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New results in the fundamental theory of synchrotron radiation: the evolution of spectral maximum

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New results referred to the fundamental theory of synchrotron radiation (SR) are to be presented. We thoroughly analyze the relation between the amount of radiation emitted by a scalar particle at the transitions to the first excited state and to the ground state. Generalizing basic expressions we can follow the evolution of spectral maximum. It turns out there is a condition for radiation maximum to stay at highest harmonic.

Keywords: *synchrotron radiation; spectral maximum; quantum transitions*

1 Introduction

The motion of a scalar particle (boson) in a constant uniform magnetic field of intensity $\mathbf{H} = (0, 0, H)$ can be described by Klein-Gordon equation from which follows that the spectrum of particle's energy is discrete (see [1–3] for details)

$$E = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1-\beta^2}},\ \gamma^2 = \frac{1}{1-\beta^2} = 1 + (2n+1)b, \quad (1)$$

$$b = \frac{H}{H_0},\ H_0 = \frac{m_0^2 c^3}{|e_0|\hbar} = \frac{Q\hbar}{e_0^2 |e_0|},\ Q = \frac{e^2 m_0^2 c^3}{\hbar^2}.$$

Here γ is the relativistic factor; m_0 is the rest mass of the particle; c is the speed of light; $\beta = v/c$, where v is the speed of the particle in classical theory; $\hbar$ is the Plank constant and e_0 is the charge of the particle. H_0 may be interpreted as the Schwinger field.

Energy levels are numbered with $n = 0,\ 1,\ 2,\ 3,$ etc., $n = 0$ for the ground state. The concept of quantum spectrum of SR bases on the quantum transitions between energy levels, i.e. at the transition from initial level n to final level s the harmonic $\nu = n - s$ of quantum spectrum is radiated. Obviously, quantum spectrum contains a finite number of harmonics $\nu \leqslant n$ in contrast to classical one which, as we know, is infinite whatever the radiation parameters are.

It is well-known that for non-relativistic limit classical theory of SR predicts the maximum of radiation to be found at the first harmonic of classical spectrum and quantum theory does not contradict this prediction. Indeed, at low values of β ($\beta \approx 0$) only the first harmonic is radiated (in classical theory the radiation associated with other harmonics is vanishingly small), so there is no other possible position for radiation maximum. Increasing β we get to relativistic domain where the shift of radiation maximum to higher harmonics is expected. In [4] L. A. Artsimovich and I. Ya. Pomeranchuk demonstrated that according to classical theory of SR the number of harmonic ν_{max} associated with the maximum amount of emitted radiation is proportional to γ^3, $\nu_{max} \sim \gamma^3$. It means that the number ν_{max} can be increased infinitely with β. This result can not be repeated in quantum theory at least because $\nu \leqslant n$, as mentioned above. However, in quantum theory one can ask about the condition for $\nu_{max} = n$, i.e. for the radiation maximum to lie on the highest possible harmonic.

2 Basic definitions and notations

Let us consider the transitions from initial quantum state n to the first excited state $s = 1$ ($\nu = n - 1$) and to the ground state $s = 0$ ($\nu = n$). The amount of unpolarized radiation may be interpreted in terms of β and θ (the angle θ gives the direction of photon emission) [1]

$$W^b(n,\beta) = \frac{Q\beta^6 \sum\limits_{\nu=1}^{n} F^b(n,\nu,\beta)}{2(2n+1)^3(1-\beta^2)},$$

$$F^b(n,\nu,\beta) = \int\limits_0^\pi f^b(n,\nu,\beta,\theta)\sin\theta d\theta, \quad (2)$$

$$f^b(n,n-1,\beta,\theta) = \frac{(n-1+x)^3 x^{n-2}}{n!(n-1-x)} e^{-x} \left([n(n-1) - \right.$$
$$\left. -(2n+1)x + x^2]^2 + (n-x)^2(n-1+x)^2\cos^2\theta\right),$$
$$\nu = n-1,\ n \geqslant 2,$$

$$f^b(n,n,\beta,\theta) = \frac{(n+x)^3 x^{n-1}}{n!(n-x)} e^{-x} \left[(n-x)^2 + (n+x)^2 \cos^2\theta\right], \ \nu = n \geqslant 1.$$

3 The evolution of spectral maximum

Now we introduce the functions $K(n,\beta)$

$$K(n,\beta) = \frac{F^b(n,n,\beta)}{F^b(n,n-1,\beta)}.$$

The graphics of these functions are shown in Fig. 1.

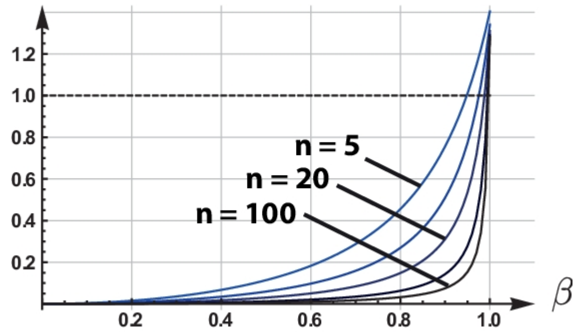

Figure 1: The functions $K(n,\beta)$ at $n = 5,\ 10,\ 20,\ 50,\ 100$.

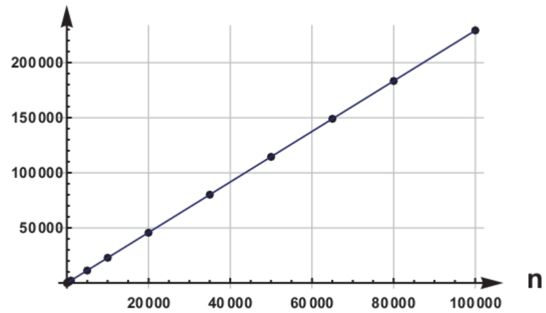

Figure 2: The $\gamma^2 = 2k_0 n$ line with the $(n,\ \gamma^2)$-roots of $K(n,\beta)$=1.

One can interpret $K(n,\beta)$ as a relation between the amount of radiation emitted at $\nu = n$ and $\nu = n-1$ transitions. It is easy to see that if $K(n,\beta) > 1$, then the maximum of radiation stays at highest harmonic, $\nu_{max} = n$ and if $K(n,\beta) < 1$, then the maximum does not shift to highest harmonic. Finally, the condition

$$K(n,\beta) = 1 \tag{3}$$

at fixed n defines such $\beta = \beta_n$ and, therefore, such $\gamma = \gamma_n = (1-\beta_n)^{-1/2}$ that the maximum in radiation spectrum shifts from harmonic $\nu = n-1$ to the highest harmonic $\nu = n$. From (1) at high initial energies n one can get an approximation

$$\gamma^2 \approx 2bn,\ n \gg 1. \tag{4}$$

So, as the solutions of (3) can be rewritten in the form (n,γ_n^2), according to (4) they must lie on the 'line' $\gamma^2 = 2k_0 n$ (Fig. 2).

The coefficient k_0 was calculated numerically, $k_0 \approx 1,146128792697$. It is important to emphasize that k_0 does not depend on n. On the other hand, $\gamma_n^2 = 2k_0 n = 2b_0 n$, i.e. $k_0 = b_0$, where b_0 is associated with a certain value of external magnetic field. Thus, whatever the initial energy level, the radiation maximum shifts to the highest harmonic only if the external field is more or equal to a certain critical value, $b \geqslant b_0$. This can be formulated differently - no matter how much we increase the energy of a particle or change other conditions, the shift of maximum to $\nu_{max} = n$ can only occur when $b \geqslant b_0$

$$\text{if } b < b_0,\ \nu_{max} \leqslant n-1,$$

$$\text{if } b \geqslant b_0,\ \nu_{max} \leqslant n.$$

Thus, we assume that there exists an ordered set of numbers $b_0 > b_1 > b_2 > ... > b_s$ such that if $b < b_s$ then the spectral maximum lies on $\nu_{max} \leqslant n - s$ and it shifts to higher harmonics with n. If $b_m < b < b_{m-1}$ $(1 \leq m \leq s)$ then for any initial state $n > m$ the spectral maximum stays at $\nu_{max} = n - s$. Finally, if $b > b_0$ then at any n we have $\nu_{max} = n$.

4 Conclusion

In the framework of quantum theory we consider the evolution of SR spectral maximum. We found the condition for the maximum to lie at the highest harmonic of the spectrum, and, therefore, increasing the energy, any other shift of the maximum for higher harmonics does not occur. The condition of its shift to the highest harmonic is that the external magnetic field intensity is higher than certain critical value. If the intensity of external field is less than this critical value, whatever the other parameters are, the spectral maximum does not move to the highest harmonic. Thus, we found the restrictions for the rule concerning the evolution of spectral maximum in classical theory.

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НОВЫЕ РЕЗУЛЬТАТЫ В ФУНДАМЕНТАЛЬНОЙ ТЕОРИИ СИНХРОТРОННОГО ИЗЛУЧЕНИЯ: ЭВОЛЮЦИЯ СПЕКТРАЛЬНОГО МАКСИМУМА

Представлены новые результаты, относящиеся к фундаментальной теории синхротронного излучения (СИ). Исследовано отношение количества испускаемого излучения при переходах в основное состояние и первое возбужденное. Обобщение основных выражений позволило изучить вопрос об эволюции спектрального максимума. Обнаружено условие смещения максимума на последнюю гармонику.

Ключевые слова: *синхротронное излучение, максимум спектра излучения, квантовые переходы*

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