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ANGULAR MOMENTUM OF THE RADIATION FROM RELATIVISTIC POSITRONS AT PLANAR CHANNELING IN SI CRYSTAL

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The calculation of the both spin and orbital angular momenta of the radiation from 350 MeV positrons at (100) planar channeling in the ultra-thin silicon crystal is carried out by means of computer simulation. The calculations are performed as a proposal for an experimental verification at the DAFNE LINAC (INFN-LNF, Italy; energy up to 500 MeV). The possibility of the usage of the channeling radiation from relativistic positrons as the source of 4-5 MeV twisted photons is discussed.

Keywords: *channeling radiation, angular momentum of electromagnetic field, twisted photon.*

1 Introduction

The works on the angular momentum of the electromagnetic field (AMEF) can be divided into three branches: the theoretical description and experimental confirmation [1–7], the practical applications [8–11] and the search of the new sources of the so-called twisted photons (i.e. photons carrying definite angular momentum) [7, 9, 12–14].

The earliest work we have found, which deals with the idea that the light (i.e. photons) has its own angular momentum belongs to Sadowsky [1]. The effect described in his works predict, that the body absorbed elliptically polarized light must be rotated, due to the transfer of angular momentum from the photons to the body. These works being published in Russian have remained unnoticed more later publications. These authors give more attention to the work of Poynting [2], who even did not mention Sadowsky in his works. For a long time, the idea that photons can possess their own orbital angular momentum (OAM) remained just a hypothesis, until found confirmation in experimental studies of Beth and Holborn [3, 4].

The Lorentz-covariant relativistic theory of AMEF was developed by Ivanenko and Sokolov [5]. In this approach, the tensor of AMEF density is determined based on the principle of least action for the electromagnetic field. However, the proposed by Ivanenko and Sokolov decomposition of the total angular momentum into the orbital (OAM) and spin (SAM) components was not relativistically invariant and the resulting tensor of the spin angular momentum was not gauge invariant. Alternatively Teitelboim [6] proposed another approach: to determine the AMEF he used a gauge invariant symmetric electromagnetic stress-energy tensor. This approach allows to get rid of significant limitations of the Ivanenko-Sokolov approach. Remarkable fact was observed in [7]: in the wave zone these two approaches [5, 6] lead to the same results. Also the expressions for the OAM and SAM of the radiation from the relativistic particle moving in given trajectory was obtained in [7].

The authors of the work [9] have shown experimentally, that it is possible to use two beams of incoherent radio waves at almost the same frequency but encoded by different orbital angular momentum states for transmission of two independent radio channels simultaneously. This technique is able to increase the information capacity of data transmission in several times in future. The authors of Ref. [10] have demonstrated that four encoded light beams signals with different values of orbital angular momentum can be multiplexed and de-multiplexed, allowing an increasing of data transmission in 4 times. In another work [11] the same technique was applied for data transmitting through the suitable optical fiber.

To produce the twisted photons two different techniques are used: to transfer the OAM to the photon having no initial OAM or to create conditions for the emission of a photon carrying necessary OAM. A simple method for producing twisted photons in the radio range is proposed in [9] – the transmitter antenna was cut radially and *physically* twisted, so that emitted radio signal initially possess OAM. The effective way for production twisted photons in near optical range is described in [12], the laser beam (800 nm, 1.55 eV) passed through a specially prepared thin foil (shaped like a spiral stairway). By adjustment of the spirial stairway steps number, it is possible to control the amount of OAM transferred to the photons. Another method for the production of the twisted photons in a near optical range, which uses the hologram of special form was suggested in Ref. [8]. Regarding X-Ray region, we can mention the theoretical paper [13], the authors of which suggested to use the helical undulator as a source of X-ray twisted photons. In the experiment [14] the dual undulator scheme was used, which allowed to observe twisted photons with energies of 99 eV. The angular momentum of the synchrotron radiation (at the maximum photon energy 75 keV) from the 10^{11} bunch of 10 GeV electrons was calculated to be equal to $9{\cdot}10^{23}h$ [7].

In this work we consider both SAM and OAM of (100) channeling radiation (CR) from 350 MeV positrons in thin Si crystal. The maxima of the spectrum is near 4-5 MeV [15]. For the SAM and OAM calculations, we use the Ivanenko-Sokolov method [5]. The trajectories of positrons at planar channeling in Si crystal are calculated using Wolfram Mathematica code BCM-1 [16]. Commonly in other works the OAM of the particles (electrons or positrons) instead of OAM of photons is considered, for the axial channeling see [17]. We made this calculations as a proposal to experimental groups at INFN-LNF for collaboration in this subject. We assume that CR from relativistic positrons can be suitable as the source of high-energy (more 1 MeV) twisted photons.

2 The equations of the motion and the trajectories of the positrons at planar channeling in crystals

The trajectories of relativistic (with the energy range from hundreds of MeV to dozens of GeV) positrons (as well as electrons) at channeling in a crystal can be determined by solution of the classical relativistic equation of motion:

$$\frac{d\boldsymbol{p}}{dt} = \frac{d}{dt}\left(\frac{m\boldsymbol{v}}{\sqrt{1-(\boldsymbol{v}^2/c^2)}}\right) = -\,\mathrm{grad}\,U\left(x,y,z\right), \qquad (1)$$

here $\boldsymbol{p}$, m, $\boldsymbol{v}$ are the positron momentum, rest mass, and velocity respectively, c is the speed of light, and $\boldsymbol{F}$ is the force applied to the positron.

Fig. 1 shows the scheme of orientation of the initial velocity of positrons in the system of atomic planes. In the case of planar channeling equations (1) reads:

$$\boldsymbol{\gamma} m\ddot{\boldsymbol{x}} = -\frac{\partial U\left(x\right)}{\partial x}. \qquad (2)$$

The x coordinate of the initial point of incidence of the positron into a planar channel of the crystal and the projection of its initial velocity onto OX axis are the initial conditions to the equation (2): $x(0) = x_0$, $v_x(0) = c\sqrt{1-\gamma^{-2}}sin(\theta)$.

the trajectories (2) of 350 MeV positrons at (100) channeling in a thin Si crystal with different incidence angles are calculated numerically using the Wolfram Mathematica code BCM-1 [16].

3 OAM and SAM of the radiation from the positrons at planar channeling in crystals

The OAM (3) and SAM (4) of the electromagnetic field being emitted (wave zone) from moving relativistic particle are determined by equations derived in [7]:

$$\frac{d\tilde{\Lambda}^{\mu\nu}}{d\tau} = \frac{2}{3}\frac{e^2}{c^5}\omega_\rho\omega^\rho\left(r^\mu v^\nu - r^\nu v^\mu\right), \qquad (3)$$

$$\frac{d\tilde{\Pi}^{\mu\nu}}{d\tau} = \frac{2}{3}\frac{e^2}{c^3}\left(v^\mu\omega^\nu - \omega^\nu v^\mu\right). \qquad (4)$$

In the case of planar channeling (Fig. 1) the expressions for OAM and SAM can be considerably simplified. As a result, only OY component of OAM and SAM in the case of planar channeling are nonzero:

$$\tilde{\Lambda}^{31} = \int\left(\frac{2}{3}\frac{e^2}{c^5}\gamma^4 {a_x}^2\left(xv_z - zv_x\right)\right)dt, \qquad (5)$$

$$\tilde{\Pi}^{31} = \int\left(\frac{2}{3}\frac{e^2}{c^3}\gamma^2 v_z a_x\right)dt, \qquad (6)$$

here Λ^{31} is the OY component of the OAM of the positron, Π^{31} is the OY component of the SAM of the positron, e is positron charge, γ is relativistic factor, x and z is transverse and longitudinal coordinates respectively, v and a is the positron velocity and acceleration.

For the determination of the total yields of radiation with OAM and SAM one needs to integrate equations (5) and (6). According to Fig. 2 and (5), (6) in the planar case SAM and OAM of CR are possible to observe only in crystals of the half-wave thickness, since the contributions to the total SAM or OAM from subsequent oscillations of positrons neglect each other. To obtain the angle-of-incidence and crystal thickness dependencies in the case of planar channeling, we calculated 200 positron trajectories choosing the angle of incidence up to $2\theta_c$ (21 angles in total) and averaged over trajectories the values of OAM (5) and SAM (6) obtained for each individual trajectory using the method [19].

4 Conclusion

The angle-of-incidence and crystal thickness dependences of OAM and SAM of radiation from 350 MeV positrons at (100) channeling in thin Si crystal is studied by means of computer simulation.

We performed representative calculations for planar channeled positrons, but the beams of relativistic positrons are available only at several high-energy accelerators. Therefore, one needs to consider relativistic electrons channeling, too. In the case of electrons, since the radiation from axially channeled electrons is more intensive than one from planar channeled electrons, one can expect larger values of angular momentum in axial channeling radiation. Also it is possible to use axial channeling instead of planar for the observation of x component of OAM in CR instead of y component. These experiments are possible at linear accelerator at SAGA-LS (Japan) for the axial channeling of 255 MeV electrons [20].

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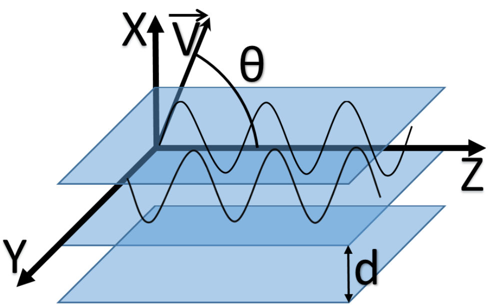

Figure 1. The scheme of orientation of the initial velocity of positrons in the system of crystallographic planes

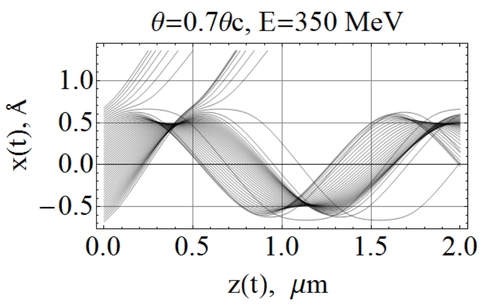

Figure 2. The 50 calculated trajectories of 350 MeV positrons at (100) channeling in Si crystal. The angle of incidence θ is 0.7 of the Lindhard critical angle (264 μrad)

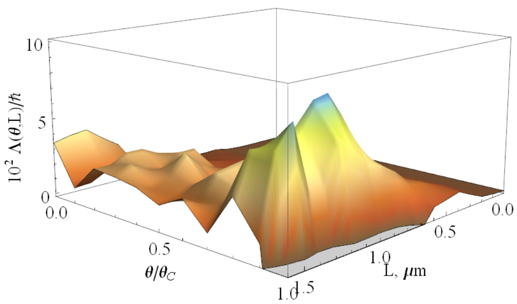

Figure 3. The OAM (5) of the radiation from 350 MeV positrons at (100) channeling Si as the function of angle of incidence and the crystal thickness. θ_c is the critical Lindhard angle

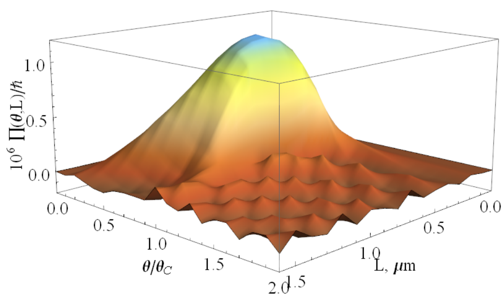

Figure 4. The SAM (6) of the radiation from 350 MeV positrons at (100) channeling Si as the function of angle of incidence and the crystal thickness. θ_c is the critical Lindhard angle

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УГЛОВОЙ МОМЕНТ ИЗЛУЧЕНИЯ РЕЛЯТИВИСТСКИХ ПОЗИТРОНОВ ПРИ ПЛОСКОСТНОМ КАНАЛИРОВАНИИ В КРИСТАЛЛЕ КРЕМНИЯ

Методом компьютерного моделирования рассчитаны спиновый и орбитальный угловые моменты излучения 350 МэВ позитронов при (100) плоскостном каналировании в ультратонком кристалле кремния. Расчеты проведены в качестве предложения для экспериментальной проверки на линейном ускорителе DAFNE (INFN-LNF, Италия; энергия позитронов до 500 МэВ). Рассмотрена возможность использования излучения релятивистских позитронов при каналировании в качестве источника закрученных фотонов с энергией 4-5 МэВ.

Ключевые слова: *излучение при каналировании, угловой момент электромагнитного поля, закрученные фотоны.*

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