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THE $1/N$ CORRECTION TO THE $D3$-BRANE DESCRIPTION OF CIRCULAR WILSON LOOPS

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We consider the one-loop correction to the probe D3-brane action in $AdS_5 \times S^5$ expanded around the classical Drukker-Fiol solution ending on a circle at the boundary. It is given by the logarithm of the one-loop partition function of an Abelian $\mathcal{N} = 4$ vector multiplet in $AdS_2 \times S^2$ geometry. This one-loop correction is expected to describe the subleading $1/N$ term in the expectation value of circular Wilson loop in the totally symmetric rank k representation in $\mathcal{N} = 4$, $SU(N)$ supersymmetric Yang-Mills theory at strong coupling. We also discuss a comparison with the matrix model.

Keywords: *The AdS/CFT correspondence, Wilson loops, D-branes, matrix models, the heat kernel technique.*

1 Introduction

According to the AdS/CFT correspondence [1] $\mathcal{N} = 4$, $SU(N)$ supersymmetric Yang-Mills theory admits a dual description as string theory on the $AdS_5 \times S^5$ background. The correspondence provides a string theory interpretation of various field theory phenomena and gives a way to perform computations in gauge theory in the regime of strong coupling where the standard field theoretic methods break down. However, since the AdS/CFT remains a conjecture finding its non-trivial checks is an important problem.

Supersymmetric circular Wilson loops represent a good lab for high precision tests of the AdS/CFT correspondence. The reason is that they admit a matrix model solution for any representation $\mathcal{R}$ as well as for any N and for any 't Hooft coupling λ [2, 3]

$$\langle W_{\mathcal{R}} \rangle = \frac{1}{Z} \int dM \frac{1}{N} \mathrm{Tr}_{\mathcal{R}} e^{M} e^{-\frac{2N}{\lambda} \mathrm{tr} M^2}, \qquad (1)$$

where M is an Hermitian matrix. Recently, a general solution (though rather inexplicit and hard to use) to (1) was found in [4] to all order in N and λ. Hence, any string calculation of $\langle W_{\mathcal{R}} \rangle$ can be compared with the matrix model result. Such a comparison is not based on any symmetry and represents a highly non-trivial test of the AdS/CFT correspondence.

In this paper, we will concentrate on circular Wilson loops in the symmetric representation. If in the large N limit the number of boxes k in the Young tableau is small comparing to N then at strong coupling such a Wilson loop is described by k coincident circular strings in $AdS_5 \times S^5$. The case $k = 1$ corresponds to the Wilson loop in the fundamental representation $\square$ and is described by a single string [5–7]. The situation becomes different when k gets of order N. It was argued in [8–13] that as the number of boxes in the Young tableau becomes of order N the Wilson loop is effectively described by D-branes rather than by strings. In particular, the Wilson loop in the symmetric representation is now described by a probe $D3$-brane in $AdS_5 \times S^5$.

The classical solution corresponding to a probe $D3$-brane ending on a circle on the boundary was found by Drukker and Fiol in [8]. The investigation of the semiclassical quantization near the Drukker-Fiol solution was initiated in [14, 15]. The action for the quadratic fluctuations was found to be that of an Abelian $\mathcal{N} = 4$ multiplet in $AdS_2 \times S^2$. In this paper we discuss the 1-loop effective action for these quadratic fluctuations following [16]. Essectially, we compute the 1-loop effective action on $AdS_2 \times S^2$ using the heat kernel technique and the ζ-function regularization. At the end we comment on a comparison with the matrix model.

2 Circular Wilson loops in the fundamental representation

Let us review the string theory description of circular Wilson loops in the fundamental representation. In the large N limit, semiclassically they are described by a solution to the Nambu-Goto action in the $AdS_5 \times S^5$ target space ending on a circle [5–7]. The solution sits in AdS_3 and in conformal gauge it is of the form

$$z = \tanh \tau, \quad x_1 = \frac{\cos \sigma}{\cosh \tau}, \quad x_2 = \frac{\sin \sigma}{\cosh \tau}. \qquad (2)$$

Here (z, x_1, x_2) are the Poincare coordinates in AdS_3 (z is the radial coordinate), $\tau \in (0, \infty), \sigma \in [0, 2\pi]$. The induced worldsheet is AdS_2. To compute $\langle W_{\square} \rangle$ semiclassically at large λ we compute the Nambu-Goto action on this solution to get

$$\ln \langle W_{\square} \rangle = -\frac{\sqrt{\lambda}}{2\pi} \mathcal{A}(AdS_2) = \sqrt{\lambda}\,, \qquad (3)$$

where $\mathcal{A}(AdS_2)$ is the regularized volume of AdS_2, $\mathcal{A}(AdS_2) = -2\pi$. Expanding the Nambu-Goto action around this solution gives the following structure of the 1-loop correction [17]

$$\langle W_\Box \rangle_{1-loop} = \frac{\mathrm{Det}^{8/2}[-\nabla^2 + 1 + R/4]}{\mathrm{Det}^{3/2}[-\nabla^2 + 2]\mathrm{Det}^{3/2}[-\nabla^2]} . \quad (4)$$

That is to find the 1-loop correction we have to study the 1-loop effective action of the following fields propagating on AdS_2: 5 scalars with $m^2 = 0$, 3 scalars with $m^2 = 2$ and 8 Majorana fermions with $m^2 = 1$. The 1-loop effective action in (4) was computed using the methods of quantum field theory in curved space [16] and by the Gelfand-Yaglom method [18,19]. Both methods give

$$\ln\langle W_\Box \rangle_{1-loop} = -\frac{1}{2}\ln(2\pi) . \quad (5)$$

Let us compare these results with the matrix model. The matrix integral (1) for the representation $\Box$ in the limit of large N and large λ gives

$$\ln\langle W_\Box \rangle = \sqrt{\lambda} + \frac{1}{2}\ln\frac{2}{\pi} + \dots . \quad (6)$$

Comparing (6) with (3) and (5) we see that the leading terms $\sqrt{\lambda}$ agree but the subleading terms do not

$$-\frac{1}{2}\ln(2\pi) \neq \frac{1}{2}\ln\frac{2}{\pi} . \quad (7)$$

Surprisingly, the resolution of this disagreement is currently unknown. It suggests that the string partition function (4) is missing a factor of $2 = e^{\ln 2}$ which presumably should come from a proper division of the 1-loop determinant of the longitudinal modes by the ghost determinant.

Now let us briefly discuss the simplest higher representation $\Box^k$, $k \ll N$. At the level of the matrix integral (1) the modification is very simple: $\lambda \to k^2\lambda$. At the level of string theory we modify the classical solution (2) as $\tau \to k\tau, \sigma \to k\sigma$. Semiclassically, $\ln\langle W_{\Box^k}\rangle = k\sqrt{\lambda}$. However, finding the 1-loop effective action becomes a very difficult problem. The reason is that the induced metric now has a conical singularity at the origin with deficit angle $2\pi(1-k)$ and we have to study quantum field theory on a singular space.

3 Wilson loops as D-branes

Now we will discuss the case when the number of boxes k becomes of order N. In this case the number of strings becomes large and Drukker and Fiol [8] proposed that the appropriate description is given in terms of $D3$-branes. Let us parametrize AdS_5 as follows

$$ds^2 = \frac{L^2}{\sin^2\eta}[d\eta^2 + \cos^2\eta d\psi^2 + d\rho^2 + \sinh^2\rho \, ds_{S^2}] , \quad (8)$$

where L is the radius of AdS_5 and ds_{S^2} is the metric on the unit 2-sphere. The circular Wilson loop is then given by the following solution of the Born–Infeld action

$$\sin\eta = \kappa^{-1}\sinh\rho , \quad F_{\psi\rho} = \frac{i\kappa\sqrt{\lambda}}{2\pi\sinh^2\rho} , \quad \kappa = \frac{k\sqrt{\lambda}}{4N} . \quad (9)$$

Note that the solution involves a non-trivial electric field propagating on the $D3$-brane worldvolume. Near the boundary $\eta = 0$ we also have $\rho = 0$ and, hence, from (8) we see that the solution at the boundary is parametrized by an angular variable ψ which means that the $D3$-brane ends on a circle. The induced metric g_{ab} on the $D3$-brane is that of $AdS_2 \times S^2$ where

$$R_{AdS_2} = L\sqrt{1+\kappa^2} , \quad R_{S^2} = L\kappa . \quad (10)$$

The vev of the Wilson loop was found to be

$$\ln\langle W \rangle = 2N[\kappa\sqrt{1+\kappa^2} + \mathrm{arcsinh}\kappa] . \quad (11)$$

The above discussion shows that the description in terms of $D3$-branes is valid in the limit of large k, N and λ but fixed κ. Note that in the limit $k \ll N$, that is $\kappa \ll 1$ we obtain $\ln\langle W \rangle \to k\sqrt{\lambda}$ which is the action of k strings.

Originally it was proposed in [8] that a probe $D3$-brane describes Wilson loops in the representation $\Box^k$ when k becomes large. However, it was later agued in [9] that $D3$-branes describe Wilson loops in the symmetric representation Sym_k. Indeed, let us take N $D3$-branes and intersect them with some other branes in a supersymmetric way so that the intersection is 1-dimensional. It was shown in [9] that there are only 2 such possibilities: a $D3-D3$ system or a $D3-D5$ system. The action describing the intersecting branes is given by $S_{\mathcal{N}=4} + S_{defect}$, where $S_{\mathcal{N}=4}$ is the action of $\mathcal{N}=4$, $SU(N)$ supersymmetric Yang-Mills theory supported on the stack of the original N $D3$-branes and S_{defect} is the defect action supported on the intersection. We can integrate the defect degrees of freedom which can be interpreted as an insertion of a Wilson loop in some representation into the path integral. In [9] it was shown that the $D3-D3$-system corresponds to inserting a Wilson loop in the symmetric representation and the $D3-D5$-system corresponds to inserting the Wilson loop in the antisymmetric representation. When we go to strong coupling limit we replace the stack of N $D3$-branes with the $AdS_5 \times S^5$ geometry and the remaining brane becomes a probe. From this viewpoint, a probe $D3$-brane should describe Wilson loops in the symmetric representation. At the semiclassical limit the matrix integral (1) for both $\Box^k$ and Sym_k representations give the same answer (11) [8,10]. However, at the subleading level we should expect to see the difference. Note that the leading semiclassical contributions scales as N, hence, the higher order corrections will go as powers of $1/N$.

4 The 1-loop correction to the $D3$-brane effective action

Expansion of the $D3$-action around the Drukker-Fiol solution was performed in [14,15]. The result reads

$$S_B = \int d^4\sigma\sqrt{M}[\frac{1}{2}G^{ab}\partial_a\Phi^I\partial_b\Phi^I + \frac{1}{4}f_{ab}f^{ab}]\,,$$
$$S_F = \int d^4\sigma\sqrt{M}\bar{\Theta}^A(iG^{ab}\Gamma_a\nabla_b)\Theta^A\,, \qquad (12)$$

where S_B and S_F are the actions for the bosonic and fermionic fluctuations respectively. In (12) Φ^I are the 6 scalar fluctuations, f_{ab} are the fluctuations of the Abelian gauge field and Θ^A are the 4 Majorana spinors. Except for the $\sqrt{M}$ the action is defined using the background metric G_{ab}

$$G_{ab} = g_{ab} + (2\pi\alpha')^2 g^{cd}F_{ac}F_{bd}\,,$$
$$G_{ab}d\sigma^a d\sigma^b = L^2\kappa^2(ds^2_{AdS_2} + ds^2_{S^2})\,,$$
$$R_{AdS_s} = R_{S^2} = L\kappa\,. \qquad (13)$$

Finally, the matrix M is given by

$$M_{ab} = g_{ab} + (2\pi\alpha')^2 F_{ab}\,. \qquad (14)$$

The metric G_{ab} is called the open string metric [20] and it is related to M as follows $G^{ab} = M^{(ab)}$. Furthermore, we have a relation

$$\sqrt{M} = c\sqrt{G}\,, \quad c = \frac{\sqrt{1+\kappa^2}}{\kappa}\,. \qquad (15)$$

To summarize, we find that

$$S_B + S_F = cS^{Abelian}_{\mathcal{N}=4}(AdS_2\times S^2), \qquad (16)$$

that is the action of the quadratic fluctuations is, up to the prefactor c, the action of an Abelian $\mathcal{N}=4$ multiplet on $AdS_2\times S^2$ where both AdS_2 and S^2 have the same radius $L\kappa$. The prefactor c, however, is important as it depends on κ. It also means that the fluctuations have a non-trivial norm given by

$$||\Phi||^2 = \int d^4\sigma\sqrt{M}\Phi^I\Phi^I = c\int d^4\sigma\sqrt{G}\Phi^I\Phi^I \qquad (17)$$

and similarly for the vector and spinor fluctuations.

Now let us explicitly compute the 1-loop effective action

$$\Gamma_1 = \Gamma_v + 6\Gamma_s + 4\Gamma_f\,, \quad \ln\langle W\rangle_{1-loop} = -\Gamma_1, \qquad (18)$$

where we have contributions from a vector, 6 scalars and 4 Majorana fermions. Let us first ignore the prefactor c and discuss the effective action on just $AdS_2\times S^2$ where both AdS_2 and S^2 have the same radius a. We can study the effective action using the heat kernel $K(s)$:

$$K(s) = \int d^4\sigma\sum_n e^{-\lambda_n s}f^*_n(\sigma)f_n(\sigma)$$
$$= \mathrm{Vol}\cdot\sum_n e^{-\lambda_n s}f^*_n(\sigma)f_n(\sigma), \qquad (19)$$

where $\{f_n(\sigma)\}$ is a full set of orthonormal functions with eigenvalues λ_n, Vol is the regularized volume of $AdS_2\times S^2$ and in the last equality we used the fact that on a homogeneous space the integrand does not depend on the point. The 1-loop effective action is then given by

$$\Gamma_1 = -\frac{1}{2}\int_{\epsilon/a^2}^{\infty}\frac{ds}{s}K(s)\,, \qquad (20)$$

where ϵ is a UV cut-off. Note that since $AdS_2\times S^2$ is conformally flat the dependence on the radius a appears only via the combination a^2/ϵ. Let us expand the heat kernel for small s which corresponds to concentrating on the UV divergences. In 4 dimensions the expansion is of the form

$$K(s) = \mathrm{Vol}\cdot[\frac{b_0}{s^2} + \frac{b_2}{s} + b_4 + \mathcal{O}(s)]\,. \qquad (21)$$

In the present case the coefficient $b_0 = 0$ because we have equal number of bosonic and fermionic degrees of freedom. The coefficient $b_2 = 0$ because we the total curvature of AdS_2 and S^2 vanishes. The coefficient b_4 has the following general structure (see [21] for a review)

$$b_4 = \frac{\tilde{b}}{(360(4\pi)^2}[2G^{ab}D_aD_bR$$
$$+ 5R^2 - 2R^2_{ab} + 2R^2_{abcd}], \qquad (22)$$

where $\tilde{b}$ equals 1 for a scalar, $11/2$ for a Majorana fermion and 62 for a vector. In the present case we obtain

$$\mathrm{Vol}\cdot b_4 = -1\,. \qquad (23)$$

This means that the 1-loop effective action is given by

$$\Gamma_1 = -\mathrm{Vol}\cdot\frac{b_4}{2}\ln\frac{a^2}{\epsilon} + \Gamma_{fin} = \frac{1}{2}\ln\frac{a^2}{\epsilon} + \Gamma_{fin}\,, \qquad (24)$$

where Γ_{fin} is finite and independent of the radius a.

The term Γ_{fin} can be computed using the ζ-function regularization [16]

$$\zeta(s) = \frac{1}{\Gamma(s)}\int_0^\infty dt\ t^{s-1}K(t)\,,$$
$$\Gamma_1 = -\frac{1}{2}\zeta(0)\ln\frac{a^2}{\epsilon} - \frac{1}{2}\zeta'(0)\,. \qquad (25)$$

With the proper zero mode treatment one can show that $\zeta(0) = \text{Vol} \cdot b_4 = -1$. Explicit calculations presented in [16] give

$$\zeta'(0) = \frac{45 - 158\ln 2 - 160\ln\pi}{120} + 4\ln A - 30\zeta_R'(-3)J_1 + J_2. \quad (26)$$

Here A is the Glaisher's constant $\ln A = \frac{1}{12} - \zeta_R'(-1)$ and $J_{1,2}$ are given by

$$J_{1,2} = 32\int_0^\infty \frac{dv\ v}{e^{2\pi v} \pm 1}\mathcal{H}_{1,2}(v)\,,$$
$$\mathcal{H}_1 = iv\frac{\Gamma(-iv+1/2)}{\Gamma(iv+1/2)} + \psi^{(-2)}(-iv+1/2) + \psi^{(-2)}(iv+1/2)\,, \quad (27)$$
$$\mathcal{H}_2 = iv\frac{\Gamma(-iv)}{\Gamma(iv)} + \psi^{(-2)}(-iv) + \psi^{(-2)}(iv)\,,$$

where $\psi^{(-2)}$ is the poly-gamma function. We were not able to evaluate the integrals $J_{1,2}$ analytically but it is straightforward to evaluate them numerically.

Finally, let us take into account the contribution from the prefactor c in (16). Let us consider

$$\text{Det}(c\cdot\Delta) = \text{Det}(c)\cdot\text{Det}(\Delta) = e^{\text{Tr}\ln c + \text{Tr}\ln\Delta}\,. \quad (28)$$

We are interested in the first factor

$$\text{Tr}\ln c = \ln c\cdot\text{Tr}_\Delta 1 = \ln c\cdot\text{Vol}\cdot\sum_n f_n^*(\sigma)f_n(\sigma)\,, \quad (29)$$

where we represented the trace of the unit operator in terms of a complete set of eigenfunctions of the corresponding differential operator. Comparing with (19) we notice that

$$(\text{Tr}_\Delta 1)_{reg} = K(s)\Big|_{s\to 0} = \text{Vol}\cdot b_4\,, \quad (30)$$

where we recalled that $b_0 = b_2 = 0$ after we sum over the bosonic and fermionic degrees of freedom. Thus, the full 1-loop contribution is

$$\Gamma_1 = \frac{1}{2}\ln\Big(\frac{a^2}{\epsilon}\frac{1}{c}\Big) + \Gamma_{fin}\,. \quad (31)$$

Recalling that $a = L\kappa, c = \frac{\sqrt{1+\kappa^2}}{\kappa}$ we obtain

$$\Gamma_1 = \frac{1}{2}\ln\frac{L^2}{\epsilon} + \frac{1}{2}\ln\frac{\kappa^3}{\sqrt{1+\kappa^2}} + \Gamma_{fin}\,. \quad (32)$$

Note that the result is not UV finite and, hence, is not well-defined. This is not surprising since the Born–Infeld action is only the effective action for massless open string modes. However, we observe that the second term in (32) is cut-off independent. Thus, we propose that it is robust and survives the proper embedding in string theory. That is up to a possible κ-independent constant the $1/N$ correction to the Wilson loop reads

$$\ln\langle W\rangle_{1-loop} = -\frac{1}{2}\ln\frac{\kappa^3}{\sqrt{1+\kappa^2}}\,. \quad (33)$$

Let us compare it with the matrix model results. First, lets us compare with the matrix integral for the representation $\Box^k$. The matrix model result in this case is [22]

$$\ln\langle W_{\Box^k}\rangle = -\frac{1}{2}\ln[\kappa^3\sqrt{1+\kappa^2}] \quad (34)$$

which is not the same as (33). Hence, we showed explicitly that $D3$-branes do not describe Wilson loops in the tensor product of k fundamental representation as was originally proposed by Drukker and Fiol in [8]. Note, however, that (33) and (34) agree in the limit of small κ. This fact can be explained as follows. The dimensions of the representations $\Box^k$ and Sym_k,

$$d_{\Box^k} = N^k\,, \quad d_{Sym_k} = \frac{(N+k-1)!}{k!(N-1)!}\,, \quad (35)$$

in the limit of large N and small k/N satisfy

$$\ln d_{\Box^k} = \ln d_{Sym_k} = k\ln N\,. \quad (36)$$

Due to this equality it is natural to expect that

$$\ln\langle W_{\Box^k}\rangle = \ln\langle W_{Sym_k}\rangle \quad (37)$$

in this limit.

Finally, let us compare (33) with the matrix integral for the representation Sym_k. The matrix model calculation of the subleading correction in the limit of large N, λ and fixed κ for this representation was recently performed in [23] and gave exactly the same result as (33). This can be viewed as a confirmation that $D3$-branes indeed describe Wilson loops in the symmetric representation and as a highly non-trivial check of the AdS/CFT correspondence.

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1/N ПОПРАВКА К D3-БРАННОМУ ОПИСАНИЮ КРУГОВЫХ ПЕТЕЛЬ ВИЛЬСОНА

Мы рассматриваем однопетлевую поправку к действию пробной D3-браны в $AdS_5 \times S^5$, разложенному около классического решения Друккера-Фиола, заканчивающегося на окружности на границе. Она дается логарифмом однопетлевой статистической суммы Абелева $\mathcal{N} = 4$ векторного мультиплета в геометрии $AdS_2 \times S^2$. Эта однопетлевая поправка описывает $1/N$ слагаемое в вакуумном среднем петли Вильсона, имеющей форму окружности, в симметричном представлении в $\mathcal{N} = 4$, $SU(N)$ теории Янга-Миллса в пределе сильной связи. Мы также обсуждаем сравнение с матричной моделью.

Ключевые слова: *AdS/CFT соответствие, петли Вильсона, Д-браны, матричные модели, ядро теплопроводности.*

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