alpha H_{\alpha\dot\alpha} = 0$ and the equation of motion for $H_{\alpha\dot\alpha}$ becomes the Klein-Gordon equation with $a_4 = 1$

$$\Box H_{\alpha\dot\alpha} = m^2H_{\alpha\dot\alpha}. \tag{20}$$

To complete the analysis we look for the consistency and non-trivial solution of the systems of equations (9,15,17) plus a_2=0 and a_4=1. A solution exists

$$\begin{aligned} &a_1 = 0\ ,\ b_1 = \text{free, can be set to zero}\ ,\ \gamma = 1\ , \\ &a_2 = 0\ ,\ b_2 = \frac{1}{6}\ , \qquad A = 1\ ,\\ &a_3 = 0\ ,\ b_3 = \frac{1}{6}\ , \qquad B = -1\ ,\\ &a_4 = 1\ ,\ b_4 = \frac{1}{12}\ , \qquad K = 4\ ,\\ &\qquad b_5 = 0\ , \qquad \Lambda = -2 \end{aligned} \tag{21}$$

and the final action takes the form

$$\begin{aligned} S = \int d^8z \Big\{ &H^{\alpha\dot\alpha}\mathrm{D}^\gamma\bar{\mathrm{D}}^2\mathrm{D}_\gamma H_{\alpha\dot\alpha}\\ &-2H^{\alpha\dot\alpha}\bar{\mathrm{D}}_{\dot\alpha}\mathrm{D}^2\chi_\alpha + \frac{1}{6}u^\alpha\bar{\mathrm{D}}^2u_\alpha + c.c.\\ &-2(\chi^\alpha\mathrm{D}^2\chi_\alpha + c.c.) + \frac{1}{6}u^\alpha\bar{\mathrm{D}}^{\dot\alpha}\mathrm{D}_\alpha\bar{u}_{\dot\alpha}\\ &+2\chi^\alpha\mathrm{D}_\alpha\bar{\mathrm{D}}^{\dot\alpha}\bar{\chi}_{\dot\alpha} + \frac{1}{12}u^\alpha\mathrm{D}_\alpha\bar{\mathrm{D}}^{\dot\alpha}\bar{u}_{\dot\alpha}\\ &+m^2H^{\alpha\dot\alpha}H_{\alpha\dot\alpha} + m(u^\alpha\chi_\alpha + c.c.)\Big\}. \end{aligned} \tag{22}$$

This is the superspace action that describes a superspin Y=$\frac{3}{2}$ system with the minimum number of auxiliary superfields and has a massless limit that gives the free linearized non-minimal supergravity. This action is a representative of a family of actions that are all equivalent and connected through superfields redefinitions of the form

$$\begin{aligned} \chi_a &\rightarrow \chi_\alpha + z_1u_\alpha + w_1\bar{\mathrm{D}}^{\dot\alpha}H_{\alpha\dot\alpha},\\ u_\alpha &\rightarrow u_\alpha + z_2\chi_\alpha + w_2\bar{\mathrm{D}}^{\dot\alpha}H_{\alpha\dot\alpha}, \end{aligned}$$

where z_i and $w_i \in \mathbb{C}$.

5 Summary and conclusions

We started with the $\frac{3}{2}$ superhelicity theory of free linearized non-minimal supergravity, formulated in terms of a real vector superfield $H_{\alpha\dot\alpha}$ and a fermionic compensator χ_α. We then added mass terms to it in an attempt to discover a theory for massive superspin $\frac{3}{2}$ system, only to find that it is not possible and we need the help of an extra fermionic auxiliary superfield u_α which must couple only to χ_α through a mass term. Finally using the equations of motion we manage to show that on-shell $u_\alpha = 0 \rightsquigarrow \chi_\alpha = 0 \rightsquigarrow \mathrm{D}^\alpha H_{\alpha\dot\alpha} = 0 \rightsquigarrow \Box H_{\alpha\dot\alpha} = m^2H_{\alpha\dot\alpha}$.

We have managed to derive yet another formulation of free massive supergravity supermultiplet and most importantly probe into the set of auxiliary superfields required for the construction of higher superspin theories. The fermionic superfield u_α is the first non-trivial auxiliary superfield needed beyond the massless theory. As we go to even higher superspin values we should discover more and more of these objects. The hope is that after the study of some non-trivial low superspin examples, such as the one demonstrated here, we will have a deeper understanding on the number, type and role of these auxiliary objects. When that happens we might be in a position to construct the arbitrary massive superspin irreducible representation in an inductive manner.

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ДИНАМИЧЕСКАЯ ТЕОРИЯ СВОБОДНОГО МАССИВНОГО СУПЕРСПИНА 3/2

Мы предлагаем новую теорию свободного массивного суперспина $Y = 3/2$ неприводимого представления $4D$, $\mathcal{N} = 1$ супер-Пуанкаре группы, которое линеаризует неминимальную супергравитацию (суперспиральность $Y = 3/2$) в безмассовом пределе. Теория представлена в терминах реального бозонного суперполя $H_{\alpha\dot{\alpha}}$ и двух фермионных суперполей χ_α , u_α. Новые результаты исключают неоднозначность структуры явных суперполей, необходимой для описания систем массивных высших суперспинов.

Ключевые слова: *суперспин, суперпространство*

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