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SUPERFIELD MODELS ON S^2 AND S^3

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We develop superfield models for constructing classical actions of various models with rigid supersymmetry on S^2 and S^3. We introduce superspaces based on supercoset manifolds $SU(2|1)/U(1)$ and $SU(2|1)/[U(1)\times U(1)]$. We show that models on S^3 with extended supersymmetry can have different versions which are invariant under different supersymmetry groups. Among the models with extended supersymmetry on S^3 we consider the $\mathcal{N}=4$ and $\mathcal{N}=8$ SYM theories, Gaiotto-Witten and ABJM models as well as their analogs on S^2.

Keywords: *superspace, supercoset, super Yang-Mills model.*

1 Introduction and summary

Recently, there have been a surge of interest to supersymmetric field theories with rigid supersymmetry on curved manifolds with topology of sphere. In [1] it was demonstrated that such models are in the core of the so-called supersymmetric localization method which allows one to compute various quantum observables in these models exactly, beyond the perturbation theory. Although the supersymmetry plays the crucial role in this method, all applications were given using the component field formulations of field models in which the supersymmetry is not manifest. It is natural to expect that the localization method can be properly extended and applied to *superfield* models which have explicit supersymmetry by construction. As a first step towards this goal we develop appropriate superfield methods for constructing classical Lagrangians of various models with rigid supersymmetry on S^2 and S^3.

Our consideration is based on the supercoset spaces $SU(2|1)/U(1)$ and $SU(2|1)/[U(1)\times U(1)]$ which contain the spheres S^3 and S^2 as their bosonic bodies. We introduce gauge and matter superfields in these superspaces and construct classical actions using standard methods of quantum field theory in a curved superspace with four supercharges [2]. In 3d case we denote this supersymmetry as $\mathcal{N}=2$ while for two-dimensional models it is $\mathcal{N}=(2,2)$. Gauge and matter field theories with this supersymmetry on S^3 were constructed for the first time in [3,4] and on S^2 in [5,6] using standard component field methods. The authors of these papers found many new exact results for these models on the quantum level.

One of the main results of our consideration is the application of superfield methods for constructing classical actions of various models with *extended* supersymmetry both on S^3 and S^2, such as the $\mathcal{N}=4$ and $\mathcal{N}=8$ SYM theories, Gaiotto-Witten and ABJM. For three-dimensional models with extended supersymmetry we show that there exist different supergroups with the same number of fermionic generators and containing S^3 in the bosonic sector. As a consequence, there are different versions of models with extended supersymmetry on S^3 which reduce to the same model in flat space limit. In the two-dimensional case there is no such ambiguity. Superfield classical actions of these theories are given explicitly together with the hidden supersymmetry transformations.

The present contribution is based essentially on our papers [7, 8].

2 Superspaces as coset realizations

The analogs of two- and three-dimensional Euclidian Poinaré groups are $SU(2|1)$ and $SU(2|1)\times SU(2)$ for models on S^2 and S^3, respectively. Therefore, it is natural to introduce superspaces as the following coset spaces

$$2d:\frac{SU(2|1)}{U(1)\times U(1)}\,,\quad 3d:\frac{SU(2|1)}{U(1)}\,. \tag{1}$$

The superalgebra $su(2|1)$ is spanned by four bosonic generators M_a, $(a=1,2,3)$ and R and four Grassmann-odd generators Q_α, $\bar Q_\alpha$, $\alpha=1,2$. There are the following non-trivial commutation relations among these generators:

$$[M_a,M_b]=\frac{2i}{r}\varepsilon_{abc}M_c\,,$$
$$[M_a,Q_\alpha]=-\frac{1}{r}(\gamma_a)_\alpha^\beta Q_\beta\,,$$
$$[M_a,\bar Q_\alpha]=-\frac{1}{r}(\gamma_a)_\alpha^\beta \bar Q_\beta\,,$$
$$\{Q_\alpha,\bar Q_\beta\}=\gamma^a_{\alpha\beta}M_a+\frac{1}{r}\varepsilon_{\alpha\beta}R\,,$$

$$[R, Q_\alpha] = -Q_\alpha\,, \quad [R, \bar{Q}_\alpha] = \bar{Q}_\alpha\,. \tag{2}$$

Let us introduce superspace coordinates $Z^M = (x^m, \theta^\mu, \bar{\theta}^\mu)$, where m takes values from one to d with $d = 2$ for S^2 and $d = 3$ for S^3 case. The geometry of the supercosets (1) is entirely encoded in the supervielbein one-forms

$$E^A = dz^M E_M{}^A(z)\,, \qquad E^A = (E^a, E^\alpha, \bar{E}^\alpha)\,. \tag{3}$$

Using the commutation relations (2), the expressions of the supervielbens can be found explicitly in a given coordinate system. Then, it is straightforvard to find the inverse supervielbein $E_A{}^M$ and construct the covariant derivatives $\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha, \bar{\mathcal{D}}_\alpha)$. The algebra (2) defines (anti)commutation relations among these derivatives. In 3d case they are

$$\begin{aligned}
&[\mathcal{D}_a, \mathcal{D}_b] = -\frac{i}{2r}\varepsilon_{abc}M_c\,,\\
&[\mathcal{D}_a, \mathcal{D}_\alpha] = -\frac{i}{2r}(\gamma_a)_\alpha^\beta \mathcal{D}_\beta\,,\\
&[\mathcal{D}_a, \bar{\mathcal{D}}_\alpha] = -\frac{i}{2r}(\gamma_a)_\alpha^\beta \bar{\mathcal{D}}_\beta\,,\\
&\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_\beta\} = i\gamma^a_{\alpha\beta}\mathcal{D}_a - \frac{1}{2}\gamma^a_{\alpha\beta}M_a + \frac{1}{r}\varepsilon_{\alpha\beta}R\,,\\
&\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = \{\bar{\mathcal{D}}_\alpha, \bar{\mathcal{D}}_\beta\} = 0\,,
\end{aligned} \tag{4}$$

while for the case of S^2 we find

$$\begin{aligned}
&[\mathcal{D}_a, \mathcal{D}_b] = \frac{i}{r^2}\epsilon_{ab}M\,,\\
&[\mathcal{D}_a, \mathcal{D}_\alpha] = -\frac{i}{2r}(\gamma_a)_\alpha^\beta \mathcal{D}_\beta\,,\\
&[\mathcal{D}_a, \bar{\mathcal{D}}_\alpha] = -\frac{i}{2r}(\gamma_a)_\alpha^\beta \bar{\mathcal{D}}_\beta\,,\\
&\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_\beta\} = i\gamma^a_{\alpha\beta}\mathcal{D}_a + \frac{1}{r}\gamma^3_{\alpha\beta}M + \frac{1}{2r}\varepsilon_{\alpha\beta}R\,,\\
&\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = \{\bar{\mathcal{D}}_\alpha, \bar{\mathcal{D}}_\beta\} = 0\,.
\end{aligned} \tag{5}$$

Here $M \equiv M_3$.

Any action in full superspace has the following form

$$S = \int d^dx d^2\theta d^2\bar{\theta}\, E\,\mathcal{L}\,, \tag{6}$$

where $\mathcal{L}$ is a superfield Lagrangian and E is the Berezinian of the supervielbein,

$$E = \mathrm{Ber} E_M{}^A\,. \tag{7}$$

The latter has the following important property

$$\int d^dx d^2\theta d^2\bar{\theta}\, E = 0\,. \tag{8}$$

As a consequence, the supercosets (1) have vanishing volume.

3 Superfield actions with minimal supersymmetry

3.1 Gauge superfield

Gauge theory in superspace is described by gauge superfield connections V_A which covariantize the superspace derivatives, $\nabla_A = \mathcal{D}_A + V_A$. The gauge connections obey superspace constraints which correspond to the following deformations of (anti)commutation relations (4) and (5), 3d:

$$\begin{aligned}
&\{\nabla_\alpha, \nabla_\beta\} = \{\bar{\nabla}_\alpha, \bar{\nabla}_\beta\} = 0\,,\\
&\{\nabla_\alpha, \bar{\nabla}_\beta\} = i\gamma^a_{\alpha\beta}\nabla_a - \frac{1}{2}\gamma^a_{\alpha\beta}M_a + \frac{1}{r}\varepsilon_{\alpha\beta}R + i\varepsilon_{\alpha\beta}G\,,\\
&[\nabla_a, \nabla_b] = -\frac{i}{2r}M_{ab} + i\mathbf{F}_{ab}\,,\\
&[\nabla_a, \nabla_\alpha] = -\frac{i}{2r}(\gamma_a)_\alpha^\beta\nabla_\beta - (\gamma_a)_\alpha^\beta\bar{W}_\beta\,,\\
&[\nabla_a, \bar{\nabla}_\alpha] = -\frac{i}{2r}(\gamma_a)_\alpha^\beta\bar{\nabla}_\beta + (\gamma_a)_\alpha^\beta W_\beta\,,
\end{aligned} \tag{9}$$

2d:

$$\begin{aligned}
&\{\nabla_\alpha, \nabla_\beta\} = \{\bar{\nabla}_\alpha, \bar{\nabla}_\beta\} = 0\,,\\
&\{\nabla_\alpha, \bar{\nabla}_\beta\} = i\gamma^a_{\alpha\beta}\nabla_a + \frac{1}{r}\gamma^3_{\alpha\beta}M + \frac{1}{2r}\varepsilon_{\alpha\beta}R\\
&\qquad + i\varepsilon_{\alpha\beta}G + \gamma^3_{\alpha\beta}H\,,\\
&[\nabla_a, \nabla_b] = \frac{i}{r^2}\varepsilon_{ab}M + i\mathbf{F}_{ab}\,,\\
&[\nabla_a, \nabla_\alpha] = -\frac{i}{2r}(\gamma_a)_\alpha^\beta\nabla_\beta - (\gamma_a)_\alpha^\beta\bar{W}_\beta\,,\\
&[\nabla_a, \bar{\nabla}_\alpha] = -\frac{i}{2r}(\gamma_a)_\alpha^\beta\bar{\nabla}_\beta + (\gamma_a)_\alpha^\beta W_\beta\,.
\end{aligned} \tag{10}$$

Here W_α ($\bar{W}_\alpha$) are (anti)chiral superfield strengths while G and H are linear,

$$\begin{aligned}
\bar{\nabla}_\alpha W_\beta &= 0\,, \quad \nabla^2 G = \nabla^2 H = 0\,,\\
\bar{\nabla}^2 G &= \bar{\nabla}^2 H = 0\,.
\end{aligned} \tag{11}$$

These superfield strengths can be expressed in terms of unconstrained gauge potential V as

$$\begin{aligned}
G &= \frac{i}{2}\bar{\mathcal{D}}^\alpha(e^{-V}\mathcal{D}_\alpha e^V)\,, \quad W_\alpha = -\frac{i}{4}\bar{\mathcal{D}}^2(e^{-V}\mathcal{D}_\alpha e^V)\,,\\
H &= -\frac{1}{2}\gamma_3^{\alpha\beta}\bar{\mathcal{D}}_\alpha(e^{-V}\mathcal{D}_\beta e^V)\,.
\end{aligned} \tag{12}$$

Gauge transformation for V has standard form

$$e^V \longrightarrow e^{i\bar{\Lambda}}e^V e^{-i\Lambda}\,, \tag{13}$$

where Λ and $\bar{\Lambda}$ are covariantly (anti)chiral local gauge parameters.

Superfield Yang-Mills action has the same form both on S^2 and S^3,

$$S_{\mathrm{SYM}} = -\frac{4}{g^2}\mathrm{tr}\int d^dx d^2\theta d^2\bar{\theta}\, E\, G^2\,, \tag{14}$$

where g^2 is the gauge coupling constant of mass dimension $[g] = 1/2$ in 3d and $[g] = 1$ in 2d.

In 3d, one can consider also the Chern-Simons action

$$S_{\rm CS} = -\frac{ik}{\pi}{\rm tr}\int_0^1 dt\int d^dx d^2\theta d^2\bar\theta\, E \times \bar{\mathcal{D}}^\alpha(e^{-tV}\mathcal{D}_\alpha e^{tV})e^{-tV}\partial_t e^{tV}\,. \quad (15)$$

For integer k this action is invariant under large gauge transformations and is topological. In 2d this action is not topological any more, but represents a BF-type interaction of component fields.

An interesting feature of 2d gauge theories is the possibility to construct the following gauge invariant action for the gauge superfield

$$S_{\rm BF} = \kappa\,{\rm tr}\int_0^1 dt\int d^2x d^2\theta d^2\bar\theta\, E \times(\gamma^3)^{\alpha\beta}\bar{\mathcal{D}}_\alpha(e^{-tV}\mathcal{D}_\beta e^{tV})e^{-tV}\partial_t e^{tV}\,, \quad (16)$$

where κ is a coupling constant of mass dimension -1. One can check that in components this action contains only BF-type interaction of fields and has no dynamical degrees of freedom.

3.2 Chiral superfield

By definition, the chiral superfield Φ and its antichiral counterpart $\bar\Phi$ obey

$$\bar{\mathcal{D}}_\alpha\Phi = 0\,,\quad \mathcal{D}_\alpha\bar\Phi = 0\,. \quad (17)$$

In general, such superfields can be charged under R generator,

$$R\Phi = -q\Phi\,,\qquad R\bar\Phi = q\bar\Phi\,. \quad (18)$$

Classical action for the chiral superfield minimally interacting with gauge superfield V in some representation reads

$$S = 4\int d^dx d^2\theta d^2\bar\theta\, E\,\bar\Phi e^V\Phi \quad (19)$$
$$+2\int d^dx d^2\theta\,\mathcal{E}\,W(\Phi) + 2\int d^dx d^2\bar\theta\,\bar{\mathcal{E}}\,\bar W(\bar\Phi)\,,$$

where $W(\Phi)$ is a superpotential. Note that for models on S^2 or S^3 the R-charge of the superpotential is fixed to be -2 to balance the R-charge of the chiral measure.

In conclusion of this section we point out that superfield classical actions on S^2 and S^3 for gauge and matter superfields have a simple form which is very similar to usual flat space actions. They respect the full symmetry under the $SU(2|1)$ group by construction.

4 Models with extended supersymmetry

Superfield models constructed in the previous section are invariant under minimal supersymmetry generated by four Killing spinors ϵ_α and $\bar\epsilon_\alpha$ which obey

$$\mathcal{D}_a\epsilon^\alpha = \frac{i}{2r}(\gamma^a)^\alpha_\beta\epsilon^\beta\,. \quad (20)$$

Models with extended supersymmetry should involve extra Killing spinors, say η^α and $\bar\eta^\alpha$. However, there is a sign ambiguity in the Killing spinor equation for these spinors,

$$\mathcal{D}_a\eta^\alpha = \pm\frac{i}{2r}(\gamma^a)^\alpha_\beta\eta^\beta\,. \quad (21)$$

It is important to note that 3d Killing spinors with opposite signs in (21) are independent while in 2d they are related to each other by the γ_3 matrix. Thus, supersymmetric field theories on S^3 with extended supersymmetry can have different versions which differ in the number of positive and negative Killing spinors. In this section we consider various such models, starting with the case of the $\mathcal{N}=4$ SYM model.

4.1 $\mathcal{N}=4$ SYM with $SU(2)\times SU(2)$ R-symmetry

The $\mathcal{N}=4$ gauge multiplet in 3d (or $\mathcal{N}=(4,4)$ in 2d) consists of the $\mathcal{N}=2$ gauge superfield V and a chiral superfield Φ. The chiral superfield, in principle, can have arbitrary R-charge. Extra supersymmetry should transform these $\mathcal{N}=2$ superfields into each other.

In this subsection we consider extra supersymmetries generated by Killing spinors η_α which obey the Killing spinor equation with the same sign as (20). Such a Killing spinor appears as a component of chiral superfield Υ,

$$\Upsilon = a + \theta^\alpha\eta_\alpha + \theta^2 b\,, \quad (22)$$

where a and b are some constants corresponding to the parameters of internal symmetry group. The form of transformations of the superfields V and Φ is fixed uniquely from the requirement of closure of such transformations to a supergroup:

$$\Delta_\Upsilon V = i(\Upsilon\bar\Phi - \bar\Upsilon\Phi)\,,$$
$$\delta_\Upsilon\Phi = \bar\nabla^\alpha G\mathcal{D}_\alpha\Upsilon + \frac{q}{r}G\Upsilon\,. \quad (23)$$

Together with the manifest supersymmetry group, these transformations form the group $SU(2|2)\times SU(2)$. The classical SYM action with this supersymmetry reads

$$S^{\mathcal{N}=4}_{\rm SYM} = -\frac{4}{g^2}{\rm tr}\int d^dx d^2\theta d^2\bar\theta\, E\big[G^2 - e^{-V}\bar\Phi e^V\Phi + \frac{q}{2r}\int_0^1 dt\bar{\mathcal{D}}^\alpha(e^{-tV}\mathcal{D}_\alpha e^{tV})e^{-tV}\partial_t e^{tV}\big]\,. \quad (24)$$

A novel feature of this action is the appearance of the last perm which has the form of the Chern-Simons action. This term is present for $q\neq 0$ and drops out in the flat limit $r\to\infty$.

We point out that in the 2d case the classical action (24) describes the SYM model with $\mathcal{N}=(4,4)$ supersymmetry.

4.2 $\mathcal{N}=4$ SYM with $U(1)\times U(1)$ R-symmetry

Now let us consider Killing spinors η_α which obey (21) with minus sign. Using such a Killing spinor we construct the following transformations of $\mathcal{N}=2$ superfields V and Φ

$$\begin{aligned}\Delta_\eta V &= (\theta^\alpha - \frac{1}{r}\theta^2\bar\theta^\alpha)\eta_\alpha\overline{\Phi} - \bar\theta^\alpha\bar\eta_\alpha\Phi\,,\\ \delta_\eta\Phi &= -i\eta^\alpha\bar\nabla_\alpha G\,, \qquad \delta_\eta\overline{\Phi} = i\bar\eta^\alpha\nabla_\alpha G\,. \end{aligned} \tag{25}$$

One can check that these transformations (together with the manifest supersymmetry) form a supergroup $SU(2|1)\times SU(2|1)$ only if the R-charge of the chiral superfield is fixed as

$$R\Phi = -\Phi\,, \qquad R\overline{\Phi} = \overline{\Phi}\,. \tag{26}$$

The classical $\mathcal{N}=4$ SYM action with this supersymmetry has simple form

$$S = -\frac{4}{g^2}\mathrm{tr}\int d^3x d^2\theta d^2\bar\theta\, E(G^2 - e^{-V}\bar\Phi e^V\Phi)\,. \tag{27}$$

This action has $U(1)\times U(1)$ R-symmetry and does not possess an S^2 analog.

4.3 $\mathcal{N}=8$ SYM

Here we consider only the $\mathcal{N}=8$ SYM model with Killing spinors obeying (20) with the same sign. In this case the $\mathcal{N}=8$ multiplet consists of the $\mathcal{N}=2$ gauge superfield V and an $SU(3)$ triplet of chiral superfields Φ_i, $i=1,2,3$. These superfields transform among each other by means of the extra $\mathcal{N}=6$ supersymmetry as

$$\begin{aligned}\delta_\Upsilon V &= i\Upsilon_i\overline{\Phi}^i - i\bar\Upsilon^i\Phi_i\,, \qquad\qquad (28)\\ \delta_\Upsilon\Phi_i &= \bar\nabla^\alpha G\mathcal{D}_\alpha\Upsilon_i + \frac{2}{3r}G\Upsilon_i + \frac{1}{2}\varepsilon_{ijk}\bar\nabla^2(\bar\Upsilon^j\overline{\Phi}^k)\,,\\ \delta_\Upsilon\overline{\Phi}^i &= -\nabla^\alpha G\bar{\mathcal{D}}_\alpha\bar\Upsilon^i - \frac{2}{3r}G\bar\Upsilon^i - \frac{1}{2}\varepsilon^{ijk}\nabla^2(\Upsilon_j\Phi_k)\,,\end{aligned}$$

where Υ_i is a triplet of superfield parameters of the form (22). The invariant $\mathcal{N}=8$ SYM action appears to be

$$\begin{aligned}S^{\mathcal{N}=8}_{\rm SYM} &= S_{\rm YM} + S_{\rm CS} + S_{\rm pot}\,, \qquad\qquad (29)\\ S_{\rm YM} &= -\frac{4}{g^2}\mathrm{tr}\int d^dx d^2\theta d^2\bar\theta\, E(G^2 - e^{-V}\bar\Phi^i e^V\Phi_i)\,,\\ S_{\rm CS} &= -\frac{4}{3rg^2}\mathrm{tr}\int_0^1 dt\int d^dx d^2\theta d^2\bar\theta\, E\\ &\quad\times\bar{\mathcal{D}}^\alpha(e^{-tV}\mathcal{D}_\alpha e^{tV})e^{-tV}\partial_t e^{tV}\,,\\ S_{\rm pot} &= -\frac{i}{3g^2}\mathrm{tr}\int d^dx d^2\theta\,\mathcal{E}\,\varepsilon^{ijk}\Phi_i[\Phi_j,\Phi_k] + c.c.\end{aligned}$$

Similarly to the $\mathcal{N}=4$ SYM model, this action has the Chern-Simons term. However, R-charges of all chiral superfields are fixed now

$$R\Phi_i = -\frac{2}{3}\Phi_i\,, \qquad R\bar\Phi^i = \frac{2}{3}\bar\Phi^i\,. \tag{30}$$

We point out that the action (29) describes the $\mathcal{N}=(8,8)$ SYM model on S^2 with the same form of the hidden supersymmetry transformations (29).

4.4 Gaiotto-Witten theory

In flat space the Gaiotto-Witten theory was introduced in [9]. In the $\mathcal{N}=2$ superspace it is described by two $\mathcal{N}=2$ gauge superfields V and $\tilde V$ corresponding to two different gauge groups and two chiral superfields (a hypermultiplet) X_+ and X_- in the bifundamental representation. The classical superfield action for this model on S^3 reads

$$\begin{aligned}S_{\rm GW} &= S_{\rm CS}[V] - S_{\rm CS}[\tilde V] + S_X\,, \qquad\qquad (31)\\ S_X &= 4\,\mathrm{tr}\int d^dx d^2\theta d^2\bar\theta\, E(\bar X_+ e^V X_+ e^{-\tilde V}\\ &\quad + X_- e^{-V}\bar X_- e^{\tilde V})\,, \qquad\qquad (32)\end{aligned}$$

where $S_{\rm CS}$ is the Chern-Simons action (15). We find that this action is invariant under the following hidden supersymmetry transformation

$$\begin{aligned}\Delta V &= \bar\Sigma\mathcal{X}_+\mathcal{X}_- + \Sigma\bar{\mathcal{X}}_-\bar{\mathcal{X}}_+\,,\\ \Delta\tilde V &= \bar\Sigma\mathcal{X}_-\mathcal{X}_+ + \Sigma\bar{\mathcal{X}}_+\bar{\mathcal{X}}_-\,,\\ \delta\mathcal{X}_\pm &= \pm\bar\nabla^2(\bar\Upsilon\bar{\mathcal{X}}_\mp)\,,\end{aligned} \tag{33}$$

where $\mathcal{X}_\pm$ and $\bar{\mathcal{X}}_\pm$ are covariantly (anti)chiral superfields, $\bar{\mathcal{X}}_+ = e^{-\tilde V}\bar X_+ e^V$, $\mathcal{X}_+ = X_+$, $\bar{\mathcal{X}}_- = e^{-V}\bar X_- e^{\tilde V}$ $\mathcal{X}_- = X_-$. In (33) the superfield parameter Υ has the form (22) while Σ is related to the latter as follows

$$\mathcal{D}_\alpha\Sigma = -\frac{8i\pi}{k}\bar{\mathcal{D}}_\alpha\bar\Upsilon\,, \qquad \bar{\mathcal{D}}_\alpha\bar\Sigma = -\frac{8i\pi}{k}\mathcal{D}_\alpha\Upsilon\,. \tag{34}$$

Note that these transformations correspond to the case of Killing spinors obeying (20) with the same sign which form together $\mathcal{N}=4$ supersymmetry. Note also that the action (31) has a two-dimensional analog which we refer to as the Gaiotto-Witten model on S^2. The latter possesses $\mathcal{N}=(4,4)$ supersymmetry.

4.5 ABJM model

Finally, let us construct the classical action of the ABJ(M) theory [10–12] on S^3. This model is similar to the Gaoitto-Witten theory, but it involves two hypermultiplets, (X_{+i}, X_-^i), $i=1,2$, where X_{+i} and X_-^i are chiral superfields in the bi-fundamental representation of the gauge group. We find the following generalization of this action on S^3:

$$\begin{aligned}S_{\rm ABJM} &= S_{\rm CS}[V] - S_{\rm CS}[\tilde V] + S_X + S_{\rm pot}\,, \qquad\qquad (35)\\ S_X &= 4\mathrm{tr}\int d^dx d^2\theta d^2\bar\theta\, E\big(\bar X_+^i e^V X_{+i} e^{-\tilde V}\\ &\quad + X_-^i e^{-V}\bar X_{-i} e^{\tilde V}\big)\,,\\ S_{\rm pot} &= -\frac{4\pi i}{k}\mathrm{tr}\int d^dx d^2\theta\,\mathcal{E}\big(X_{+i}X_-^i X_{+j}X_-^j\\ &\quad - X_-^i X_{+i}X_-^j X_{+j}\big) + c.c.\end{aligned}$$

This action is invariant under the following hidden supersymmetry

$$\begin{aligned}
\Delta V &= -\frac{8i\pi}{k}(\bar\Upsilon^i{}_j\mathcal{X}_{+i}\mathcal{X}_-^j + \Upsilon_i{}^j\bar{\mathcal{X}}_{-j}\bar{\mathcal{X}}_+^i)\,,\\
\Delta\tilde V &= -\frac{8i\pi}{k}(\bar\Upsilon^j{}_i\mathcal{X}_-^i\mathcal{X}_{+j} + \Upsilon_i{}^j\bar{\mathcal{X}}_+^i\bar{\mathcal{X}}_{-j})\,,\\
\delta\mathcal{X}_{+i} &= \bar\nabla^2(\bar\Upsilon_i{}^j\bar{\mathcal{X}}_{-j})\,,\\
\delta\bar{\mathcal{X}}_+^i &= \nabla^2(\Upsilon^i{}_j\mathcal{X}_-^j)\,.
\end{aligned} \qquad (36)$$

Here $\mathcal{X}_{\pm i}$ and $\bar{\mathcal{X}}_{\pm i}$ are covariantly (anti)chiral superfields while $\Upsilon^i{}_j$ is a quartet of chiral superfield parameters each of which is similar to (22).

In two-dimensional case the action (35) corresponds to a reduction of the ABJM model to S^2 which possesses $\mathcal{N}=(6,6)$ supersymmetry.

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СУПЕРПОЛЕВЫЕ МОДЕЛИ НА S^2 И S^3

Развиваются суперполевые методы для построения классических действий различных моделей с глобальной суперсимметрией на S^2 и S^3. Вводятся суперпространства, основанные на факторпространствах вида $SU(2|1)/U(1)$ и $SU(2|1)/[U(1)\times U(1)]$. Показывается, что модели с расширенной суперсимметрией на S^3 имеют различные неэквивалентные версии, обладающие одинаковым числом суперсимметрий, но основанные на различных супералгебрах. В частности, построены классические действия для $\mathcal{N}=4$ и $\mathcal{N}=8$ суперсимметричных моделей Янга-Миллса, а также теории Гайотто-Виттена и ABJM на S^3. Для всех этих случаев рассмотрены аналогичные модели на S^2.

Ключевые слова: *суперпространство, суперполе Янга-Миллса.*

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