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Post-Newtonian effects in the motion of the nearest satellites of Jupiter

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In this article the possibility of measurement of post-Newtonian effects in the motion of close satellites of Jupiter is discussed. On an example of Jupiter's fifth satellite Amalthea we study a question of, whether can be isolated the PN component of orbital precession of the satellite from much bigger Newtonian components. Results of researches have shown that all larger contributions of Newtonian perturbations can be modeled and subtracted out.

Keywords: *celestial mechanics, perturbation theory, satellites dynamics, relativistic effects, Jupiter's satellites.*

1 Introduction

It is known that gravitation fields of the gas giant planets are the best laboratories for measuring the effects of Post-Newtonian (PN) gravity on trajectories of natural satellites and spacecrafts [1]. In this work the problem connected with the PN shift of pericenters of orbits of close satellites of Jupiter is considered. Theoretically predicted the PN shift of pericenters of orbits of the close satellites of Jupiter much more, than for the Mercury orbit ($\dot{\varpi} = 43''$/per century) [1], [2].

Table 1. *PN precession rates $\dot{\varpi}$ (arcsec/per century)*

MetisJ16	AdrasteaJ15	AmaltheaJ5	ThebeJ14
$5286''.64$	5283.65	2212.85	1335.57

Although the post-Newtonian effects are very large for these satellites (Table 1), there are serious problems to measure and separate out such effects. In article J.D. Anderson, et al. "Gravitation and Celestial Mechanics Investigations with Galileo" [3] it is possible to read: "It is unknown whether the relativistic components of orbital precession for the inner satellites can be isolated from the far larger Newtonian precessions". In this work the attempt of studying of this question is undertaken.

2 Perturbing factors in movement of of the Jupiter's close satellites

The Hamiltonian function of a considered problem within the PN formalism can be written down in a form:

$$H = H_0 + H_{\rm N} + H_{\rm PN}, \tag{1}$$

where H_0 is the Hamiltonian function of a separable system (two-body problem), $H_{\rm N}$ is the disturbing Newtonian potential function and $H_{\rm PN}$ is the relativistic disturbing part of H. Function $H_{\rm N}$ is

$$H_{\rm N} = H_{01} + H_{02} + H_{03}. \tag{2}$$

Here H_{01}, H_{02}, H_{03} are perturbations caused respectively by the Jupiter's oblateness, Galilean satellites and the Sun.

The orbit of the fifth satellite of Jupiter Amalthea is the most studied. Therefore we use it as the test.

The first term of Hamiltonian function can be introduced in form:

$$H_0 = \frac{\mu_{\rm J}}{2a}, \;\; \mu_{\rm J} = G\, m_{\rm J}, \tag{3}$$

where a is the semi-magor axis of an orbit of the satellite, G is the Newtonian gravitational constant, $m_{\rm J}$ is mass of Jupiter. This term establishes the zero order level for the Hamiltonian function.

The most considerable disturbing influence is defined by the second zonal harmonic of the Jupiter's potential. Considering only a leading factor in expression for a zonal harmonic, we receive

$$\epsilon = \frac{\mu_{\rm J} R_{\rm J}^2 J_2}{H_0 r_{\rm A}^3}, \;\; J_2 = 1473.6 \cdot 10^{-5}. \tag{4}$$

Here $R_{\rm J}$ is the equatorial radius of Jupiter, $r_{\rm A}$ is the mean orbit radius of a satellite, J_2 is the dimensionless coefficients of the second zonal harmonic. The ratio (4) defines the small parameter ϵ of our problem. The same way the valuations of perturbations from Galilean satellites and from the Sun were made.

The term $H_{\rm PN}$ is treated as the additional potential to the Newtonian disturbing function which describes the geodesic motion of a test particle (satellite) in the gravitational field of a spherically symmetric body (Jupiter) of mass $m_{\rm J}$. In the framework of General Relativity (GR) the dominant GR effect can be described by the following expression

$$\tau_{\rm PN} = \left| -\frac{\mu h^2}{c^2 r^3} \right|. \tag{5}$$

For a case which is considered in this work , the right part of (5) is interpreted as follows: $\mu = \mu_{\rm J}, r =$

r_A. $h = r^2\dot{\theta} = \sqrt{\mu a \cdot (1-e^2)}$ is the orbital angular momentum of the satellite per unit mass, θ is the polar angle, e is the eccentricity of satellite's orbit. The contributions from H_{PN} is τ_{PN}/H_0.

The numerical values of contributions from the disturbing potential terms are given in Table 2:

Table 2. *The disturbing function of the problem.*

Disturbing action	(Leading factor)/H_0	Order
Zonal harmonic(J_2)	$4.55866 \cdot 10^{-3}$	ϵ
Galilean satellites)	$9.43476 \cdot 10^{-6}$	ϵ^2
Zonal harmonic(J_4)	$2.81962 \cdot 10^{-5}$	ϵ^2
Zonal harmonic(J_6)	$2.30768 \cdot 10^{-7}$	ϵ^3
Sun	$2.64792 \cdot 10^{-8}$	ϵ^3
H_{PN}	$1.55476 \cdot 10^{-8}$	ϵ^3

Analysis of results shows that the influence of the PN effects of the gravity are rather big and it is necessary to take them into consideration in the process modeling of movement of the inner satellites of Jupiter. The contribution from H_{PN} is commensurable with Solar perturbations. And, if it is possible to separate Solar perturbations, it can be possible to separate and PN effects also. Influence of other bodies of the Solar system on movement of the close satellites of Jupiter is negligible.

3 Secular perturbations

Usually the disturbing function is represented in the form of the truncated series at which there are century and periodic terms. Our problem (1) is nonintegrable. However, by means of suitable approximations it is possible to find an analytical solution for (1) with sufficient accuracy. In this work we concentrate only on studying of the secular motions of satellites.

With help of canonical transformations and the averaging method the function H_{01} (oblateness part) was presented in a form:

$$\begin{aligned}\langle H_{01}\rangle = &\frac{1}{2}n^2a^2\left[\frac{3}{2}J_2\left(\frac{R}{a}\right)^2 - \frac{9}{8}J_2^2\left(\frac{R}{a}\right)^4\right]e^2 - \\ &-\frac{1}{2}n^2a^2\left[\frac{15}{4}J_4\left(\frac{R}{a}\right)^4 + 15J_6\left(\frac{R}{a}\right)^6\right]e^2 - \\ &-\frac{1}{2}n^2a^2\left[\frac{3}{2}J_2\left(\frac{R}{a}\right)^2 - \frac{27}{8}J_2^2\left(\frac{R}{a}\right)^4\right]\sin^2 i + \\ &+\frac{1}{2}n^2a^2\left[\frac{15}{4}J_4\left(\frac{R}{a}\right)^4 + 15J_6\left(\frac{R}{a}\right)^6\right]\sin^2 i \quad (6)\end{aligned}$$

Here $n = 2\pi/T_s$ is the mean motion of a satellite, where T_s is its orbital period, i is the orbital inclination of a satellite to the equatorial plane of Jupiter. Expression (6) by accuracy to J4 can be found in work [4].

Then, using the equations of motion of the satellite in the orbital elements, we received expressions for the rate of change of pericentre $\dot{\varpi}_{01}$ (Newtonian part from oblateness):

$$\begin{aligned}\dot{\varpi}_{01} = &n\left[\frac{3}{2}J_2\left(\frac{R}{a}\right)^2 - \frac{9}{8}J_2^2\left(\frac{R}{a}\right)^4\right] - \\ &-n\left[\frac{15}{4}J_4\left(\frac{R}{a}\right)^4 + 15J_6\left(\frac{R}{a}\right)^6\right] \quad (7)\end{aligned}$$

The full velue of the Newtonian shift of a pericenter is determined by a formula:

$$\dot{\varpi}_N = \dot{\varpi}_{01} + \dot{\varpi}_{02} + \dot{\varpi}_{03}, \quad (8)$$

Calculation of perturbations in $\dot{\varpi}_{02}$ (from Galilean satellites) and in $\dot{\varpi}_{03}$ (from the Sun)was carried out on known analytical expressions [4].

4 Numerical experiment

Observable value of shift of a pericenter of Amalthea was received by P.V. Sudbury from the analysis of a large number of observations of the satellite [5] :

$$\dot{\varpi}_S = 3''.30264001 \cdot 10^8 \, per\, century. \quad (9)$$

At first we received total value of the Newtonian shift of a pericenter without having included the sixth harmonica:

$$\dot{\varpi}_N^{(4)} = 3''.30426244 \cdot 10^8 \, per\, century. \quad (10)$$

The similar calculations made by S. Breiter [6] , give quite close value of the shift of a pericenter:

$$\dot{\varpi}_N^{(4)} = 3''.30409477 \cdot 10^8 per\, century\, (Breiter). \quad (11)$$

The difference between the observable value and the calculated value of the shift of a pericenter is:

$$\Delta\dot{\varpi}^{(4)} = \dot{\varpi}_S - \dot{\varpi}_N^{(4)} = -162242''. \quad (12)$$

The difference has negative value, and the problem becomes uncertain.

Then the sixth harmonica was included. The following result was received:

$$\dot{\varpi}_N^{(6)} = 3''.30261439 \cdot 10^8 \, per\, century. \quad (13)$$

and

$$\Delta\dot{\varpi}^{(6)} = \dot{\varpi}_S - \dot{\varpi}_N^{(6)} = 2566''.5. \quad (14)$$

Predicted relativistic shift of pericenter of an orbit of Amalthea has the following value (Table 1):

$$\dot{\varpi}_{PN} = 2212''.85 \, per\, century. \quad (15)$$

Thus the value $\Delta\dot{\varpi}^{(6)}$ is commensurable with a value of the predicted relativistic shift of a pericenter of the satellite.

5 Conclusion

The presented analytical solution for the secular shift of the pericenter of a satellite includes the sixth harmonic of the gravitational potential of Jupiter. The difference between the observable value and the calculated value of the Newtonian part of shift of a pericenter is commensurable with the value of the predicted relativistic shift of the pericenter of a satellite. The received result says that, apparently, possibility of isolation of a relativistic part of pertutbations from Newtonian exists. But this problem is very difficult and it is necessary to consider the received result preliminary.

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ПОСТНЬЮТОНОВСКИЕ ЭФФЕКТЫ В ДВИЖЕНИИ БЛИЗКИХ СПУТНИКОВ ЮПИТЕРА

В статье обсуждается проблема измерения постньютоновских (ПН) эффектов в движении близких спутников Юпитера. На примере пятого спутника Юпитера Амальтеи решается вопрос об отделении ПН компоненты орбитальной прецессии спутника от сравнительно большой по величине ньютоновской части. Результаты исследования показали, что все нерелятивистские возмущения в вековом движении перицентра спутника могут быть получены с достаточно высокой степенью точности, позволяющей отделить их от релятивистской компоненты.

Ключевые слова: *небесная механика, теория возмущений, релятивистские эффекты, спутники Юпитера.*

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