UDC 530.1; 539.1

ACCELERATION AND DECELERATION IN COSMOLOGY WITH SPINOR AND SCALAR FIELDS NON-MINIMALLY COUPLED TO F(R) GRAVITY

K. E. Osetrin, A. N. Makarenko, Yu. A. Rybalov

Tomsk State Pedagogical University, Kievskaya str., 60, 634061 Tomsk, Russia.

E-mail: osetrin@tspu.edu.ru; andre@tspu.edu.ru; ribalovyua@tspu.edu.ru

The solution to the current extending Universe problem, and the description of all stages of evolution compels scientists to consider various cosmological models. We combine two different approaches to the description of dark energy: modified gravity theory and introduction of the additional fields. We investigate the accelerating and decelerating cosmological models with non-linear spinor fields and non-minimal interaction of gravity with a scalar field. Scalar - tensor models are rather simple and also allow us to clearly define the separate stages of evolution. We explained the role of scalar and spinor potentials for emergence of accelerating or decelerating cosmology.

Keywords: *cosmological model, non-minimally interactions, scalar field, spinor field.*

The problem of the dark energy and dark matter is one of the main challenges of modern cosmology. Astrophysical data indicates that the observed universe is in an accelerated phase. This acceleration could be caused by this so-called dark energy (see [1] for a recent review). On the other hand, astrophysical observations provide evidence [2] for the existence of a non-baryonic, non-interacting and pressure-less component of the Universe, dubbed dark matter. This leads us to the need to revise the standard cosmology. The spectrum of models, having been postulated and explored in recent years, is extremely wide and includes, in particular, Quintessence, K-essence, Ghost Condensates, Dvali-Gabadadze-Porrati gravity, Galileon gravity, and $f(R)$ gravity [3] (see [4] for detailed reviews of these and other models). Such cosmological models tend to describe not only the accelerated expansion at this stage of evolution, but also at all other stages of evolution. All these models are divided into two main classes: $f(R)$ theory and alike (see [4–6]), and models which use various objects: scalars, spinor, cosmological constant, liquid with the difficult state equation.

The cosmological constant models are the simplest candidates for the solution of the problem of the universe acceleration. However, these models have still problems with the consistent description of the different evolution stages of the Universe. Scalar theory is most popular to describe the current accelerating expansion and early-time inflation. However, to describe the dark matter we have to introduce additional fields. One can consider a model with two scalar fields [7] (or scalar field and lagrange multiplier(s) [8]), or, for example, models with additional spinor field to describe dark energy and dark matter.

The spinor fields have been used as a source of a gravitational field in a number of works, and the interaction of the spinor and scalar fields as a factor of inflation expansion has been considered. Spinor fields can be used to describe the primordial inflation [9] and current expansion [10]. However, the exact solutions in the presence of the spinor field is difficult to build (for example, see [11]). A significant number of attempts have been made to construct the cosmological models with a spinor field for description of dark energy, where a non-canonical kinetic term was considered, such as k-inflation and k-essence models [12]. In [13, 14], the properties of one of the foregoing models with self-interacting spinor with the noncanonical kinetic term were studied.

The scalar invariant constructed from two spinor fields dynamically develops a nonvanishing value in Quantum Chromodynamics (QCD) theory [15]. There is another way to solve the problem of dark energy that does not require the introduction of the dark component. The modified theory of gravity may be quite realistic to describe the different phase of evolution of the Universe (see recent review [6]).

We considered a cosmological model with a spinor field and a scalar field couples with an arbitrary function of the curvature. Of course, such models are not standard ones, in the sense that they are not multiplicatively renormalizable in curved spacetime [16]. Hence, they should be considered as kind of effective theories (without clear understanding of their origin and their relation with more fundamental string/M-theory). The paper is devoted to study of non-minimally coupled scalar theory introduced in [3] with self-interacting spinor field. We study the Friedman-Robertson-Walker (FRW) equations of motion for such non-linear and non-minimal system with scalar and spinor fields. In this work we will study cosmological models similar to those offered in the works [14, 17], but with non-minimally interacted scalar field. Specific choice of scalar and spinor potentials is made in the process of the search of explicit accelerating/decelerating cosmological solutions. Several power-law solutions for current dark energy epoch are constructed. It is known that these cosmologies are quite realistic and pass the observational bounds.

The models allowing non-minimal interaction of scalar field derivatives and curvature are of particular interest. As Amendola showed, the theory of such kind, cannot be brought to a form of Einstein gravitation by conformal transformation. Note that usually field equations in models with non-minimal derivative interaction are the differential equations above the second order. However, the order goes down to the second in a special case when the kinetic term is connected only to Einstein's tensor, i.e. $\kappa G_{\mu\nu}\phi^{\mu}\phi^{\nu}$ (see, for example [18]). In works [19, 20] authors have studied cosmological scenarios from non-minimal interaction of the derivative $\kappa G_{\mu\nu}\phi^{\mu}\phi^{\nu}$, concentrating on models with zero and constant potential. According to the parameter choices, we have obtained a variety of behaviors including the Big Bang, an expanding universe without a beginning, a cosmological turnaround, an eternally contracting universe, a Big Crunch, and a cosmological bounce. In this cosmological model the non-minimal interaction of gravitation and a matter (a scalar field) is considered in a combination with the spinor field.

Let us consider two models with the action in the form:

$$S = \int d^4x\sqrt{-g}\{\frac{R}{8\pi} - (g_{\mu\nu} + \kappa(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}))\nabla^{\mu}\phi\nabla^{\nu}\phi - V(\phi) - L_D\}, \quad (1)$$

where R is scalar curvature, and the action in the form:

$$S = \int d^4x\sqrt{-g}\{\frac{R}{8\pi} - f(R)L_{\phi} - L_D\}. \quad (2)$$

The Lagrangian of a scalar field of mass m is given by:

$$L_{\phi} = \frac{1}{2}g^{\mu\nu}\nabla_{\mu}\phi\nabla_{\nu}\phi - V(\phi). \quad (3)$$

The Dirac Lagrangian L_D of fermion mass field m_f has the form:

$$L_D = \frac{i}{2}\{\bar{\psi}\Gamma^{\mu}D_{\mu}\psi - D_{\mu}\bar{\psi}\Gamma^{\mu}\psi\} - m_f\langle\bar{\psi}\psi\rangle - F(\psi\bar{\psi}). \quad (4)$$

In the expression (4), $F(\psi\bar{\psi})$ describes the potential of fermion field and $\bar{\psi} = \psi^{\dagger}\gamma^0$ denotes the conjugate spinor. $\Gamma^{\mu} = e^{\mu}_{a}\gamma^{a}$ are generalized Dirac-Pauly matrices in a curved spacetime (where e^{μ}_{a} is tetrad).

A consideration of the spinor field is carried out in works [14, 17], we have used the necessary results. Let us now consider a FRW universe with the flat spatial metric:

$$ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2). \quad (5)$$

Einstein's equations can be written as

$$\frac{1}{8\pi}(R_{\mu\sigma} - \frac{1}{2}g_{\mu\nu}R) = -T_{\mu\nu}, \quad (6)$$

where

$$T_{\mu\nu} = (T_f)_{\mu\nu} + (T_{\phi})_{\mu\nu},$$

$(T_f)_{\mu\nu}$ is the energy-momentum tensor of the fermion fields and $(T_{\phi})_{\mu\nu}$ is the contribution of the variation of the scalar field which interacts non-minimally.

We choose the potential of a fermionny field $F(\bar{\psi}\psi)$ in the following form:

$$F(\bar{\psi}\psi) = \alpha_1(\bar{\psi}\psi)^2 + \alpha_2(\bar{\psi}\psi)^4 + \alpha_3(\bar{\psi}\psi)^6,$$
$$\bar{\psi}\psi = \frac{c}{a(t)^3}. \quad (7)$$

We choose the action in form (1). We will notice that for $\alpha_1 = \alpha_2 = \alpha_3 = m_f = 0$ the equation is similar to the equation formulated in work [20]. The set of similar scalar models is considered in [4], but in our work the model is not only considered with the scalar, but also with the spinor field. We will contemplate solutions of the field equations.

Equation shows us that the existence of a special value for $H = \frac{\dot{a}}{a}$ follows, namely at $H = \frac{1}{3\sqrt{\kappa}}$ coefficient at ϕ'^2 addresses in zero. Furthermore, it is shown that, in this case, there is a solution.

I) Solution $H = const = \pm\sqrt{1/(3\kappa)}$. In this case the potential of a scalar field plays a role of a cosmological constant Λ.

II) Solution $H = const = \pm\sqrt{1/(9\kappa)}$.

We will now consider now solutions of the field equations in the absence of scalar potential ($V(\phi) = 0$). Asymptotic solutions are of greater interest in the equation at small a ($\tau \to -\infty$) and big a ($\tau \to +\infty$), and as transition from $\tau \to -\infty$ to $\tau \to +\infty$ ($\tau = \ln a$).

A) We will consider a case $\tau \to -\infty$. For the equation there are two partial solutions $Y = 1/9$, $Y = 1/3$ ($Y = \kappa H^2$). The solution $Y = 1/9$ – stable, $Y = 1/3$ – unstable. There is a special value of $Y = -1/9$ for which Y' does not exist.

At $\tau \to -\infty$ for Y from an $(-\infty,\ -1/9)$ interval at approach for special value of Y' tends to $-\infty$, for Y from an $(0,\ 1/9)$ interval at approach for special value of Y' tends to $+\infty$, for Y from interval of $(1/9,\ +\infty)$ the solution tends to $Y = 1/9$, $a(t) = \alpha e^{\frac{t}{\sqrt{9\kappa}}}$. In the field of $Y \in (-\infty,\ 0)$ the main contribution to the equation gives the summands of the spinor field, generally summed with α_3. If in the equation we consider solutions of higher order of smallness, then the most important are summands with α_2, α_3 and the behavior of function Y qualitatively doesn't change. If we put m_f, α_2, $\alpha_3 = 0$ we will receive singular solutions $Y = 0$, $Y = -1/9$. Note that the results calculated in works [19,20], in our case, are impossible, as in a limit at $\tau \to -\infty$ the spinor field renders considerable influence. If we exclude the spinor field, we will receive solutions given in the work above. We can see that the accelerated expansion is observed, and the prevalence (existence) of the summand spinor field, which causes faster expansion.

B) We will consider $\tau \to +\infty$ case. For the equation there are three partial solutions $Y = 0$, $Y = 1/3$, $Y = 1/9$. Solution $Y = 0$, $Y = 1/3$ – stable, $Y = 1/9$ – unstable.

At $\tau \to +\infty$ for Y from an $(-\infty,\ 1/9)$ interval solution aspire at big τ to $Y = 0$, $a(t) = const$, for Y from an $(1/9,\ +\infty)$ interval solution aspire big τ to $Y = 1/3$, $a(t) \to \alpha e^{\frac{t}{\sqrt{3\kappa}}}$.

By considering an interval of positive values τ at $\tau \to +\infty$ we can observe a gradual weakening of the spinor field influence, and within the limit we receive the prevalence of the summands for a scalar field and the Einstein summand.

Note that in work [19] solutions, with additional asymptotic restrictions, for a conformal factor, in operation without the spinor field are received. In our case, however, the scalar field behaves differently, which is caused by the influence of the spinor field summands. As the existence of a spinor excludes a number of asymptotic solutions if we do not impose additional approximations (for example degree).

We choose the action in form (2), where the Lagrangian of the scalar field is (3) and $a(t) = a_0 t^n$, $f(R) = r_0 R^p$. We assume reconstruction of solutions. Thus, in our model we have a different types of behavior of the universe expansion. The presence of the spinor field leads to a slowing of the universe expansion, when the scale factor is positive and less than one ($n = 1/9$, $1/6$, $1/3$ and $2/3$). For the case of a free scalar field if $n = 2/3$ then we get the scalar field decreasing over time. Otherwise, the scalar field increases with time ($\phi \sim t^{1/2}$, $t^{3/4}$ and $t^{5/6}$).

If we consider the model in the absence of a spinor field the situation is changing. The presence of non-minimal interaction allows to obtain solutions for any value of the degree in the scale factor. The degree of a scalar field is arbitrary. We have only one restriction - $k = p$ (where $\phi = f_0 t^k$).

All the solutions we obtained for the power-law scalar field ($\phi \sim t^k$). Choosing a different type of fields, such as logarithmic function of time, lead us to an equation without explicit solution.

Consider as an example the case of the scalar potential field set to $1/2m\phi^2$. In this case, the equation of motion of the scalar field gets the form

$$2m\, t\, \phi + (3n - 2p)\dot{\phi} + t\ddot{\phi} = 0.$$

$$\phi = t^{\frac{1}{2} - \frac{3n}{2} + p}(\text{BesselJ}\left[\frac{1}{2} - \frac{3n}{2} + p, \sqrt{2}\sqrt{m}t\right] c_1$$

$$+\text{BesselY}\left[\frac{1}{2} - \frac{3n}{2} + p, \sqrt{2}\sqrt{m}t\right] c_2),$$

where c_1 and c_2 are constant, BesselJ is the Bessel function of the first kind and BesselY is the Bessel function of the second kind. We see that in this case it would be difficult to check the compatibility of the solutions with the Einstein equations. For this reason, we restricted ourselves to the power dependence of the scalar field on time.

If the degree of the scale factor is positive, we get the quintessence-type universe. However, we can consider the case of a negative power. One can do a replacement $t \to t - t_s$ (t_s is a constant) and we obtain the phantom universe with the singularity of the future such as the Big Rip.

We see that the presence of a spinor field of specific type does not permit the universe to expand with acceleration. The introduction of a scalar field does not change the situation. However, in the absence of a spinor field the non-minimal interaction leads us to arbitrary powers of the scale factor and the scalar (in power form), but fixing potential and function $f(R)$.

For the action in form (2). The behavior of the cosmological model with the spinor field and non-minimally interacted scalar field has been considered. In the work a number of solutions addressing the exponential behavior of a large-scale factor for model with a scalar and spinor field have been calculated. That partial solution are solutions of dS (de-Sitter), or solutions $a(t) = const$. Asymptotic solutions are generally more difficult to comprehend and partial asymptotic solutions are the de Sitter's solutions or solutions $a(t) = const$. If we impose additional conditions on the asymptotic equations it is possible to receive also the series solutions for a large-scale factor. The existence of the spinor field has an impact on evolution in the initial stage and further weakens its influence. Towards the latter stages of evolution, a scalar field and $R/2$ term are very influential. Our following research will include the equivalent model with interaction of the scalar field and $f(R)$ gravitation.

Acknowledgements

The work was supported by the grant of Russian Ministry of Education and Science project No. 867.

References

[1] S. Perlmutter et al., Nature 404, 955 (2000); C. L. Bennett et al., Astrophys. J. Suppl. 148, 1 (2003) arXiv: astro-ph/0302207; N.W. Halverson et al., Ap.J. 568, 38 (2002), arXiv: astro-ph/0104489; I. Brevik, V. V. Obukhov, A. V. Timoshkin, TSPU Bulletin. 13, (128), p. 42-45, (2012) arXiv: gr-qc/1303.5669.

[2] D. Clowe, M. Bradac, A. H. Gonzalez, M. Markevitch, S. W. Randall, C. Jones and D. Zaritsky, Astrophys. J. 648, L109 (2006), astro-ph/0608407; G. Bertone, D. Hooper and J. Silk, Phys. Rept. 405, 279 (2005), hep-ph/0404175.

[3] S. Nojiri and S. D. Odintsov, Phys. Lett. B599, 137 (2004) arXiv: astro-ph/0403622; S. Nojiri and S. D. Odintsov, Phys. Rev. D 68, 123512 (2003) arXiv: hep-th/0307288; G. Allemandi, A. Borowiec, M. Francaviglia, and S. D. Odintsov, Phys. Rev. D 72, 063505 (2005) arXiv: gr-qc/0504057; K. Bamba, A. Lopez-Revelles, R. Myrzakulov, S. D. Odintsov and L. Sebastiani. TSPU Bulletin. 13, (128), p. 22-27, (2012) arXiv: gr-qc/1301.3049.

[4] K. Bamba, S. Capozziello, S. Nojiri and S. D. Odintsov, Astrophys. and Space Sc., 342:155-228 (2012) arXiv: 1205.3421 [gr-qc].

[5] S. Nojiri and S. D. Odintsov, Int. J. Geom. Meth. Mod. Phys. 4, 115-146 (2007) arXiv:hep-th/0601213; S. Nojiri and S. D. Odintsov, J. Phys. Conf. Ser. 66, 012005 (2007) arXiv: hep-th/0611071.

[6] S. Nojiri, S. D. Odintsov, Phys. Rept., 505, 59-144, (2011) arXiv: 1011.0544 [gr-qc].

[7] Emilio Elizalde, Shin'ichi Nojiri, Sergei D. Odintsov, Phys.Rev. D 70, 043539 (2004), [arXiv:hep-th/0405034].

[8] S. Capozziello, J. Matsumoto, S. Nojiri, S. D. Odintsov, Phys. Lett. B 693, 198 (2010), arXiv:1004.3691 [hep-th]; S. Capozziello, A. N. Makarenko, S. D. Odintsov, Phys. Rev. D 87, 084037 (2013), arXiv:1302.0093 [gr-qc]; S. Capozziello, M.Francaviglia, A. N. Makarenko, arXiv:1304.5440 [gr-qc].

[9] C. Armendariz-Picon and P. B. Greene, Gen. Rel. Grav. 35, 1637 (2003).

[10] M. O. Ribas, F. P. Devecchi and G. M. Kremer, Phys. Rev. D 72, 123502 (2005), arXiv:0511099 [gr-qc].

[11] A. N. Makarenko, V. V. Obukhov, Entropy 14, 1140 (2012), arXiv:1207.0589 [gr-qc].

[12] R. Rakhi, G. V. Vijayagovindan, N. P. Abraham and K. Indulekha, Int. J. Mod. Phys. A 25, 1267 (2010), arXiv:0910.3761 [hep-ph]; T. Watanabe, arXiv:0902.1392 [astro-ph.CO]; V. G. Bagrov, A. N. Makarenko, V. V. Obukhov, K. E. Osetrin, Grav.Cosmol.Suppl. 8, N2, 3-5 (2002); A. N. Makarenko, V. V. Obukhov, Russ. Phys. J. 41, 1124-1133 (1998).

[13] T. Inagaki, Yu. Rybalov, Eur. Phys. J. C 71, 1656 (2011), arXiv:1102.5036 [hep-ph].

[14] K. E. Osetrin, Yu. A. Rybalov. Rus. Phys. J., 55, 1416 - 1424, (2013).

[15] Y. Q. Gu, Int. J. Mod. Phys. A 22, 4667 (2007), arXiv:0610147[gr-qc]; C. G. Boehmer and D. F. Mota, Phys. Lett. B 663, 168 (2008), arXiv:0710.2003 [astro-ph].

[16] I. L. Buchbinder, S. D. Odintsov and I. L. Shapiro, Effective Action in Quantum Gravity, IOP 1992.

[17] Yu. A. Rybalov, A. N. Makarenko, K. E. Osetrin. Astroph. Space Sc., 349, N1, 561-566 (2014) arXiv: 1310.2505.

[18] S. V. Sushkov, Phys. Rev. D80, 103505 (2009) arXiv: 0910.0980 [gr-qc].

[19] E. N. Saridakis and S. V. Sushkov, Phys. Rev. D81, 083510 (2010) arXiv: 1002.3478 [gr-qc].

[20] M. A. Skugoreva, S. V. Sushkov and A. V. Toporensky, Phys. Rev. D88, 083539 (2013) arXiv: 1306.5090 [gr-qc].

Received 24.11.2014

К. Е. Осетрин, А. Н. Макаренко, Ю. А. Рыбалов

УСКОРЕНИЕ И ЗАМЕДЛЕНИЕ В КОСМОЛОГИИ СО СПИНОРНЫМ ПОЛЕМ И НЕМИНИМАЛЬНЫМ ВЗАИМОДЕЙСТВИЕМ f(R) ГРАВИТАЦИИ СО СКАЛЯРНЫМ ПОЛЕМ

Решение проблемы текущего расширения Вселенной и описание всех стадий эволюции Вселенной заставляет ученых рассматривать различные космологические модели. В нашей работе объединены два различных подхода к описанию темной материи: модифицированные теории гравитации и введение дополнительных полей. Исследовано ускорение и замедление в космологических моделях с нелинейным спинорным полем и неминимальное взаимодействие гравитации со скалярным полем. Скалярно-тензорные модели довольно просты и также позволяют ясно определить различные этапы эволюции Вселенной. В исследовании объяснена роль скалярного и спинорного потенциалов для появления ускорения или замедления космологии.

Ключевые слова: *космологические модели, скалярное поле, спинорное поле, неминимальное взаимодействие.*

Осетрин К. Е., доктор физико-математических наук, профессор.
Томский государственный педагогический университет.
Ул. Киевская, 60, 634061Томск, Россия.
E-mail: osetrin@tspu.edu.ru

Макаренко А. Н., кандидат физико-математических наук, доцент.
Томский государственный педагогический университет.
Ул. Киевская, 60, 634061 Томск, Россия.
E-mail: andre@tspu.edu.ru

Рыбалов Ю. А., кандидат физико-математических наук.
Томский государственный педагогический университет.
Ул. Киевская, 60, 634061 Томск, Россия.
E-mail: ribalovyua@tspu.edu.ru