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$U(N)$ spinning particles and higher spin fields on Kähler backgrounds

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In this short contribution we will review the quantization of $U(N)$ spinning particles with complex target spaces, producing equations for higher spin fields on complex backgrounds. We will focus first on flat complex space, and subsequently present our model on an arbitrary Kähler manifold. In the final section, we will specialize to (p, q) -forms on arbitrary Kähler spaces and present their one-loop effective actions as well as issues related to Hodge duality.

Keywords: *sigma models, higher spins, gauge symmetry.*

1 Introduction

Spinning particle models [1–4] have been a useful framework to study fields of different spins in a first-quantized approach. Worldline techniques can indeed give manageable representations of one-loop quantities in quantum field theories, such as effective actions, amplitudes and anomalies (see for a review [5, 6] and references therein), and allow to describe ordinary higher spin fields in first quantization. We will present here a class of spinning particles enjoying a $U(N)$ -extended supersymmetry on the worldline [7, 8], that naturally live on complex manifolds and give rise to complex higher spin equations. Although a direct spacetime interpretation of these models is prevented by the complex nature of the target space, they are useful playgrounds to study issues related to the quantization of higher spins. They can as well provide insights in the study of Kähler geometries, supersymmetric field theories and QFT's in curved backgrounds.

More precisely, spinning particles are quantum mechanical models, enjoying local supersymmetries on the worldline. For instance, particles with N local real supersymmetries describe spin $N/2$ fields in four dimensional spacetime. The constraints on the Hilbert space of the particle theory, arising from the gauging, translate into a set of differential equations for the spacetime field, viewed as the wave function of the quantum mechanical model. Interactions with scalars, gauge fields and gravity can be achieved, when allowed, by coupling the particle theory to suitable backgrounds. Quantizing the particle, one can recover in a rather simple way various useful objects of the related QFT, such as effective actions, n-point functions, anomalies and so on. The path integral quantization for these non-linear sigma models, arising in gravitational backgrounds, requires regularization [6, 9, 10]. This is essentially related to ill-defined products of distributions in perturbative computations, and we will use Time Slicing regulariza-

tion (TS) in all the computations that will be showed.

We organize the paper as follows: in the next section we will present the $U(N)$ spinning particle and its Dirac quantization in flat complex space. The end of the section will be devoted to the coupling to a curved Kähler background. In the last section we will focus on the $U(2)$ model, studied in [11], describing (p, q) -forms on an arbitrary Kähler space. The local proper time expansion of the one-loop effective action will be briefly sketched, along with the first Seeley-DeWitt coefficients. We will comment at the end on exact relations between Hodge dual forms that have been extracted from the particle model.

2 The $U(N)$ spinning particle

The graded phase space of the model is spanned by complex coordinates of \mathbb{C}^d and momenta: $x^\mu(t)$, $\bar{x}^{\bar{\mu}}(t)$ and $p_\mu(t)$, $\bar{p}_{\bar{\mu}}(t)$, along with the fermionic superpartners $\psi_i^\mu(t)$ and $\bar{\psi}_\mu^i(t)$, with $i = 1, 2, \dots, N$. They obey canonical (anti)-commutation relations:

$$[x^\mu, p_\nu] = i \delta_\nu^\mu, \quad [\bar{x}^{\bar{\mu}}, \bar{p}_{\bar{\nu}}] = i \delta_{\bar{\nu}}^{\bar{\mu}}, \quad \{\psi_i^\mu, \bar{\psi}_\nu^j\} = \delta_\nu^\mu \delta_i^j. \quad (1)$$

Quadratic operators constructed from the basic variables provide generators for the $U(N)$ -extended worldline SUSY:

$$\begin{aligned} H &= p_\mu \bar{p}^\mu, \quad Q_i = \psi_i^\mu p_\mu, \quad \bar{Q}^i = \bar{\psi}^{\bar{\mu}i} \bar{p}_{\bar{\mu}}, \\ J_j^i &= \frac{1}{2} [\psi_j^\mu, \bar{\psi}_\mu^i] = \psi_j^\mu \bar{\psi}_\mu^i - \frac{d}{2} \delta_j^i, \end{aligned} \quad (2)$$

where we raise and lower indices by means of the flat complex metric $\delta_{\mu\bar{\nu}}$ and its inverse. H , Q_i and \bar{Q}^i are the hamiltonian and supercharges, respectively generating worldline translations and supersymmetries, while J_j^i generates $U(N)$ R -symmetry rotations. Given the (anti)-commutation relations (1), the above generators obey an extended supersymmetry algebra with

R -symmetry group $U(N)$:

$$\begin{aligned} [J_j^i, Q_k] &= \delta_k^i Q_j, & [J_j^i, \bar{Q}^k] &= -\delta_j^k \bar{Q}^i, \\ [J_j^i, J_l^k] &= \delta_l^i J_j^k - \delta_j^k J_l^i, & \{Q_i, \bar{Q}^j\} &= \delta_i^j H \end{aligned} \quad (3)$$

the other (anti)-commutators being zero. In order to construct a worldline action that is invariant under *local* symmetries generated by (2), we couple the generators to one-dimensional gauge fields: an einbein $e(t)$ for local worldline translations, complex gravitinos $\chi_i(t)$ and $\bar{\chi}^i(t)$ for local supersymmetries and a one-dimensional $U(N)$ gauge field $a_j^i(t)$. Together with the symplectic kinetic terms, we obtain the phase space action for the $U(N)$ spinning particle:

$$S = \int_0^1 dt \left[p_\mu \dot{x}^\mu + \bar{p}_\mu \dot{\bar{x}}^\mu + i \bar{\psi}_\mu^i \dot{\psi}_i^\mu - eH - i \bar{\chi}^i Q_i - i \chi_i \bar{Q}^i - a_j^i \left(J_j^i - s \delta_j^i \right) \right], \quad (4)$$

where we added a Chern-Simons coupling s that is quantized as $s = m - \frac{d}{2}$ for integer m . The equations of motion for the one-dimensional gauge fields constrain the classical generators to vanish. At the quantum level, the operators (2) impose constraints on the Hilbert space, by requiring that they annihilate physical states:

$$|\Phi\rangle \in \mathcal{H}_{\text{phys}} \quad \Leftrightarrow \quad T_A |\Phi\rangle = 0,$$

where $T_A = (H, Q_i, \bar{Q}^i, J_j^i - s \delta_j^i)$. This is allowed since (3) is a first class superalgebra, and amounts to Dirac quantization. The above constraints will be the aforementioned higher spin equations obeyed by spacetime fields, that sit as x -dependent coefficients inside $|\Phi\rangle$.

2.1 Dirac quantization in flat space

In a Schroedinger picture, we shall realize the fermionic oscillator algebra in (1) treating the ψ_μ^i operators as Grassmann-odd variables, and the ψ_μ^i as odd derivatives thereof: $\bar{\psi}_\mu^i \sim \frac{\partial}{\partial \psi_\mu^i}$. The states in the Hilbert space will have a finite Taylor expansion in powers of ψ 's, so that the coefficients of the expansion are multi-form spacetime fields with only holomorphic indices. The $J_j^i - s \delta_j^i$ constraints are purely algebraic on the multi-forms, and impose irreducibility conditions. After imposing them, the only states that survive in the Hilbert space correspond to tensors with N blocks of m antisymmetric indices, with $s = m - \frac{d}{2}$:

$$|F\rangle \sim F_{\mu^1[m], \dots, \mu^N[m]}(x, \bar{x}),$$

where we denoted $\mu[k] := [\mu_1 \dots \mu_k]$. Symmetry between block exchanges ensures they belong to a $U(d)$ rectangular Young tableaux with m rows and N columns. On

the spacetime fields the supercharges Q_i and \bar{Q}^i act as Dolbeault operators¹ and their hermitian conjugates, generalized to multi-forms such that $Q_k \sim -i \partial_{(k)}$ meaning that it antisymmetrizes the derivative only among the indices of the k -th block. The remaining constraints have then the form of generalized Bianchi and Maxwell equations, *i.e.*

$$\begin{aligned} Q_i |F\rangle &= 0 \sim \partial_{[\mu} F_{\mu^1 \dots \mu_m], \dots, \nu_1 \dots \nu_m} = 0, \\ \bar{Q}^i |F\rangle &= 0 \sim \bar{\partial}^\mu F_{\mu \mu^2 \dots \mu_m, \dots, \nu_1 \dots \nu_m} = 0. \end{aligned} \quad (5)$$

It is natural to interpret the F fields as higher spin curvatures that obey Maxwell-like equations. It is indeed possible to solve the first of (5) by introducing a gauge potential ϕ as $|F\rangle = Q_1 Q_2 \dots Q_N |\phi\rangle$ or, in tensor language:

$$F_{\mu^1[m], \dots, \mu^N[m]} = \partial_{\mu^1} \dots \partial_{\mu^N} \phi_{\mu^1[m-1], \dots, \mu^N[m-1]}, \quad (6)$$

where each set of indices μ^k is antisymmetrized: $\mu^k \mu^k[m-1] := [\mu_1^k \dots \mu_{m-1}^k]$.

Let us notice that for $N > 1$ the Maxwell equation (5) on the potential is higher derivative. We also mention that for $N = 1$ we have ordinary $(p, 0)$ -forms $\phi_{(p,0)}$, carefully studied in [12], while for $m = 2$ the gauge potentials are completely symmetric tensors $\phi_{\mu^1 \dots \mu^N}$. Since $(Q_i)^2 = 0$ for each i , it is easy to see that the curvature F is indeed invariant under a gauge transformation of the form $\delta |\phi\rangle = Q_i |\Lambda^i\rangle$, that is

$$\begin{aligned} \delta \phi_{\mu^1[p], \dots, \mu^N[p]} &= \partial_{\mu^1} \Lambda_{\mu^1[p-1], \dots, \mu^N[p]}^{(1)} \\ &+ \dots + \partial_{\mu^N} \Lambda_{\mu^1[p], \dots, \mu^N[p-1]}^{(N)} \end{aligned} \quad (7)$$

We already noticed that the Maxwell-like equations are higher derivative in terms of the gauge field: $\bar{Q}^i Q_1 \dots Q_N |\phi\rangle = 0$. It is actually possible to introduce a second order wave operator, analogous to the Fronsdal-Labastida one, and reduce the field equations to second order: $(-H + Q_i \bar{Q}^i) |\phi\rangle = Q_i Q_j |\rho^{ij}\rangle$ by means of an auxiliary compensator $|\rho^{ij}\rangle$. In tensor language it reads

$$\begin{aligned} \partial_\nu \bar{\partial}^\nu \phi_{\mu^1[p], \dots, \mu^N[p]} - \sum_{i=1}^N \partial_{\mu^i} \bar{\partial}^\nu \phi_{\mu^1[p], \dots, \nu \mu^i[p-1], \dots, \mu^N[p]} &= \\ = \sum_{i \neq j} \partial_{\mu^i} \partial_{\mu^j} \rho_{\mu^1[p], \dots, \mu^i[p-1], \dots, \mu^j[p-1], \dots, \mu^N[p]}^{(ij)}. \end{aligned} \quad (8)$$

The field equations are invariant under the gauge transformations (7), provided that the compensator field transforms as the divergence of the gauge parameter. It turns out that it is possible to gauge fix the compensators to zero, at the price of having transverse gauge parameters²: $\bar{\partial} \cdot \Lambda = 0$.

¹On a complex (p, q) -form $A_{\mu^1 \dots \mu_p \bar{\nu}^1 \dots \bar{\nu}_q} dz^{\mu^1} \wedge \dots \wedge dz^{\mu^p} \wedge d\bar{z}^{\bar{\nu}^1} \wedge \dots \wedge d\bar{z}^{\bar{\nu}_q}$ the Dolbeault operator acts as an holomorphic exterior derivative: $\partial := dz^\mu \partial_\mu$

²For related issues on real spacetime, see for instance [13, 14]

2.2 Coupling to curved space

We analyze here the changes needed to couple the spinning particle to an arbitrary background metric. Let us consider as target space a $D = 2d$ dimensional Kähler manifold, equipped with a metric $g_{\mu\bar{\nu}}(x, \bar{x})$ in holomorphic coordinates. Having in mind the minimal coupling, it is sufficient to replace suitably covariantized constraints in the action (4). To this aim, we define $U(d)$ ‘‘Lorentz’’ generators $M_{\nu}^{\mu} = \frac{1}{2}[\psi_{\nu}^{\mu}, \bar{\psi}_{\nu}^{\mu}]$ so that we can construct covariant momenta and supercharges³:

$$\pi_{\mu} = p_{\mu} + i\Gamma_{\mu\nu}^{\lambda} M_{\lambda}^{\nu}, \quad \bar{\pi}_{\bar{\mu}} = \bar{p}_{\bar{\mu}}, \quad (9)$$

$$Q_i = \psi_i^{\mu} g^{\frac{1}{2}} \pi_{\mu} g^{-\frac{1}{2}}, \quad \bar{Q}^i = \bar{\psi}_{\bar{\mu}}^i g^{\mu\bar{\nu}} g^{\frac{1}{2}} \bar{\pi}_{\bar{\nu}} g^{-\frac{1}{2}} \quad (10)$$

The superalgebra (3) is deformed by the target space geometry, namely one has

$$\{Q_i, \bar{Q}^j\} = \delta_i^j H_0 - \psi_i^{\mu} \bar{\psi}^{j\bar{\nu}} R_{\mu\bar{\nu}\lambda\bar{\sigma}} M^{\lambda\bar{\sigma}}, \quad (11)$$

where the minimally covariantized hamiltonian reads $H_0 = g^{\frac{1}{2}} g^{\mu\bar{\nu}} \bar{\pi}_{\bar{\nu}} \pi_{\mu} g^{-\frac{1}{2}}$. One can see from (11) that the algebra is no longer first class, and hence the model is inconsistent, on general Kähler backgrounds. Important exceptions are the cases $N = 1, 2$, presented in the next section, that can be quantized on any curved background, and represent differential forms. For $N > 2$ one can still quantize the model on particular backgrounds. For instance, it is possible to quantize the spinning particle for general N , that is for arbitrary ‘‘spin’’, on Kähler spaces with constant holomorphic curvature [8], *i.e.*

$$R_{\mu\bar{\nu}\lambda\bar{\sigma}} = \Lambda (g_{\mu\bar{\nu}} g_{\lambda\bar{\sigma}} + g_{\mu\bar{\sigma}} g_{\lambda\bar{\nu}}).$$

3 N=2, (p,q)-forms on Kähler spaces

We shall focus in this section to the model with $N = 2$ on an arbitrary Kähler manifold with metric $g_{\mu\bar{\nu}}$. We decide to realize the fermionic operators in a slightly different way: here we will treat ψ_1^{μ} and $\bar{\psi}^{2\bar{\mu}}$ as odd coordinates, and $\bar{\psi}_{\bar{\mu}}^1, \psi_{2\bar{\mu}}$ as derivatives thereof. In this way, states in the Hilbert space are (p, q) -forms. In [11], several theories of differential forms were investigated by using the $U(2)$ spinning particle. Here we restrict ourselves to present one of those models. To obtain the model we are interested in, it is sufficient to gauge only the $U(1) \times U(1)$ subgroup of the R -symmetry $U(2)$ generated by J_1^1 and J_2^2 . In such a case one is free to have two different Chern-Simons couplings s_1 and s_2 , compared to s in (4). This allows to fix independently the eigenvalues of the J_1^1 and J_2^2 constraints, and physical states will be forms $F_{(m,n)}$, for given and arbitrary m, n . The four supercharges

are realized now as Dolbeault operators and their hermitian conjugates: $\partial, \bar{\partial}$ and $\partial^{\dagger}, \bar{\partial}^{\dagger}$, while the superalgebra (3) closes on the hamiltonian that acts as the Hodge laplacian

$$\Delta = -\{\partial, \partial^{\dagger}\} = \frac{\nabla^2}{2} + \frac{1}{2} R_{\mu\bar{\nu}\lambda\bar{\sigma}} M^{\mu\bar{\nu}} M^{\lambda\bar{\sigma}}.$$

Bianchi equations can be locally integrated by introducing a potential: $F_{(p+1, q+1)} = \partial\bar{\partial}A_{(p, q)}$. In this particular model, even if we are dealing with differential forms, Maxwell equations ($\partial^{\dagger}F = \bar{\partial}^{\dagger}F = 0$) are higher derivative with respect to the potential A . As we did in the general case, it is possible to have second order field equations by introducing a compensator:

$$(\Delta + \partial\partial^{\dagger} + \bar{\partial}\bar{\partial}^{\dagger}) A_{(p, q)} = \partial\bar{\partial}\rho_{(p-1, q-1)}. \quad (12)$$

The equations (12) are gauge invariant under the combined transformations of the gauge field and compensator

$$\begin{aligned} \delta A_{(p, q)} &= \partial\Lambda_{(p-1, q)}^1 + \bar{\partial}\Lambda_{(p, q-1)}^2, \\ \delta\rho_{(p-1, q-1)} &= \bar{\partial}^{\dagger}\Lambda_{(p-1, q)}^1 - \partial^{\dagger}\Lambda_{(p, q-1)}^2, \end{aligned} \quad (13)$$

and one can see again that gauge fixing the compensator to zero would restrict the gauge parameters to be transverse.

Let us turn now to the path integral quantization of the $U(2)$ spinning particle. If the particle is quantized on a circle with external gravity, one finds an expansion for the QFT one-loop effective action of the (p, q) -form in a gravitational background. We will present the heat kernel expansion in terms of local Seeley-DeWitt coefficients. In order to quantize the spinning particle, we have to take into account the gauge fixing of local worldline symmetries and Faddeev-Popov determinants. Because of the topology of the worldline circle, one is left with three modular integrals: the usual one over β , being the proper length of the circle, and two angular integration over θ and ϕ , taking into account the gauge invariant Wilson loops of the two $U(1)$ factors. The resulting partition function is given by

$$\begin{aligned} Z_{p, q}[g] &= \int_0^{\infty} \frac{d\beta}{\beta} \int_0^{2\pi} \frac{d\theta}{2\pi} \int_0^{2\pi} \frac{d\phi}{2\pi} \mu(\phi, \theta) \times \\ &\times \int \frac{d^d x_0 d^d \bar{x}_0}{(2\pi\beta)^d} g(x_0, \bar{x}_0) \langle e^{-S_{\text{int}}} \rangle, \end{aligned} \quad (14)$$

where x_0 is an arbitrary fixed spacetime point, S_{int} is the interaction part of the spinning particle action and $\mu(\phi, \theta)$ is the modular measure given by

$$\begin{aligned} \mu(\phi, \theta) &= e^{-i(p+1-d/2)\phi} e^{i(q+1-d/2)\theta} \times \\ &\times \left(2 \cos \frac{\phi}{2}\right)^{d-2} \left(2 \cos \frac{\theta}{2}\right)^{d-2}. \end{aligned} \quad (15)$$

³The g factors ensure that $(Q_i)^{\dagger} = \bar{Q}^i$

⁴In the modular integration one encounters poles along the integration path. Detailed explanation of the prescription to deal with such poles can be found in [11]

After performing the worldline perturbative computation, and evaluating the modular integrals⁴ we organize the effective action expansion, up to order β^2 , as follows:

$$Z_{p,q}[g] = \int_0^\infty \frac{d\beta}{\beta} \int \frac{d^d x_0 d^d \bar{x}_0}{(2\pi\beta)^d} g(x_0, \bar{x}_0) v_1 \{1 + v_2 \beta R + \beta^2 [v_3 (R_{\mu\bar{\nu}\lambda\bar{\sigma}})^2 + v_4 (R_{\mu\bar{\nu}})^2 + v_5 R^2 + v_6 \nabla^2 R]\}. \quad (16)$$

The Seeley-DeWitt coefficients v_i are given by

$$\begin{aligned} v_1 &= \left(\frac{d-2}{p}\right) \left(\frac{d-2}{q}\right), \quad v_2 = \frac{1}{6} - 12k_1, \\ v_3 &= \frac{1}{180} - k_2 + k_3, \quad v_4 = -\frac{1}{360} - k_1 + 4k_2 - 2k_3, \\ v_5 &= \frac{1}{72} + k_1 - 3k_2 + k_3, \quad v_6 = \frac{1}{60} - k_1, \end{aligned} \quad (17)$$

where the numerical factors k_i read

$$\begin{aligned} k_1 &= \frac{p(d-2-q) + q(d-2-p)}{24(d-2)^2}, \\ k_2 &= \frac{p(d-2-p) + q(d-2-q)}{24(d-2)(d-3)}, \\ k_3 &= \frac{p(d-2-p)q(d-2-q)}{2(d-2)^2(d-3)^2}. \end{aligned} \quad (18)$$

A few comments are now in order. First of all, let us stress that the overall coefficient v_1 gives the number of propagating degrees of freedom. For p or q greater than $d-2$ it vanishes and cannot be factored out in (16). Other coefficients can and indeed are non-vanishing, and represent the topological contribution of a non-propagating form. As a second remark we should notice that the result (18) holds only for $d > 3$. In $d = 2$ only scalars propagate, and all the k_i vanish. In $d = 3$ instead, they have the form

$$k_1 = \frac{p(1-q) + q(1-p)}{24}, \quad k_2 = \frac{p+q}{24}, \quad k_3 = \frac{pq}{2} \quad (19)$$

Finally, we stress that the k_i coefficients in (18) were used to make manifest the symmetry under the exchanges $p \leftrightarrow q$ and $p \leftrightarrow d-2-q$. The first exchange is related to the symmetry under complex conjugation, that states the equivalence $Z_{p,q}[g] = Z_{q,p}[g]$ and is exact. The second exchange relates Hodge dual forms and is more subtle. Despite the manifest symmetry in the coefficients (18), it is not an exact symmetry at the quantum level, and one can already see that in $d = 3$ the k_i (19) are not invariant under $p \leftrightarrow 1-q$. In general, for higher dimension d the mismatch appears in higher order Seeley-DeWitt coefficients. The spinning particle model, however, allows us to find an exact non-perturbative result for the mismatch, that is

⁴We denoted $Z_{p,q} = \int_0^\infty \frac{d\beta}{\beta} \mathcal{Z}_{p,q}(\beta)$.

purely topological. The derivation can be found in [11], and one has⁵

$$\begin{aligned} \mathcal{Z}_{d-2-q,d-2-p}(\beta) - \mathcal{Z}_{p,q}(\beta) &= (-)^{q+d} \mathcal{Z}_{d-1,q}^{\text{top}}(\beta) \\ &+ (-)^{p+d} \mathcal{Z}_{p,d-1}^{\text{top}}(\beta) + (-)^{p+q} \mathcal{Z}_{d-1,d-1}^{\text{top}}(\beta) \\ &+ (d-1-p)(-)^{p+q} \sum_{m=0}^q (-)^m (q+1-m) \text{ind}(\Omega^{m,0}, \bar{\partial}) \\ &+ (d-1-q)(-)^{p+q} \sum_{n=0}^p (-)^n (p+1-n) \text{ind}(\Omega^{n,0}, \bar{\partial}) \\ &+ (-)^{p+q} \left[\left(p+1 - \frac{d}{2}\right) \left(q+1 - \frac{d}{2}\right) - \frac{d^2}{4} \right] \chi(\mathcal{M}). \end{aligned} \quad (20)$$

In the above formula $\text{ind}(\Omega^{m,0}, \bar{\partial})$ is the Dolbeault index twisted by the $(m,0)$ -form bundle, whose expression can be found in [11, 15]. $\chi(\mathcal{M})$ is the Euler characteristics of the manifold, and the top forms $Z_{p,d-1}$, $Z_{d-1,q}$, $Z_{d-1,d-1}$ are related to the Ray-Singer analytic torsion [16] via

$$Z_{p,d-1}^{\text{top}} = 2 \sum_{n=0}^p (-)^{n+1} (n+1) \ln T_{d-p+n}(\mathcal{M}). \quad (21)$$

Having presented the expansion for the effective actions up to order β^2 , it is possible to check the mismatch (20) only in $d = 2$ and $d = 3$. These cases have been considered in [11] and agree with (20), giving a nontrivial check.

4 Conclusion

In this short contribution, we presented the $U(N)$ spinning particles, introduced in [7] and subsequently studied in [8], where they were shown to describe a class of gauge invariant higher spin equations, close in form to Fronsdal-Labastida equations for real spacetimes and to Maxwell-like equations recently introduced in [13]. We briefly described the possibility of defining the model on a Kähler manifold, and finally focused on the $U(2)$ model on an arbitrary Kähler space. We presented field equations for differential (p,q) -forms as well as the heat kernel expansion of their effective actions and issues related to Hodge duality. Even if a genuine spacetime interpretation is somehow prevented by the complex target space, these models can provide interesting insights in the general problem of higher spin field theories, sharing crucial properties such as the appearance of constrained gauge invariance and compensator fields [17]. When quantized on Kähler backgrounds, they show quite a rich structure, such as the topological issues related to Hodge duality, or the coupling to the $U(1)$ part of the Kähler holonomy, and could be a useful instrument in Kähler geometry.

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$U(N)$ СПИНОВЫЕ ЧАСТИЦЫ И ПОЛЯ ВЫСШИХ СПИНОВ НА КЭЛЕРОВОМ ФОНЕ

Даётся краткий обзор квантования $U(N)$ спиновых частиц на комплексном фоне. В начале рассматривается плоское комплексное пространство и затем рассматривается модель на произвольном Кэлеровом многообразии. В последнем разделе рассматриваются (p, q) -формы на произвольных Кэлеровых пространствах и даётся их однопетлевое эффективное действие а также вопросы связанные с дуальностью Ходжа.

Ключевые слова: *спиновые частица, поля высших спинов, Кэлеровы многообразия, дуальность.*

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