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AUXILIARY SUPERFIELDS IN $\mathcal{N}=2$ BORN-INFELD THEORY

E. A. Ivanov, B. M. Zupnik
Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Moscow Reg., Russia.
E-mail: eivanov,zupnik@theor.jinr.ru

Some time ago the authors proposed a new approach to self-dual electrodynamics and its superextensions, based on introducing auxiliary (super)fields which ensure linearization of the duality transformations. In the talk this universal method is applied for analysis of the self-duality properties of the $\mathcal{N}=2$ Born-Infeld (BI) action with the nonlinearly realized "hidden" $\mathcal{N}=4$ supersymmetry. Its self-duality is proved up to the 10th order in the superfield strengths. The conjecture about the general structure of the $\mathcal{N}=2$ BI action modified by the auxiliary chiral superfields is put forward.

Keywords: *supersymmetry, duality, superfield.*

1 Motivations: why Born-Infeld?

The BI theory is the renowned nonlinear extension of Maxwell theory:

$$L^{BI}(\varphi,\bar\varphi)=1-\sqrt{1+\varphi+\bar\varphi+(1/4)(\varphi-\bar\varphi)^2}\,,$$

$$\varphi+\bar\varphi=\frac{1}{2}(F_{mn})^2\,,\ \varphi-\bar\varphi=\frac{i}{2}F_{mn}\tilde F^{mn}.$$

It possesses many remarkable properties.

It is the first known example of nontrivial self-dual model of nonlinear electrodynamics.

The BI actions are necessary ingredients of the static-gauge D brane worldvolume actions.

The super D branes actions enjoy linearly realized worldvolume supersymmetries, the relevant worldvolume supermultiplets being the vector ones, with the gauge field among the component fields. This poses the problem of supersymmetrizing the BI action.

The spontaneously broken part of the full D brane supersymmetry should manifest itself as the second nonlinear supersymmetry of such super BI actions.

To date, two superextended BI actions with the second nonlinearly realized supersymmetry are known in the superfield approach.

First, it is $\mathcal{N}=1$ supersymmetric BI action [1, 2]. It describes the "space-filling" $D3$ brane and possesses the second nonlinear $\mathcal{N}=1$ supersymmetry building up the manifest one to $\mathcal{N}=2$. So it can be abbreviated as "$\mathcal{N}=2/\mathcal{N}=1$" BI action.

Secondly, it is $\mathcal{N}=2$ supersymmetric BI action [3–6]. It describes $D3$ brane in $D=6$ and exhibits a nonlinearly realized $\mathcal{N}=4/\mathcal{N}=2$ supersymmetry.

While the $\mathcal{N}=2/\mathcal{N}=1$ BI action is known in a closed superfield form, no such a formulation was given for the $\mathcal{N}=4/\mathcal{N}=2$ BI action so far. There exists another $\mathcal{N}=2$ BI action admitting the closed form [7], but it does not possess any extra supersymmetry.

It seems very interesting to further elaborate on the $\mathcal{N}=4/\mathcal{N}=2$ BI action and try to find a closed form for it. One more intriguing problem is as follows.

The $\mathcal{N}=2/\mathcal{N}=1$ BI action was shown to be self-dual like its bosonic counterpart. The same self-duality for the $\mathcal{N}=4/\mathcal{N}=2$ BI action was checked up to the 8th order in the $\mathcal{N}=2$ superfield strengths. The question is whether this self-duality extends to all orders and how it is related to the hidden supersymmetry.

Recently, a new framework was developed for self-dual electrodynamics models and their $\mathcal{N}=1,\mathcal{N}=2$ extensions [8–10]. It proceeds from the formulation proposed in [11] and includes, as a new ingredient, auxiliary tensorial fields and their superfield counterparts. It is natural to analyze the $\mathcal{N}=4/\mathcal{N}=2$ BI action in this general approach, with the hope that it will allow to give a general proof of self-duality of this action and find an ansatz for the latter beyond the perturbative expansion over $\mathcal{N}=2$ superfield strengths. Some steps toward this goal were recently made in [12]. Major part of the talk will be based on this work.

We will start with recalling the salient features of self-duality in electrodynamics and explain basics of the new approach with auxiliary tensorial fields. Then we will discuss how to put the $\mathcal{N}=4/\mathcal{N}=2$ BI theory into this framework and to which new understanding this gives rise.

2 Self-duality: general setting

$U(N)$ duality invariance is an on-shell symmetry of a wide class of the nonlinear electrodynamics models including the renowned Born-Infeld theory. It generalizes the free-case $O(2)$ symmetry between the equations of motion and Bianchi identities ($F_{mn}=$

$\partial_m A_n - \partial_n A_m, \tilde{F}_{mn} = \frac{1}{2}\epsilon_{mnpq}F^{pq}$):

$$\text{E.O.M.} : \partial^m F_{mn} = 0 \iff \text{Bianchi} : \partial^m \tilde{F}_{mn} = 0,$$

$$\delta F_{mn} = \omega \tilde{F}_{mn}, \ \delta \tilde{F}_{mn} = -\omega F_{mn},$$

In the nonlinear case: $P_{mn} = 2\frac{\partial L(F)}{\partial F^{mn}}$,

$$\text{E.O.M.} : \partial^m P_{mn} = 0 \iff \text{Bianchi} : \partial^m \tilde{F}_{mn} = 0\,,$$

$$\delta P_{mn} = \omega \tilde{F}_{mn}\,, \ \delta \tilde{F}_{mn} = -\omega P_{mn}.$$

There should be valid the self-consistency condition (the so called GZ condition) [13, 14]:

$$P\tilde{P} - F\tilde{F} = 0.$$

Recently, there came about a rebirth of interest in the duality-invariant theories [15–17]. The basic reason is the hypothetical important role of the generalized duality symmetry in analyzing the UV properties of extended $4D$ supergravities.

We will use the bispinor formalism:

$$F_{mn} \Rightarrow (F_{\alpha\beta},\ \bar{F}_{\dot\alpha\dot\beta}),\ \varphi = F^{\alpha\beta}F_{\alpha\beta}\,,\ \bar\varphi = \bar{F}^{\dot\alpha\dot\beta}\bar{F}_{\dot\alpha\dot\beta}\,,$$

$$L(\varphi, \bar\varphi) = -\frac{1}{2}(\varphi + \bar\varphi) + L^{int}(\varphi, \bar\varphi)\,,$$

$$\partial^{\dot\beta}_{\alpha}\bar{P}_{\dot\alpha\dot\beta}(F) - \partial^{\beta}_{\dot\alpha}P_{\alpha\beta}(F) = 0\,,\ \partial^{\dot\beta}_{\alpha}\bar{F}_{\dot\alpha\dot\beta} - \partial^{\beta}_{\dot\alpha}F_{\alpha\beta} = 0\,,$$

$$P_{\alpha\beta} = i\frac{\partial L}{\partial F^{\alpha\beta}}\,,$$

$$\delta_\omega F_{\alpha\beta} = \omega P_{\alpha\beta}, \qquad \delta_\omega P_{\alpha\beta} = -\omega F_{\alpha\beta},$$

$$F_{\alpha\beta}F^{\alpha\beta} + P_{\alpha\beta}P^{\alpha\beta} - \text{c.c} = 0\,,\ \Leftrightarrow\ \varphi - 4\varphi(L_\varphi)^2 - \text{c.c} = 0\,,$$

$$L^{sd} = \frac{i}{2}(\bar{P}\bar{F} - PF) + I(\varphi, \bar\varphi)\,, \quad \delta_\omega I(\varphi, \bar\varphi) = 0\,.$$

How to determine the invariant $I(\varphi, \bar\varphi)$? Our approach with the auxiliary tensorial fields provides an answer.

3 Formulation with bispinor auxiliary fields

Let us introduce the auxiliary unconstrained fields $V_{\alpha\beta}$ and $\bar{V}_{\dot\alpha\dot\beta}$ and write the extended Lagrangian in the (F, V)-representation as

$$\mathcal{L}(V, F) = \mathcal{L}_2(V, F) + E(\nu, \bar\nu),$$

$$\mathcal{L}_2(V, F) = \frac{1}{2}(\varphi + \bar\varphi) + \nu + \bar\nu - 2\,(V \cdot F + \bar{V} \cdot \bar{F}),$$

with $\nu = V^2,\ \bar\nu = \bar{V}^2$. Here, $E(\nu, \bar\nu)$ is the nonlinear interaction involving only auxiliary fields.

Dynamical equations of motion

$$\partial^{\dot\beta}_{\alpha}\bar{P}_{\dot\alpha\dot\beta}(V, F) - \partial^{\beta}_{\dot\alpha}P_{\alpha\beta}(V, F) = 0\,,$$

$$P_{\alpha\beta}(F, V) = i\frac{\partial \mathcal{L}(V, F)}{\partial F^{\alpha\beta}} = i(F_{\alpha\beta} - 2V_{\alpha\beta}),$$

together with Bianchi identity, are covariant under $O(2)$ transformations

$$\delta V_{\alpha\beta} = -i\omega V_{\alpha\beta},\ \delta F_{\alpha\beta} = i\omega(F_{\alpha\beta} - 2V_{\alpha\beta}),\ \delta\nu = -2i\omega\nu.$$

The algebraic equation of motion for $V_{\alpha\beta}$,

$$F_{\alpha\beta} = V_{\alpha\beta} + \frac{1}{2}\frac{\partial E}{\partial V^{\alpha\beta}} = V_{\alpha\beta}(1 + E_\nu)\,,$$

is $O(2)$ covariant *if and only if*

$$\nu E_\nu - \bar\nu E_{\bar\nu} = 0 \quad \Rightarrow \quad E(\nu, \bar\nu) = \mathcal{E}(a)\,, \ \ a := \nu\bar\nu\,.$$

The meaning of this constraint is that $E(\nu, \bar\nu)$ should be $O(2)$ invariant function

$$\delta_\omega E = 2i\omega(\bar\nu E_{\bar\nu} - \nu E_\nu) = 0\,.$$

This is none other than the GZ constraint:

$$F^2 + P^2 - \bar{F}^2 - \bar{P}^2 = 0 \iff \nu E_\nu - \bar\nu E_{\bar\nu} = 0\,.$$

The auxiliary equation can be now written as

$$F_{\alpha\beta} - V_{\alpha\beta} = V_{\alpha\beta}\bar\nu\,\mathcal{E}'\,.$$

It serves to express $V_{\alpha\beta}, \bar{V}_{\dot\alpha\dot\beta}$ in terms of $F_{\alpha\beta}, \bar{F}_{\dot\alpha\dot\beta}$:

$$V_{\alpha\beta}(F) = F_{\alpha\beta}G(\varphi, \bar\varphi),\ G(\varphi, \bar\varphi) = \frac{1}{2} - L_\varphi = (1 + \bar\nu\mathcal{E}_a)^{-1}.$$

After substituting these expressions back into $\mathcal{L}$ we obtain the corresponding selfdual $L^{sd}(\varphi, \bar\varphi)$

$$L^{sd}(\varphi, \bar\varphi) = -\frac{1}{2}\,\frac{(\varphi + \bar\varphi)(1 - a\mathcal{E}_a^2) + 8a^2\mathcal{E}_a^3}{1 + a\mathcal{E}_a^2} + \mathcal{E}(a)\,,$$

where a is related to $\varphi, \bar\varphi$ by the algebraic equation

$$(1 + a\mathcal{E}_a^2)^2\varphi\bar\varphi = a[(\varphi + \bar\varphi)\mathcal{E}_a + (1 - a\mathcal{E}_a^2)^2]^2\,.$$

We find $I(\varphi, \bar\varphi) = \mathcal{E}(a) - 2a\mathcal{E}_a,\ a = \nu(\varphi, \bar\varphi)\bar\nu(\varphi, \bar\varphi)$.

To summarize, *all* $O(2)$ duality-symmetric systems of nonlinear electrodynamics without derivatives on the field strengths are parametrized by the $O(2)$ invariant off-shell interaction $\mathcal{E}(a)$ which is a function of the real quartic combination of the auxiliary fields. This universality is the basic advantage of the approach with tensorial auxiliary fields.

As an instructive example, we consider the Born-Infeld action in the new setting. The BI model has a more simple description in terms of the new variables:

$$\mathcal{E}^{BI}(a) = I^{BI}(b) - 2b\,I^{BI}_b(b)\,,$$

$$I^{BI} = \frac{2b}{b-1},\ I^{BI}_b = -\frac{2}{(b-1)^2}\,,\ a = \frac{4b}{(1-b)^4}\,.$$

The equation for b becomes quadratic:

$$\varphi\bar\varphi\, b^2 + [2\varphi\bar\varphi - (\varphi + \bar\varphi + 2)^2]\, b + \varphi\bar\varphi = 0 \qquad \Rightarrow$$

$$b = \frac{4\varphi\bar\varphi}{[2(1+Q) + \varphi + \bar\varphi]^2}\,,$$

$Q(\varphi)=\sqrt{1+\varphi+\bar{\varphi}+(1/4)(\varphi-\bar{\varphi})^2}\,.$

After substituting this into the general formula for $L^{cd}(\varphi,\bar{\varphi})$ the standard BI Lagrangian is recovered

$$L^{BI}(\varphi,\bar{\varphi})=1-\sqrt{1+\varphi+\bar{\varphi}+(1/4)(\varphi-\bar{\varphi})^2}\,.$$

How to supersymmetrize the bispinor formulation? The basic idea [9, 10] is to embed tensorial auxiliary fields into chiral auxiliary $\mathcal{N}=1,2$ superfields:

$$V_{\alpha\beta}(x)\;\Rightarrow\;U_\alpha(x,\theta,\bar{\theta})=v_\alpha(x)+\theta^\beta V_{\alpha\beta}(x)+\ldots\,,$$

$$V_{\alpha\beta}(x)\;\Rightarrow\;U(x,\theta^i,\bar{\theta}_i)=v(x)+(\theta^\beta_k\theta^{\alpha k})V_{\alpha\beta}(x)+\ldots\,,$$

$$\bar{D}_{\dot\gamma}U_\alpha(x,\theta,\bar{\theta})=0\,,\quad \bar{D}_{\dot\gamma i}U(x,\theta^j,\bar{\theta}_k)=0\,.$$

The (W,U) representation for the $\mathcal{N}=1$ self-dual actions is defined as

$$\begin{aligned} S(W,U)\;\;&=\;\;\int d^6\zeta\left(UW-\frac{1}{2}U^2-\frac{1}{4}W^2\right)+\text{c.c.}\\ &+\frac{1}{4}\int d^8z\,U^2\bar{U}^2\,E(u,\bar{u},g,\bar{g}),\\ &u=\frac{1}{8}\bar{D}^2\bar{U}^2,\;\bar{u}=\frac{1}{8}D^2U^2,\;g=D^\alpha U_\alpha\,. \end{aligned}$$

Duality-invariant $\mathcal{N}=1$ systems amount to the $U(1)$ invariant interaction

$$E_{inv}=\mathcal{F}(B,A,C)+\bar{\mathcal{F}}(\bar{B},A,C)\,,$$
$$A:=u\bar{u}\,,\;C:=g\bar{g}\,,\;B:=ug^2\,,\;\bar{B}:=\bar{u}\bar{g}^2\,.$$

4 $\mathcal{N}=2$ BI theory: the $(\mathcal{W},\bar{\mathcal{W}})$ formulation

The $\mathcal{N}=2$ BI action can be written through the chiral Lagrangian density $\mathcal{A}_0$ as [4]:

$$S_{BI}(\mathcal{W})=S_2(\mathcal{W})+I_{BI}(\mathcal{W})=\frac{1}{4}\int d^8\mathcal{Z}\,\mathcal{A}_0+\text{c.c.}\,,$$
$$I_{BI}(\mathcal{W})=\int d^{12}Z\,L_{BI}(\mathcal{W})\,,$$

where

$$L_{BI}=\sum_{n=2}^\infty L^{(2n)},\;\mathcal{A}_0(\mathcal{W})=\sum_{n=1}^\infty\mathcal{A}_0^{(2n)}=\mathcal{W}^2+2\bar{D}^4L_{BI}.$$

The superfield strength $\mathcal{W}$, together with $\mathcal{A}_0$, belong to an infinite-dimensional multiplet of the spontaneously broken $\mathcal{N}=4$ supersymmetry

$$\begin{aligned} \delta_f\mathcal{W}&=f(1-\frac{1}{2}\bar{D}^4\bar{\mathcal{A}}_0)+\frac{1}{2}\bar{f}\Box\mathcal{A}_0+\frac{i}{4}\bar{D}^{\dot\alpha}_k\bar{f}D^{k\alpha}\partial_{\alpha\dot\alpha}\mathcal{A}_0,\\ \delta_f\mathcal{A}_0&=2f\mathcal{W}+\frac{1}{2}\bar{f}\Box\mathcal{A}_1+\frac{i}{4}\bar{D}^{\dot\alpha}_k\bar{f}D^{k\alpha}\partial_{\alpha\dot\alpha}\mathcal{A}_1\,,\\ \delta_f\mathcal{A}_n&=2f\mathcal{A}_{n-1}+\frac{1}{2}\bar{f}\Box\mathcal{A}_{n+1}+\frac{i}{4}\bar{D}^{\dot\alpha}_k\bar{f}D^{k\alpha}\partial_{\alpha\dot\alpha}\mathcal{A}_{n+1}\,,\\ f&:=c+2i\theta^\alpha_k\xi^k_\alpha\,,\quad n\geq1\,. \end{aligned}$$

The density $\mathcal{A}_0$, together with other superfields $\mathcal{A}_n$, can be expressed in terms of $\mathcal{W},\bar{\mathcal{W}}$ by imposing an infinite set of $\mathcal{N}=4$ covariant constraints

$$\mathcal{A}_0-\mathcal{W}^2-\frac{1}{2}\mathcal{A}_0\bar{D}^4\bar{\mathcal{A}}_0-\bar{D}^4\sum_{n=1}\frac{(-1)^n}{2^{2n+1}}\mathcal{A}_n\Box^n\bar{\mathcal{A}}_n=0\,,$$
$$\Box\mathcal{A}_1+\ldots=0\,,\ldots\ldots\Box^n\mathcal{A}_n+\ldots=0\,,$$

where dots stand for some nonlinear terms. Solving it by recursions, one can restore $\mathcal{A}_0$ to any order, e.g.,

$$\begin{aligned} \mathcal{A}_0^{(4)}&=\frac{1}{2}\bar{D}^4(\mathcal{W}^2\bar{\mathcal{W}}^2),\\ \mathcal{A}_0^{(6)}&=\frac{1}{4}\bar{D}^4\Big[\mathcal{W}^2\bar{\mathcal{W}}^2(D^4\mathcal{W}^2+\text{c.c.})-\frac{2}{9}\mathcal{W}^3\Box\bar{\mathcal{W}}^3\Big]. \end{aligned}$$

In this way, the interaction I_{BI} was found in [4] up to the 8th order. Recently [12], the next, 10th order term, was explicitly computed.

Inspecting the explicit structure of the perturbative terms in $\mathcal{A}_0$ and the respective terms in I_{BI}, we found that $\mathcal{A}_0$ has the following general splitting

$$\mathcal{A}_0=\mathcal{X}+\mathcal{R}+\mathcal{Y}\,. \tag{1}$$

Here, $\mathcal{X}$ is defined by the equation [5–7]

$$\mathcal{X}=\mathcal{W}^2+\frac{1}{2}\mathcal{X}\bar{D}^4\bar{\mathcal{X}} \tag{2}$$

and accounts for all terms without $\Box$. The part $\mathcal{R}$ accounts for all terms involving only box operator

$$\mathcal{R}=2\bar{D}^4\sum_{n=3}^\infty(-1)^n\frac{1}{(n!)^2}\mathcal{W}^n\Box^{n-2}\bar{\mathcal{W}}^n.$$

The remaining piece $\mathcal{Y}$ collects, in its perturbative expansion, the mixed terms with $D^4,\bar{D}^4,\Box$, which are not combined into any obvious series.

Is it possible to give some general formulas for all these pieces? Is it possible to prove, at least up to the 10th order, that the $\mathcal{N}=4/\mathcal{N}=2$ BI action is simultaneously self-dual? The latter property was proved in [5, 6] up to the 8th order. The reformulation of the $\mathcal{N}=2$ BI theory through auxiliary superfields turns out to be helpful for getting the answers.

5 Auxiliary superfields for $\mathcal{N}=2$ self-duality

The $(\mathcal{W},\mathcal{U})$ representation of the general action of the nonlinear $\mathcal{N}=2$ electrodynamics reads

$$\begin{aligned} &\mathcal{S}(\mathcal{W},\mathcal{U})=\mathcal{S}_b(\mathcal{W},\mathcal{U})+\mathcal{I}(\mathcal{U})\,,\\ &\mathcal{I}(\mathcal{U})=\int d^8\mathcal{Z}\,\mathcal{L}(\mathcal{U})+\text{c.c.}\,,\\ &\mathcal{S}_b(\mathcal{W},\mathcal{U})=\int d^8\mathcal{Z}\,(\mathcal{U}\mathcal{W}-\frac{1}{2}\mathcal{U}^2-\frac{1}{4}\mathcal{W}^2)+\text{c.c.}\,. \end{aligned}$$

The interaction $\mathcal{L}(\mathcal{U})$ is a local functional of $\mathcal{U},\bar{\mathcal{U}}$. The $\mathcal{U}$ equation of motion

$$\mathcal{U}=\mathcal{W}+\frac{\delta\mathcal{I}}{\delta\mathcal{U}}\,,\qquad\frac{\delta\mathcal{I}}{\delta\mathcal{U}}:=\mathcal{J}(\mathcal{U},\bar{\mathcal{U}})=\bar{D}^4J(\mathcal{U},\bar{\mathcal{U}})\,, \tag{3}$$

allows one to eliminate auxiliary superfields and recover the nonlinear $\mathcal{W}$ action. The $\mathcal{N}=2$ self-duality condition [5] becomes

$$\int d^8\mathcal{Z}\,\mathcal{U}\frac{\delta\mathcal{I}}{\delta\mathcal{U}} = \int d^8\bar{\mathcal{Z}}\,\bar{\mathcal{U}}\frac{\delta\mathcal{I}}{\delta\bar{\mathcal{U}}}\,.$$

This is just the condition of the invariance of the functional $\mathcal{I}(\mathcal{U})$ under the $U(1)$ transformations $\delta_\omega\mathcal{U} = -i\omega\mathcal{U}$, $\delta_\omega\bar{\mathcal{U}} = i\omega\bar{\mathcal{U}}$.

Thus **any** self-dual system of $\mathcal{N}=2$ electrodynamics can be reformulated as a system with the off-shell action $\mathcal{S}(\mathcal{W},\mathcal{U}) = \mathcal{S}_b(\mathcal{W},\mathcal{U}) + \mathcal{I}(\mathcal{U})$, so that the interaction $\mathcal{I}(\mathcal{U})$ is $U(1)$ duality invariant.

Conversely, if some $\mathcal{N}=2$ system **admits** such a reformulation, **it is self-dual**.

So, one way to prove that the $\mathcal{N}=4/\mathcal{N}=2$ BI action is self-dual is to put it into the $(\mathcal{U},\mathcal{W})$ form and to show that $\mathcal{I}_{BI}(\mathcal{U})$ is $U(1)$ invariant.

The goal is to write the $\mathcal{N}=4/\mathcal{N}=2$ BI action as

$$\mathcal{S}_{BI}(\mathcal{W},\mathcal{U}) = \mathcal{S}_b(\mathcal{W},\mathcal{U}) + \mathcal{I}_{BI}(\mathcal{U}),$$

where, in accordance with the triple decomposition (1) of the $(\mathcal{W},\bar{\mathcal{W}})$ BI action,

$$\mathcal{I}_{BI}(\mathcal{U}) = \mathcal{I}_\mathcal{X}(\mathcal{U}) + \mathcal{I}_\mathcal{R}(\mathcal{U}) + \mathcal{I}_\mathcal{Y}(\mathcal{Y})\,.$$

The first term generates the action associated with the chiral superfield $\mathcal{X}$. It is self-dual [6]. The term

$$\mathcal{I}_\mathcal{R}(\mathcal{U}) = \int d^8\mathcal{Z}\,\mathcal{L}_\mathcal{R}(\mathcal{U}) + \text{c.c.}\,,$$

$$\mathcal{L}_\mathcal{R} = \frac{1}{2}\bar{D}^4\sum_{n=3}^{\infty}(-1)^n\frac{1}{(n!)^2}\mathcal{U}^n\Box^{n-2}\bar{\mathcal{U}}^n$$

is the only structure capable to reproduce the highest-derivative $\mathcal{R}$ contributions in the $\mathcal{W}$ representation. At last, $\mathcal{I}_\mathcal{Y}(\mathcal{U})$ stands for possible further corrections.

The term $\mathcal{I}_\mathcal{X}(\mathcal{U})$ should reproduce the action

$$S_\mathcal{X}(\mathcal{W}) \;=\; \frac{1}{4}\int d^8\mathcal{Z}\mathcal{X}(\mathcal{W}) + \frac{1}{4}\int d^8\bar{\mathcal{Z}}\bar{\mathcal{X}}(\mathcal{W})\,.$$

After some work with introducing auxiliary superfields, this action can be equivalently represented as

$$\begin{aligned}
&\mathcal{S}_\mathcal{X}(\mathcal{W},\mathcal{U},N) = \mathcal{S}_b(\mathcal{W},\mathcal{U}) + \mathcal{I}_\mathcal{X}(\mathcal{U}^2,N)\,,\\
&\mathcal{I}_\mathcal{X}(\mathcal{U}^2,N) = \frac{1}{4}\int d^{12}Z\Big\{\mathcal{U}^2\bar{N} + \bar{\mathcal{U}}^2N - \frac{N\bar{N}}{1-\frac{1}{4}n\bar{n}}\Big\},\\
&n = \bar{D}^4\bar{N}\,.
\end{aligned}$$

This interaction is $U(1)$ invariant, provided that $\delta_\omega N = -2i\omega N$. Thus indeed $S_\mathcal{X}(\mathcal{W})$ corresponds to self-dual $\mathcal{N}=2$ system. This action is reproduced after eliminating both superfields $\mathcal{U}$ and N from $\mathcal{S}_\mathcal{X}(\mathcal{W},\mathcal{U},N)$. On the other hand, eliminating N,

$$N - (1-\frac{1}{4}n\bar{n})\mathcal{U}^2 + \frac{1}{4}(1-\frac{1}{4}n\bar{n})\bar{D}^4\left[\frac{N\bar{N}\bar{n}}{(1-\frac{1}{4}n\bar{n})^2}\right] = 0\,,$$

we obtain the sought $\mathcal{U}$ interaction

$$\mathcal{I}_\mathcal{X}(\mathcal{U}^2) = \mathcal{I}_\mathcal{X}(\mathcal{U}^2, N(\mathcal{U}^2)) = \sum_{n=1}^{\infty}\int d^{12}Z\mathcal{L}_\mathcal{X}^{(4n)}(\mathcal{U},\bar{\mathcal{U}})\,.$$

A few lowest recursive terms are

$$\begin{aligned}
&\mathcal{L}_\mathcal{X}^{(4)} = \frac{1}{4}\mathcal{U}^2\bar{\mathcal{U}}^2, \quad \mathcal{L}_\mathcal{X}^{(8)} = -\frac{1}{16}\mathcal{U}^2\bar{\mathcal{U}}^2\,A,\\
&\mathcal{L}_\mathcal{X}^{(12)} = \frac{1}{64}\mathcal{U}^2\bar{\mathcal{U}}^2\left(B\bar{B} + B^2 + \bar{B}^2\right),\\
&A := (D^4\mathcal{U}^2)(\bar{D}^4\bar{\mathcal{U}}^2),\ B := \bar{D}^4D^4(\bar{\mathcal{U}}^2\mathcal{U}^2).
\end{aligned}$$

They are capable to restore the original action $S_\mathcal{X}(\mathcal{W})$ up to the 18th order upon eliminating $\mathcal{U},\bar{\mathcal{U}}$[1]. The fact that all $\mathcal{U}$ terms are expressed through the same superfields $A, B, \bar{B}$ makes it probable that the whole $\mathcal{I}_\mathcal{X}(\mathcal{U})$ can be written as a sum of the well defined terms related by some general recurrence formula.

6 $(\mathcal{U},\mathcal{W})$ form of the $\mathcal{N}=2$ BI action up to 10th order

Our aim is to present the auxiliary interaction $\mathcal{I}_{BI}(\mathcal{U})$ which reproduces the $(\mathcal{W},\bar{\mathcal{W}})$ form of the BI action up to the 10th order, i.e. the sum of four terms

$$\hat{I}_{BI} = I_{BI}^{(4)} + I_{BI}^{(6)} + I_{BI}^{(8)} + I_{BI}^{(10)}\,.$$

The corresponding $(\mathcal{U},\mathcal{W})$ BI action is [12]

$$\hat{\mathcal{S}}_{BI} = \mathcal{S}_b + \hat{\mathcal{I}}_\mathcal{X} + \hat{\mathcal{I}}_\mathcal{R} + \hat{\mathcal{I}}_\mathcal{Y}\,,$$

$$\begin{aligned}
&\hat{\mathcal{I}}_\mathcal{X}(\mathcal{U}) = \frac{1}{4}\int d^{12}Z\,\mathcal{U}^2\bar{\mathcal{U}}^2\left[1 - \frac{1}{4}(D^4\mathcal{U}^2)(\bar{D}^4\bar{\mathcal{U}}^2)\right],\\
&\hat{\mathcal{I}}_\mathcal{R}(\mathcal{U}) = \frac{1}{8}\int d^{12}Z\Big(-\frac{2}{9}\mathcal{U}^3\Box\bar{\mathcal{U}}^3 + \frac{1}{72}\mathcal{U}^4\Box^2\bar{\mathcal{U}}^4\\
&\qquad - \frac{1}{1800}\mathcal{U}^5\Box^3\bar{\mathcal{U}}^5\Big),\\
&\hat{\mathcal{I}}_\mathcal{Y}(\mathcal{U}) = \frac{1}{72}\int d^{12}Z\,\Big[(\mathcal{U}^3\bar{\mathcal{U}}^2D^4\mathcal{U}^2\Box\bar{D}^4\bar{\mathcal{U}}^3\\
&+\mathcal{U}^2\bar{\mathcal{U}}^3\bar{D}^4\bar{\mathcal{U}}^2\Box D^4\mathcal{U}^3) + \frac{1}{2}\mathcal{U}^3\bar{D}^4\bar{\mathcal{U}}^2\Box(\bar{\mathcal{U}}^3D^4\mathcal{U}^2)\Big].
\end{aligned}$$

We observe that the first contributions from $\hat{\mathcal{I}}_\mathcal{Y}$ appear only in the 10th order. All these $\mathcal{U}$ terms are manifestly $U(1)$ invariant, so the $\mathcal{N}=4/\mathcal{N}=2$ BI action is self-dual up to the 10th order. The number of terms in this interaction is much smaller compared with the standard $\mathcal{W}$ representation. This gives a hope that the $\mathcal{U},\mathcal{W}$ representation will allow one to sum up all possible terms and give a general proof of the self-duality of the $\mathcal{N}=4/\mathcal{N}=2$ BI action.

[1] In [18] it was restored up to the 14th order by directly solving eq. (2).

7 Summary and outlook

All the duality invariant systems of nonlinear electrodynamics and its $\mathcal{N}=1,2$ superextensions admit an off-shell formulation with the auxiliary bispinor fields or their superfield counterparts. This formulation reduces the self-duality constraints to the condition of $U(1)$ invariance of the auxiliary interaction.

This universal method was applied for analysis of the self-duality properties of the $\mathcal{N}=2$ BI action with the nonlinearly realized $\mathcal{N}=4$ supersymmetry. Its self-duality was proved up to the 10th order in the superfield strengths by constructing the new $(\mathcal{U},\mathcal{W})$ representation of this action to the same order. The conjecture about the general structure of the $(\mathcal{U},\mathcal{W})$ form of the BI action was put forward.

The new closed auxiliary-superfield representation was found for the action $S_{\mathcal{X}}(\mathcal{W})$ as a necessary constituent of the full $\mathcal{N}=4/\mathcal{N}=2$ BI action.

Some further lines of study:

(a) It would be interesting to inquire whether the 10th order-truncated $(\mathcal{U},\mathcal{W})$ BI action can be somehow promoted to all orders in the auxiliary superfields $\mathcal{U},\bar{\mathcal{U}}$, thus providing the $(\mathcal{U},\mathcal{W})$ form of the complete $\mathcal{N}=4/\mathcal{N}=2$ BI action.

(b) The closely related problem is to understand how the hidden spontaneously broken $\mathcal{N}=4$ supersymmetry is realized in the $(\mathcal{U},\mathcal{W})$ formulation.

(c) It is tempting to find some general framework for the "irregular" $\mathcal{Y}$ terms of the $\mathcal{N}=2$ BI action. Perhaps, they could be understood as a recursive solution of some nonlinear superfield equation.

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Е. А. Иванов, Б. М. Зупник

ВСПОМОГАТЕЛЬНЫЕ СУПЕРПОЛЯ В $\mathcal{N} = 2$ ТЕОРИИ БОРНА-ИНФЕЛЬДА

Некоторое время назад в работах авторов был развит новый подход к описанию само-дуальной электродинамики и ее суперрасширений на основе введения вспомогательных (супер)полей, обеспечивающих линейность преобразований дуальности. В данном докладе этот универсальный метод применяется для анализа свойств само-дуальности суперполевого действия $\mathcal{N} = 2$ теории Борна-Инфельда (Б.-И.) с нелинейно реализованной “скрытой” $\mathcal{N} = 4$ суперсимметрией. Самодуальность этого действия доказана до 10-го порядка по суперполевым напряженностям. Высказана гипотеза об общей структуре $\mathcal{N} = 2$ Б.-И. действия, модифицированного киральными вспомогательными суперполями.

Ключевые слова: *суперсимметрия, дуальность, суперполе.*

Иванов Е. А., доктор физико-математических наук, профессор.
Объединенный институт ядерных исследований, Лаборатория теоретической физики им. Н. Н. Боголюбова.
Ул. Жолио-Кюри, 08, 141980 Дубна, Московская область, Россия.
E-mail: eivanov@theor.jinr.ru

Зупник Б. М., доктор физико-математических наук, профессор.
Объединенный институт ядерных исследований, Лаборатория теоретической физики им. Н. Н. Боголюбова.
Ул. Жолио-Кюри, 08, 141980 Дубна, Московская область, Россия.
E-mail: zupnik@theor.jinr.ru