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ABOUT ONE APPROACH TO CONDENSED SYSTEMS NONSTEADY BURNING RATE ESTIMATION

In the present paper the new algorithm of non-stationary burning rate estimation within the framework phenomenological theory is offered. Comparison of the received results with known calculation results have shown their satisfactory qualitative coincidence. Advantage of the offered approach consists in reduction of the experimental information involved for problem statement.

Key words: *nonsteady burning rate, solid propellant, deep regulation, phenomenological theory, temperature of a burning surface, transient processes.*

Creation of adequate methods of solid propellants nonsteady burning rate (NSBR) research is actual not only in respect of the further development of the non-stationary burning theory, but also at designing of solid propellants rocket motors and gas generators with deep regulation of the rocket thrust and mass flow rate through the nozzle.

Within the frames of the phenomenological theory developed in works of Zel'dovich, Vilyunov, Novozhilov [1–3], the one-dimensional nonsteady equation of energy for a condensed phase (*k*-phase) is possible representing in the form

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2} + u\frac{\partial T}{\partial x} + (1-c)Qz\exp\left(-\frac{E}{RT}\right), \qquad (1)$$

where t is time; $x \in [0, \infty)$ is normal coordinate to moving with speed flat surface; *a* is thermal diffusivity; *T*, *c* are temperature and decomposition fraction of *k*-phase substance; *R*, *z*, *E* are universal gas constant, the relation of a preexponential factor to material density, its activation energy; *Q* is the relation of a heat generation to a *k*-phase thermal capacity.

Two boundary conditions and value of linear speed of burning rate *u* are necessary for the solving of the equation (1). At $x = 0$ conditions are formulated depending on a reality of a solved problem, at $x \to \infty$ the condition of equality of temperature to initial value T_0 is exposed: $T = T_0$.

Concentration (decomposition fraction of *k*-phase substance) change *c* is described by the equation

$$\frac{\partial c}{\partial t} = u\frac{\partial c}{\partial x} + (1-c)z\ \exp\left(-\frac{E}{RT}\right). \qquad (2)$$

For the solving of the equation (2) one boundary condition which for $u > 0$ is formulated as $c|_{x\to\infty} = 0$ is required.

Models of NSBR estimation differ presence or absence of Arrhenius law and, accordingly, the equation (2) and also what attraction or parities for rate *u*, allowing correctly to formulate a problem.

At the formulation of model except traditional assumptions (homogeneity, uniformity, anisotropies, planes of a burning surface, etc. [1–4]) the dependences defined in full or in part experimentally are involved, for example:

– Dependence of linear burning rate on pressure *p* and initial temperature T_0:

$$u = u_0(T_0)\,p^{\nu}; \qquad (3)$$

– The set level of surface temperature $T_S = T|_{x=0}$ such as $T_S = const$ or its dependence on pressure $T_S = T_S$ (p);

– Burning rate of *k*-phase destruction chemical process in the form of Arrhenius law

$$u = z_* \cdot \exp\left(-\frac{E_*}{RT_s}\right), \qquad (4)$$

where z_*, E_* are the certain constants.

Resulted above dependences lean against the additional assumptions connected with hypotheses of a surface temperature constancy, balance of a heat generation in the condensed and gas phases and so on.

Generally it is supposed that burning process "is supervised" by a heat generation in a gas phase, and the *k*-phase is considered as partially or completely inert body, thermal inertia which nonsteady burning of substance at sharp pressure changes defines.

In history of NSBR researches it is possible to mention one of the very first model [5] (including the equation (1) without Arrhenius law, the equation (3) and hypothesis $T_S = const$); model [6] on the basis of the equation (1) without Arrhenius law and attraction of equation (3), (4).

In this paper the technique of an estimation of nonsteady burning rate, based on following assumptions is presented.

– The burning temperature does not depend on pressure. Pressure change at which there is a burning, cause change the heat flux *q* from gas in the condensed phase for distance between a burning surface and a heat generation zone in a gas phase change (instead for the burning temperature).

– At constant external (intrachamber) pressure *p* on a burning surface thermal balance is formed, namely: heat flux *q*(*p*) from gas to the condensed phase is equal to a heat flux from a burning surface at $x = 0$ into *k*-phase ($x > 0$) and is defined by the boundary condition:

$$-\lambda\left.\frac{\partial T}{\partial x}\right|_{x=+0} = q(p), \qquad (5)$$

where λ is the heat conductivity coefficient of *k*-phase.

– At change of intrachamber pressure the heat flux from gas into the condensed phase is defined by its value for corresponding “quasistationary” pressure level $p(t)$.

– The temperature of a burning surface is constant.

The mathematical model of NSBR calculation includes:

– The equation (1) with the boundary condition defined by the equation (5) and $T|_{x\to\infty} = T_0$.

– The equation (2) with the boundary condition $c|_{x\to\infty} = 0$.

Linear burning rate, representing speed of isothermal surface $T_S = const$ to *k*-phase moving, is defined from the heat balance equation on a burning surface at ($x = 0$):

$$\left.\frac{\partial T}{\partial t}\right|_{x=0} = a\left.\frac{\partial^2 T}{\partial x^2}\right|_{x=0} + u\left.\frac{\partial T}{\partial x}\right|_{x=0} + \left(1 - c|_{x=0}\right)Qz\exp\left(-\frac{E}{RT_s}\right) = 0. \qquad (6)$$

For the solving of formulated above problem dependence $q(p)$ is defined through calculated value $(\partial T/\partial x)|_{x=0}$ by the solution of a series of stationary analogues of the equations (1) and (2) under boundary conditions

$$T|_{x=0} = T_s; T|_{x\to\infty} = T_0; c|_{x\to\infty} = 0.$$

For burning rate law $u = u_0 \cdot p^v$ in a examine pressure range. We will notice that dependence $q(p)$ is obtained for the burning rate law for concrete initial temperature T_0 at $u_0 = const$ and $v = const$.

Change of intraballistic parameters in the combustion chamber in the volume V, supplied with a nozzle with the nozzle throat area S_*, is defined from the averaged mass and energy balance equations:

$$V\frac{d\varrho}{dt} = \varrho_p Su - S_*\Gamma(\gamma)\sqrt{p\varrho},$$

$$\frac{V}{\gamma-1}\frac{dp}{dt} = Q_p\varrho_p Su - \frac{\gamma}{\gamma-1}\frac{p}{\varrho}S_*\Gamma(\gamma)\sqrt{p\varrho},$$

$$\Gamma(\gamma) = \sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}},$$

where ϱ, ϱ_p are gas and a condensed material density, S is a burning surface, Q_p is the heat of combustion, γ is the specific heat ratio.

Results of NSBR calculations of a sudden pressure drop are presented on Fig. 1 in dimensionless variables – under the relation up the relaxation time of a heated *k*-phase layer. At carrying out of calculations the parameter χ (the relation of free volume relaxation time of the combustion chamber to relaxation time of a heated *k*-phase layer) was varied:

$$\chi = \frac{p_0 u_s}{RTa\varrho_p}\frac{V}{S},$$

where p_0, u_S are pressure and burning rate stationary values before pressure drop.

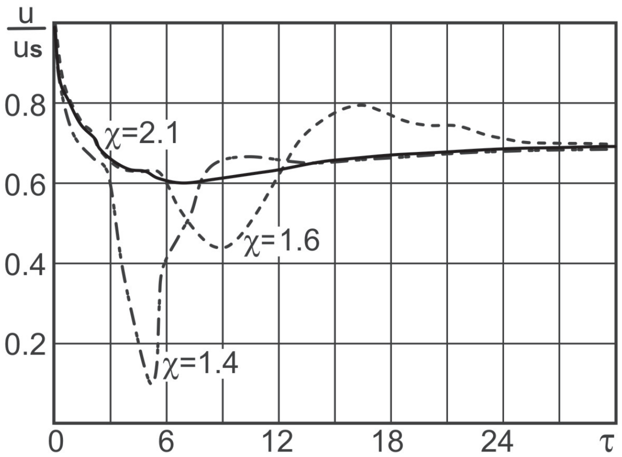

Fig. 1. Dependence of burning rate on time

Calculations were spent for the system which physical characteristics are given in [3, 6] for similar conditions. On Fig. 2 results of the similar calculations spent within the phenomenological theory frames with a variable burning surface temperature [6] are presented.

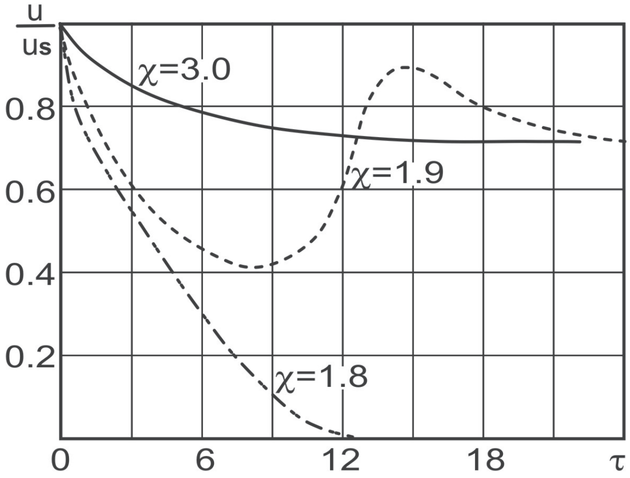

Fig. 2. Dependence of burning rate on time [6]

The dependences presented in Fig. 1, 2 are qualitatively agreemented. Oscillatory character of burning rate dependence on time is observed at certain values of parameter χ. Absence of propellant extinction at small values of parameter χ (fig. 1) is connected, apparently, with taking into account in offered model of an additional heat generation at the reactions expense in a *k*-phase.

Thus, in the present paper the new algorithm of nonsteady burning rate estimation within the frame-

work of phenomenological theory was offered. Comparison of the received results with calculations data [6] has shown their satisfactory qualitative coincidence. Advantage of the offered approach consists in reduction the experimental information involved for problem statement and solving.

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ОБ ОДНОМ ПОДХОДЕ К ОЦЕНКЕ НЕСТАЦИОНАРНОЙ СКОРОСТИ ГОРЕНИЯ КОНДЕНСИРОВАННЫХ СИСТЕМ

Предложен новый алгоритм оценки нестационарной скорости горения в рамках феноменологической теории. Сравнение полученных результатов с параметрами вычислений по ZN-модели показали их удовлетворительное качественное совпадение. Преимущество предложенного подхода заключается в сокращении объема привлекаемой для замыкания задачи экспериментальной информации.

Ключевые слова: *нестационарная скорость горения, твердое ракетное топливо, глубокое регулирование, феноменологическая теория, температура поверхности горения, нестационарные процессы.*

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