UDC 530.1; 539.1

GRAVITATIONAL COLLISION OF PARTICLES WITH DOMAIN WALLS AND RELATIVISTIC POTENTIALS

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We consider collision of a point particle with an infinitely thin planar domain wall within the linear and post-linear approximations of Einstein classical gravity in Minkowski space-time of arbitrary dimension. Both colliding objects are treated dynamically and the branon excitation of the domain wall is taken into account. The energy balance in this process is non-trivial since the interaction force does not fall with distance and the particle and the domain wall are never free. We show that contribution of the gravitational stresses effectively localizes on the particle and the wall world-volumes and gives rise to the relativistic potential energies of each object in the gravitational field of the partner. The contribution of the branons to the energy of the domain wall in the lowest order in gravitational constant is shown to be zero.

Keywords: *gravitation, branes, domain walls, conservation laws.*

1 Introduction

Existence of asymptotically free states is crucial in the standard theory of particle collisions, both classical and quantum. For this to be valid, the interaction force between the colliding objects must fall down with the distance sufficiently fast. Meanwhile, in various physical systems, like two quarks mediated by the gluon string, this is not so and the question arises, whether one can sensibly define the notion of the potential energy in the relativistic two-body problem. To this aim we consider here a model problem with non-decaying interaction force, namely, collision of the point particle with the Nambu-Goto brane immersed into space-time with the codimension one. Such a problem may have physical applications in cosmology [1–3], in particular, perforation of the domain walls by black holes was suggested as novel mechanism of domain walls destruction in the Early Universe [4–6]. It may be of interest also in the context of the Rundall-Sundrum scenario [7–10], in studying the brane – black hole composites [11, 12], black hole escape from branes [13–18], in dynamical description of branes in supergravity/string theory [19, 20]. Recently we have considered a simpler problem of interaction of the point particle with the domain wall in the linearized gravity and have shown [21, 22] that the perforation of the domain wall by the particle can be well described in terms of distributions. Here we will discuss some peculiar features of the energy balance in this collision.

If the static domain wall gravity is viewed as fixed background, the particle moves along the geodesic line, and the total energy is defined contracting the tangent vector with the time-translation Killing vector. This total energy contains the potential energy of the particle in the static gravitational field. When we consider the interacting two-body system mediated by gravity, the notion of the potential energy seems to fail, since there is no more the necessary Killing field and gravity enters as the third participant possessing an infinite degrees of freedom. In other words, in order to establish the energy conservation low one has to include contribution of the gravitational stresses as separate quantity. These stresses generically are non-local, so it seems impossible to describe the momentum balance as usual in terms of momenta associated with the colliding objects only. We will show, however, that treating the particle – domain wall problem perturbatively, expanding dynamical variables in terms of the gravitational coupling, one observes, that in the leading order the stresses effectively localize at the particle and wall world-volumes, leading to possibility to define the potential energy of each object in the gravitational field of the partner in relativistic way.

It is worth noting that gravitational interaction of branes is an essentially relativistic problem even if their relative velocity is small, since the brane tension, causing gravitational repulsion, contributes to interaction on equal footing with the energy density. The net effect of gravitational interaction of two branes therefore varies with dimensionality of the world-volumes and codimension of their embedding into space-time. It is repulsive for codimension one (domain walls) [23, 24], locally vanishes for codimension two and attractive in other cases. Another new feature due to the extended nature of branes is possibility of their free oscillations which may accompany the generic collision process. While two point particles under collision just change their momenta, but remain in the same intrinsic state, the brane will get excited and will not remain in the initial state even asymptotically. Fortunately, within the linearized gravity the piercing collision of point particle with the domain wall may be treated analytically. In fact, the degree of singularity of the linearized gravitational field at the location of the brane essentially depends on its codimension: the metric diverges as a negative power of the distance for codimension greater than two, as logarithm for codimension two, but it remains finite in the case of the domain wall. Therefore, though generically gravitational collision of two infinitely thin branes is a singular problem, the collision particle – domain wall turns out to be tractable.

2 Linearized gravitational interaction of static branes

For more generality we start with an arbitrary p-brane propagating in $D-$dimensional curved space-time (the bulk). We denote the bulk metric as g_{MN}, $M,N=0,1,2,...,D-1$, the signature is $+--\ldots$, and define the brane world-volume $\mathcal{V}_{p+1}$ by the embedding equations $x^M = X^M(\sigma^\mu)$, parameterized by arbitrary coordinates σ^μ, $(\mu=0,...,p)$ on $\mathcal{V}_{p+1}$. The corresponding action in the Polyakov form is a functional of $X^M(\sigma^\mu)$ and the metric $\gamma_{\mu\nu}$ on $\mathcal{V}_{p+1}$:

$$S_p = -\frac{\mu}{2}\int\Big[X^M_\mu X^N_\nu g_{MN}\gamma^{\mu\nu}-(p-1)\Big] \times \sqrt{|\gamma|}\, d^{p+1}\sigma\,. \tag{1}$$

Here μ is the brane tension, $X^M_\mu = \partial X^M/\partial\sigma^\mu$ are the tangent vectors and $\gamma^{\mu\nu}$ is the inverse metric on $\mathcal{V}_{p+1}$, $\gamma=\det\gamma_{\mu\nu}$. Variation of (1) with respect to X^M gives the brane equation of motion

$$\partial_\mu\left(X^N_\nu g_{MN}\gamma^{\mu\nu}\sqrt{|\gamma|}\right) = \frac{1}{2}\, g_{NP,M} X^N_\mu X^P_\nu \gamma^{\mu\nu}\sqrt{|\gamma|}\,, \tag{2}$$

which is covariant with respect to both the space-time and the world-volume diffeomorphisms. Variation over $\gamma^{\mu\nu}$ gives the constraint equation

$$\left(X^M_\mu X^N_\nu - \frac{1}{2}\gamma_{\mu\nu}\gamma^{\lambda\tau}X^M_\lambda X^N_\tau\right) \times g_{MN} + \frac{p-1}{2}\gamma_{\mu\nu} = 0\,, \tag{3}$$

whose solution defines $\gamma_{\mu\nu}$ as the induced metric on $\mathcal{V}_{p+1}$:

$$\gamma_{\mu\nu} = X^M_\mu X^N_\nu g_{MN}\big|_{x=X}\,. \tag{4}$$

Adding to (1) the Einstein action

$$S_E = -\frac{1}{\varkappa_D^2}\int R_D\sqrt{|g|}\, d^D x\,, \tag{5}$$

where $\varkappa_D^2 \equiv 16\pi G_D$, and varying S_p+S_E with respect to the space-time metric g_{MN} we obtain Einstein equations

$$R_{MN} - \frac{1}{2}g_{MN}R = \frac{\varkappa_D^2}{2}T_{MN} \tag{6}$$

with the source term

$$T^{MN} = \mu\int X^M_\mu X^N_\nu \gamma^{\mu\nu}\,\frac{\delta^D\big(x-X(\sigma)\big)}{\sqrt{|g|}} \times\sqrt{|\gamma|}\, d^{D-1}\sigma\,. \tag{7}$$

Consider static solutions of the system (2, 3, 6) for planar branes described by the linear embedding functions

$$X^M = \Sigma^M_\mu\,\sigma^\mu \tag{8}$$

with constant system of linearly independent vectors Σ^M_μ. In what follows we will mostly use the internal coordinates σ^μ coinciding with the bulk coordinates x^μ, so that $\Sigma^M_\mu=\delta^M_\mu$, but in some cases σ^μ will be still used to avoid confusion.

Consistency of the above coupled system involving singular delta sources depends on codimension $\tilde{d} = D-p-1$ of the embedding of the brane world-volume into the bulk. Strictly speaking, for $\tilde{d}\geqslant 3$ the use of distributions in the full non-linear gravity is not legitimate, though the presence of delta-sources in classical p-brane solutions in supergravities sometimes still can be detected [19]. The case $\tilde{d}=2$ as it is well-known from an example of the cosmic string in four-dimensional space-time [3], is exceptional: in this case the cylindrically symmetric field configurations exist for which Einstein equations reduce to two-dimensional Laplace equation with the delta-source leading to static locally flat conical transverse space. The case $\tilde{d}=1$ (domain wall) is legitimate too, but has a peculiar feature: exact solutions of Einstein equations are non-static [1,2,25,26] (the static solutions exist if one adds a negative cosmological constant of some special value [27,28]). Here we will restrict to the linearized theory expanding the metric as

$$g_{MN} = \eta_{MN} + \varkappa_D h_{MN}\,. \tag{9}$$

All subsequent operations with indices of h_{MN} will be performed with respect to the Minkowski metric, e.g., $g^{MN}\approx\eta^{MN}-\varkappa_D h^{MN}$. In the Lorentz gauge

$$\partial_N h^{MN} = \frac{1}{2}\,\partial^M h, \quad h = h^M_M\,, \tag{10}$$

the linearized Einstein equations reduce to

$$\Box\, h_{MN} = -\,\varkappa_D \left(T_{MN} - \frac{1}{D-2}\, T\, \eta_{MN} \right),$$
$$T = T^M_M\,, \tag{11}$$

with $\Box \equiv \partial_M \partial^M$. Consider again an arbitrary plane unexcited p-brane described by the embedding functions (8), choose the coordinates on $\mathcal{V}_{p+1}$ as $\sigma^0 = x^0 \equiv t\,, \sigma^i = x^i\,, i = 1, \ldots, p$ and denote the coordinate transverse to the brane as $z^n\,, n = 1, \ldots, \tilde{d}$. Then the brane stress-tensor T_{MN} will have non-zero only the components $\mu\,, \nu = 0, i$ given by

$$T_{\mu\nu} = \mu \eta_{\mu\nu} \delta^{\tilde{d}}(\mathbf{z})\,, \tag{12}$$

where $\eta_{\mu\nu}$ is Minkowski metric on the brane (and unity in the case $p = 0$), leading to

$$ds^2 = \left(1 + 4k(\tilde{d}-2)\Phi_{\tilde{d}}\right)\, \eta_{\mu\nu} dx^\mu dx^\nu$$
$$-\left(1 - 4k(p+1)\Phi_{\tilde{d}}\right) dz_k{}^2\,, \tag{13}$$

where

$$k = \frac{\mu \kappa_D}{2(D-2)}\,. \tag{14}$$

Here $\Phi_{\tilde{d}}$ is the solution of the transverse Poisson equation

$$\Delta_{\tilde{d}} \Phi_{\tilde{d}}(\mathbf{z}) = \delta^{\tilde{d}}(\mathbf{z})\,, \tag{15}$$

which reads explicitly

$$\Phi_{\tilde{d}}(\mathbf{z}) = \begin{cases} |z|/2, & \tilde{d} = 1 \\ (2\pi)^{-1} \ln|\mathbf{z}|, & \tilde{d} = 2 \\ -(\tilde{d}-2)^{-1} \Omega^{-1}_{\tilde{d}-1} |\mathbf{z}|^{(2-\tilde{d})}, & \tilde{d} \geqslant 3 \end{cases}\,, \tag{16}$$

where $\Omega_{\tilde{d}-1}$ is the volume of the $\tilde{d}-1-$dimensional unit sphere in $\tilde{d}-$dimensional euclidean space: $\Omega_{\tilde{d}-1} = 2\pi^{\tilde{d}}/\Gamma(\tilde{d})$.

Consider now the second $\bar{p}$-brane, (with $\bar{p} \leqslant p$) sitting parallel to the first at some finite distance. We split the space-time coordinates as $x^M = (t, \mathbf{x}, \mathbf{y}, \mathbf{z})$, where $\mathbf{x} \in \mathbb{R}^{\bar{p}}$, $\mathbf{y} \in \mathbb{R}^{p-\bar{p}}$, $\mathbf{z} \in \mathbb{R}^{\tilde{d}}$. Let the first p-brane occupy the sector $x^A = (t, \mathbf{x}, \mathbf{y})$ and located at $\mathbf{z} = \mathbf{0}$ in the overall transverse space, while the second extends in the sector $x^a = (t, \mathbf{x})$ at the position $\mathbf{z} = \bar{\mathbf{z}}$. To extract the effective interaction potential we start with the action

$$S_{\rm int} = -\frac{\varkappa_D}{2} \int h_{MN} \bar{T}^{MN} d^D x\,, \tag{17}$$

where h_{MN} is the linearized metric of the p-brane and $\bar{T}^{MN}$ is the stress-tensor of the $\bar{p}$ brane (or vice-versa) and insert as h_{MN} the solution of the corresponding d'Alembert equation. Using the scalar Green's function of the d'Alembert equation

$$\Box_D G(x, x') = \delta^D(x - x')\,, \tag{18}$$

we obtain the bilinear form of the stress-energy tensors

$$S_{\rm int} = -\frac{\varkappa_D^2}{2} \int G(x,x') \big(T^{MN}(x)\, \bar{T}_{MN}(x')$$
$$-\, \frac{1}{D-2}\, T(x)\, \bar{T}(x') \Big)\, d^D x\, d^D x'\,. \tag{19}$$

Substituting here the corresponding quantities for both branes at rest, we find that the integral (19) reduces to that over time and the spatial coordinates $\mathbf{x}$ of the $\bar{p}$-brane, allowing for introduction of the effective potential $U_{\rm eff}$ per unit volume of the smaller brane:

$$S_{\rm int} = -\int U_{\rm eff}(\bar{\mathbf{z}})\, dt d\mathbf{x}, \tag{20}$$

which explicitly reads

$$U_{\rm eff} = \frac{\varkappa_D^2 \mu \bar{\mu} (\bar{p}+1)(\tilde{d}-2)}{2(D-2)}\, \Phi_{\tilde{d}}(\bar{\mathbf{z}})\,. \tag{21}$$

Inserting here the transverse potential (16) we finally obtain

$$U_{\rm eff} = -\,\frac{\varkappa_D^2 \mu \bar{\mu} (\bar{p}+1)}{2(D-2)} \begin{cases} \bar{z}/2, & \tilde{d} = 1 \\ 0, & \tilde{d} = 2 \\ \Omega^{-1}_{\tilde{d}}\, |\bar{\mathbf{z}}|^{(2-\tilde{d})}, & \tilde{d} \geqslant 3 \end{cases}\,. \tag{22}$$

Thus, the character of interaction depends on codimension of the embedding of the bigger p-brane into the bulk: the potential is repulsive for $\tilde{d} = 1$, there is no force for $\tilde{d} = 2$ and it is attractive for $\tilde{d} > 2$. This simple picture, however, holds only in the static case. As we will see, situation becomes more sophisticated when branes are in motion. Somewhat unexpectedly, however, we will still be able to introduce the notion of the relativistic potential energy as well.

3 Interaction of domain wall with moving particle

Now we pass to the system of the gravitationally interacting domain wall $p = D-2$ and a moving point particle ($\bar{p} = 0$), adding to the sum $S_p + S_E$ the particle action

$$S_0 = -\frac{1}{2} \int \left(e\, g_{MN} \dot{z}^M \dot{z}^N + \frac{m^2}{e} \right) d\tau\,, \tag{23}$$

where $e(\tau)$ is the ein-bein of the particle world-line and dots denote derivatives with respect to τ. Varying S_0 with respect to $z^M(\tau)$ and $e(\tau)$ one obtains the geodesic equation in arbitrary parametrization

$$\frac{d}{d\tau} \left(e \dot{z}^N g_{MN} \right) = \frac{e}{2}\, g_{NP,M}\, \dot{z}^N \dot{z}^P\,, \tag{24}$$

and the constraint equation

$$e^2 g_{MN}\dot{z}^M\dot{z}^N = m^2 \,. \tag{25}$$

The corresponding energy-momentum tensor reads

$$\bar{T}^{MN} = \int \frac{e\dot{z}^M\dot{z}^N\delta^D\big(x - z(\tau)\big)}{\sqrt{|g|}}\, d\tau \,. \tag{26}$$

Both the domain wall and the point particle will be treated on equal footing in the framework of the linearized gravity on Minkowski background. So we expand the total metric similarly to (9) adding to the metric perturbation h_{MN}, which will be still associated with the brane, the metric perturbation $\bar{h}_{MN}$ due to the particle (preserving the notation of sec. 2D):

$$g_{MN} = \eta_{MN} + \varkappa_D\left(h_{MN} + \bar{h}_{MN}\right) \,. \tag{27}$$

The Lorentz gauge condition (10) will be assumed for both components independently.

To treat the interaction problem in terms of formal expansions in the gravitational coupling we have to substitute $X^M + \delta X^M$, $z^M + \delta z^M$, $\gamma_{\mu\nu} + \delta\gamma_{\mu\nu}$ and $e + \delta e$ and treat the deviations iteratively. Here $X^M, z^M, \gamma_{\mu\nu}, e$ are assumed to describe *free* motion of both objects in Minkowski space-time, while δX^M, $\delta\gamma_{\mu\nu}$ are the perturbations of the brane variables due to the gravitational field of the particle $\bar{h}_{MN}$, and $\delta z^M, \delta e$ are the perturbations of the particle variables due to the gravitational field of the brane h_{MN} (and we omit singular self-interaction terms). The unperturbed domain wall is thus described by the embedding functions (8) and the corresponding induced metric is $\gamma_{\mu\nu} = \eta_{\mu\nu}$. The unperturbed stress tensor is

$$T_0^{MN} = \mu\Sigma^M_\mu\Sigma^N_\nu\eta^{\mu\nu}\delta(z) \,, \tag{28}$$

and the corresponding metric deviation can be read off from Eq. (11):

$$h_{MN} = \frac{\varkappa_D\mu}{2}\left(\Xi_{MN} - \frac{D-1}{D-2}\,\eta_{MN}\right)|z| = \frac{\varkappa_D\mu|z|}{2(D-2)}\,\mathrm{diag}\,(-1, 1, ..., 1, D-1) \,, \tag{29}$$

where $\Xi_{MN} \equiv \Sigma^\mu_M\Sigma^\nu_N\eta_{\mu\nu}$. From here one derives the validity condition for our iteration scheme. Smallness of $\varkappa_D h_{MN}$ implies

$$kz \ll 1 \,, \tag{30}$$

where k is given by (14). This parameter has meaning of the inverse curvature radius of the bulk, so our approximation is valid at the distance from the brane small with respect to this curvature radius.

Assuming the particle to move orthogonally to the wall, we parameterize the unperturbed world-line as

$$z^M(\tau) = u^M\tau \,, \quad u^M = \gamma\,(1, 0, ..., 0, v) \,, \quad \gamma = 1/\sqrt{1-v^2} \,. \tag{31}$$

This trajectory intersects the domain wall at the moment of proper time $\tau = 0$, the corresponding coordinate time also being zero, $t = 0$. Using (29) and (31) in the Eqs. (25) and (24) one obtains for δe and δz^M the system of equations

$$\delta e = -\frac{m}{2}\left(\varkappa_D\, h_{MN}u^M u^N + 2\,\eta_{MN}\,u^M\,\delta\dot{z}^N\right) \tag{32}$$

and

$$\frac{d}{d\tau}\left(\delta e\, u_M + m\,\delta\dot{z}_M\right) = -\varkappa_D m\left(h_{PM,Q} - \frac{1}{2}\,h_{PQ,M}\right)u^P u^Q \,, \tag{33}$$

which upon the elimination of δe gives for δz^M:

$$\bar{\Pi}^{MN}\delta\ddot{z}_N = -\varkappa_D\,\bar{\Pi}^{MN} \times\left(h_{PN,Q} - \frac{1}{2}\,h_{PQ,N}\right)u^P u^Q \,, \tag{34}$$

where

$$\bar{\Pi}^{MN} = \eta^{MN} - u^M u^N \tag{35}$$

is a projector onto the subspace orthogonal to u^M. Let us now choose the overall gauge condition

$$g_{MN}\dot{z}^M\dot{z}^N = 1 \,, \tag{36}$$

with z^M including the perturbation. In view of the zero order parametrization assumed (31), this amounts to the condition $\delta e = 0$, i.e.,

$$\frac{m}{2}\left(\varkappa_D\, h_{MN}u^M u^N + 2\,\eta_{MN}\,u^M\,\delta\dot{z}^N\right) = 0 \,. \tag{37}$$

Going back to eq. (33) one has thereby

$$\delta\ddot{z}_M = -\varkappa_D\left(h_{PM,Q} - \frac{1}{2}\,h_{PQ,M}\right)u^P u^Q \,, \tag{38}$$

or, in components,

$$\delta\ddot{z}^{\,0} = 2kv\,\gamma^2\,\mathrm{sgn}(\tau) \,, \quad \delta\ddot{z} \equiv \ddot{z}^{D-1} = k\left(D\gamma^2 v^2 + 1\right)\mathrm{sgn}(\tau) \,, \tag{39}$$

so, the force is repulsive as expected.

Integrating (39) twice with initial conditions $\delta z^M(0) = 0$, $\delta\dot{z}^M(0) = 0$, one has

$$\delta z^0 = kv\tau^2\,\gamma^2\,\mathrm{sgn}(\tau) \,, \tag{40}$$

$$\delta z = \frac{1}{2}\,k\tau^2\left(D\gamma^2 v^2 + 1\right)\mathrm{sgn}(\tau) \,. \tag{41}$$

Substituting (40) into (32) one can check that the gauge condition (37) holds.

According to (40), the perturbation of the particle D-velocity $\delta\dot{z}^0$ has no discontinuity at the location of the brane $z = 0$, but its derivative has. The discontinuity of acceleration has simple physical meaning: the repulsive force changes its sign at the moment of perforation.

4 Excitation of branons

Denoting the stress-tensor and the gravitational field of the particle by bar, we will have:

$$\Box_D \bar{h}_{MN} = -\varkappa_D \left(\bar{T}_{MN} - \frac{1}{D-2} \bar{T} \eta_{MN} \right), \qquad (42)$$

where the source term

$$\bar{T}_0^{MN}(x) = \frac{m}{\gamma} u^M u^N \delta(z - vt)\, \delta^{D-2}(\mathbf{r}), \qquad (43)$$

has only $t, z-$ components non-zero. The solution (for $D \geqslant 4$) reads:

$$\bar{h}_{MN}(x) = -\frac{\varkappa_D m \Gamma\left(\frac{D-3}{2}\right)}{4\pi^{\frac{D-1}{2}}} \left(u_M u_N - \frac{1}{D-2} \eta_{MN} \right) \times \frac{1}{\left[\gamma^2 (z - vt)^2 + r^2\right]^{\frac{D-3}{2}}}, \qquad (44)$$

where $r = \sqrt{\delta_{ij} x^i x^j}$ is the radial distance on the wall from the perforation point. This is just the Lorentz-contracted D-dimensional Newton field of the uniformly moving particle.

Perturbations of the Nambu-Goto branes in the external gravitational field were expensively studied in the literature, see e.g. [29, 30]. On the Minkowski background the derivation is particularly simple. First, from Eq. (4) we find the perturbation of the induced metric

$$\delta\gamma_{\mu\nu} = 2\, \delta^M_{(\mu}\, \delta X^N_{\nu)} \eta_{MN} + \varkappa_D \bar{h}_{MN} \Sigma^M_\mu \Sigma^N_\nu, \qquad (45)$$

where brackets denote symmetrization over indices with the factor 1/2. Then linearizing the rest of the Eq. (2), after some rearrangements one obtains the following equation for deformation of the wall:

$$\Pi_{MN}\, \Box_{D-1}\, \delta X^N = \Pi_{MN}\, J^N, \quad \Pi^{MN} \equiv \eta^{MN} - \Sigma^M_\mu \Sigma^N_\nu \eta^{\mu\nu}, \qquad (46)$$

where $\Box_{D-1} \equiv \partial_\mu \partial^\mu$ and Π^{MN} is the projector onto the (one-dimensional) subspace orthogonal to $\mathcal{V}_{D-1}$. The source term in (46) reads:

$$J^N = \varkappa_D\, \Sigma^\mu_P\, \Sigma^\nu_Q\, \eta_{\mu\nu} \left(\frac{1}{2} \bar{h}^{PQ,N} - \bar{h}^{NP,Q} \right)\Bigg|_{z=0}. \qquad (47)$$

Using the aligned coordinates on the brane $\sigma^\mu = (t, \mathbf{r})$, we will have $\delta^M_\mu = \Sigma^M_\mu$, so the projector Π^{MN} reduces the system (46) to a single equation for $M = z$ component. Thus only the z-components of δX^M and J^M are physical. Generically, the transverse coordinates of the branes can be viewed as Nambu-Goldstone bosons (branons) which appear as a result of spontaneous breaking of the translational symmetry [31]. These are coupled to gravity and matter on the brane in the brane-world models via the induced metric (for a recent discussion see [32, 33]). In our case of co-dimension one there is only one such branon. The remaining components of the perturbation δX^M can be removed by suitable transformation of the coordinates on the world-volume, so $\delta X^\mu = 0$ is nothing but the choice of gauge. Note that in this gauge the perturbation of the induced metric $\delta\gamma_{\mu\nu}$ does not vanish, as it was for the perturbation of the particle ein-bein e.

Denoting the physical component as $\Phi(\sigma^\mu) \equiv \delta X^z$ we obtain the branon $(D-1)$-dimensional wave equation:

$$\Box_{D-1} \Phi(\sigma^\mu) = J(\sigma^\mu), \qquad (48)$$

with the source term $J \equiv J^z$. Substituting (44) into the eq. (47) we obtain the source term for the branon:

$$J(\sigma) = -\varkappa_D \left[\frac{1}{2} \eta_{\mu\nu} \bar{h}^{\,\mu\nu,z} - \bar{h}^{\,z\,0,0} \right]_{z=0} = -\frac{\lambda v t}{\left[\gamma^2 v^2 t^2 + r^2\right]^{\frac{D-1}{2}}}, \qquad (49)$$

where

$$\lambda = \frac{\varkappa_D^2 m \gamma^2 \Gamma\left(\frac{D-1}{2}\right)}{4\pi^{\frac{D-1}{2}}} \left(\gamma^2 v^2 + \frac{1}{D-2} \right). \qquad (50)$$

The retarded solution of the branon wave equation (48) reads

$$\Phi(x^\mu) = -\frac{1}{(2\pi)^{D-1}} \int \frac{\mathrm{e}^{-ikx}}{\omega^2 - k^2 + 2i\epsilon\omega} \times J(k^\mu)\, d^{D-1}k, \qquad (51)$$

where $J(k^\mu)$ is the Fourier-transform of the source :

$$J(k^\mu) = -\frac{2\pi^{\frac{D-1}{2}} \lambda}{\gamma \Gamma\left(\frac{D-1}{2}\right)} \frac{i\omega}{\gamma^2 v^2 k^2 + \omega^2}. \qquad (52)$$

Expanding the product of two pole factors as

$$\frac{1}{(\gamma^2 v^2 k^2 + \omega^2)(\omega^2 - k^2 + 2i\epsilon\omega)} = \left(\frac{1}{\omega - k + i\epsilon} + \frac{1}{\omega + k + i\epsilon} - \frac{2\omega}{\omega^2 + \gamma^2 v^2 k^2} \right) \frac{1}{2\gamma^2 \omega k^2}, \qquad (53)$$

and integrating over angles we present the solution as the sum

$$\Phi = \Phi_{\mathrm{a}} + \Phi_{\mathrm{b}}\,, \quad \Phi_{\mathrm{a}} \equiv -\Lambda\,\mathrm{sgn}(t) I_{\mathrm{a}}\,,$$
$$\Phi_{\mathrm{b}} \equiv 2\,\Lambda\,\theta(t) I_{\mathrm{b}}\,, \quad \Lambda \equiv \frac{\sqrt{\pi}\,\lambda}{2^{\frac{D-2}{2}}\gamma^3\Gamma\left(\frac{D-1}{2}\right)}\,, \tag{54}$$

where the remaining integrals involve Bessel functions:

$$I_{\mathrm{a}}(t,r) = \frac{1}{r^{\frac{D-4}{2}}}\int\limits_0^\infty dk\, J_{\frac{D-4}{2}}(kr)\,k^{\frac{D-6}{2}}\,\mathrm{e}^{-k\gamma v|t|}\,, \tag{55}$$

$$I_{\mathrm{b}}(t,r) = \frac{1}{r^{\frac{D-4}{2}}}\int\limits_0^\infty dk(t,r) J_{\frac{D-4}{2}}(kr)\,k^{\frac{D-6}{2}}\cos kt\,. \tag{56}$$

The first term Φ_{a} is time antisymmetric and present the "action at a distance" interaction of the particle and the domain wall. The second term Φ_{b} is the shock-wave branon starting at the moment of perforation. Details of integration ar given in [22]. The result for odd $D \geqslant 5$ is

$$\Phi_{\mathrm{a}} = -\frac{2\Lambda}{\sqrt{2\pi}}\left(-\frac{1}{r}\frac{\partial}{\partial r}\right)^{\frac{D-5}{2}}\frac{1}{r}\arctan\frac{r}{\gamma v|t|}\,,$$
$$\Phi_{\mathrm{b}} = \sqrt{2\pi}\Lambda\,\theta(t)\left(-\frac{1}{r}\frac{\partial}{\partial r}\right)^{\frac{D-5}{2}}\frac{\theta(r-t)}{r}\,. \tag{57}$$

For even $D \geqslant 6$ one finds [34]

$$I_{\mathrm{a}} = \left(-\frac{1}{r}\frac{\partial}{\partial r}\right)^{\frac{D-6}{2}}\left[\frac{1}{r^2}\left(1 - \frac{\gamma v|t|}{\sqrt{\gamma^2 v^2 t^2 + r^2}}\right)\right], \tag{58}$$

$$I_{\mathrm{b}} = \left(-\frac{1}{r}\frac{\partial}{\partial r}\right)^{\frac{D-6}{2}}\left(\frac{\theta(r-|t|)}{r^2} - \frac{\theta(|t|-r)}{\sqrt{t^2-r^2}(|t|+\sqrt{t^2-r^2})}\right). \tag{59}$$

These expressions were obtained as exact solutions of the branon equations and as such they are valid for all t. But it is important to understand that our perturbation is valid for small enough z, see (30), and since the unperturbed particle world-line is $z = vt$, this amounts to the condition on time

$$t \ll \frac{1}{kv}\,, \tag{60}$$

provided $v \neq 0$; in the static case one has simply the condition (30) on z. Thus the formal expansions in terms of $\varkappa_D$ are convergent only in bounded region of z and t.

5 Energy conservation

We would like to check the energy momentum balance in our collision problem in lowest non-trivial order in $\varkappa_D$. First we have to construct the divergence-free energy-momentum tensor (in Minkowski sense) which in zero order approximation is the sum of (28) and (43). This sum is obviously divergence free.

The first order particle stress tensor is obtained expanding the general expression (26) in $\varkappa_D$:

$$\bar{T}_1^{MN}(x) = \frac{m}{2}\int\Big[4\,\delta\dot{z}^{(M}u^{N)} - \varkappa_D\,u^M u^N \times\Big(h + 2\,\delta z^P\partial_P\Big)\Big]\,\delta^D(x-u\tau)\,d\tau\,, \tag{61}$$

where h is the trace of the first order metric deviation due to the wall (13); the symmetrization over the indices (MN) as well as the anti-symmetrization $[MN]$ below is defined with $1/2$. The delta-function indicates on the localization of the integrand at the non-perturbed particle world-line.

The first order stress-tensor of the wall is obtained substituting the first-order metric deviation (44) due to the particle and the first-order perturbations of the wall world-volume into the Eq. (7):

$$T_1^{MN}(x) = \frac{\mu}{2}\int\Big[4\,\delta_\mu^{(M}\delta X_\nu^{N)}\eta^{\mu\nu} - 2\,\delta_\mu^M\delta_\nu^N \times\Big(\bar{h}^{\mu\nu} + 2\,\eta^{LR}\delta_R^{(\mu}\delta X_L^{\nu)}\Big) + \delta_\mu^M\delta_\nu^N\eta^{\mu\nu}\left(\bar{h}^\lambda_\lambda - \bar{h}\right. \tag{62}$$
$$\left. + \,2\,\delta X_\lambda^L\delta_L^\lambda - 2\,\delta X^L\partial_L\Big)\right]\,\delta^{D-1}(x-\sigma)\,\delta(z)\,d^{D-1}\sigma\,.$$

Again, the delta-functions in the integrand indicate on its localization on the unperturbed wall world-volume.

The sum of (61) and (62) is not divergence free in the Minkowski sense, since in this order the gravitational effective stress tensor obtained as the quadratic term in the expansion of the Einstein tensor in $\varkappa_D$ enters into play:

$$G^{MN} = -\frac{\varkappa_D}{2}\Box\Big(H^{MN} - \frac{1}{2}\eta^{MN}H\Big) - \frac{\varkappa_D^2}{2}\,\mathsf{S}^{MN} + O(H^3)\,, \tag{63}$$

where $H = H_M^M$, $\Box = \eta^{MN}\partial_M\partial_N$ and S^{MN} stands for

$$\mathsf{S}^{MN} = 2\,H^{MP,Q}H^N{}_{[Q,P]} + H_{PQ}\left(H^{MP,NQ} + H^{NP,MQ} - H^{PQ,MN} - H^{MN,PQ}\right)$$
$$- 2\,H_P^{(M}\Box H^{N)P} - \frac{1}{2}\,H^{PQ,M}H_{PQ}{}^{,N} + \frac{1}{2}\,H^{MN}\Box H + \frac{1}{2}\eta^{MN}\Big(2H^{PQ}\Box H_{PQ} - H_{PQ,L}H^{PL,Q} + \frac{3}{2}\,H_{PQ,L}H^{PQ,L}\Big)\,. \tag{64}$$

To exclude the divergent self-action terms (we do not intend to investigate radiation reaction aspects of he problem) one has to substitute here the sum

$$H_{MN} = h_{MN} + \bar{h}_{MN}\,, \tag{65}$$

and keep only the product terms of h_{MN} and $\bar{h}_{MN}$, the resulting quadratic form will be denoted $S^{MN}(h,\bar{h})$. Then is straightforward, though rather tedious, to check the conservation equation

$$\partial_N \tau^{MN} = 0\,, \tag{66}$$

for the current

$$\tau^{MN} = T_1^{MN} + \bar{T}_1^{MN} + S^{MN}(h,\bar{h})\,. \tag{67}$$

Actually only the sum of three terms is invariant under gauge transformations of the linear theory

$$h_{MN} \to h_{MN} + \partial_M \xi_N + \partial_N \xi_M\,, \tag{68}$$

but we can by definition associate with the first two terms the kinetic momenta of the wall and the particle respectively since these quantities are directly localized on them. The third term looks essentially non-local and a priori can be associated neither with the particle, nor with the wall. Our main result is demonstration of the fact that within the perturbation theory it *can* be split into the particle and the wall "potential" terms.

Omitting the trivial zero order contributions, we thus define the kinetic energies of the brane and the particle as

$$\mathcal{E}(t) = \int_{\Sigma_t} T_1^{00} d^{D-1}x\,, \quad \bar{\mathcal{E}}(t) = \int_{\Sigma_t} \bar{T}_1^{00} d^{D-1}x\,, \tag{69}$$

where Σ_t is the $D-1$ space hyper-surface chosen to be orthogonal to the time axis at the moment t, with the measure $d^{D-1}x = dz d^{D-2}\mathbf{r}$. The particle kinetic energy is calculated substituting the wall metric deviation $h_{MN}(\tau)$ given by (29) and the particle world-line deviation $\delta z^M(\tau)$ given by (40) into (61) and integrating with the help of the delta-function:

$$\bar{\mathcal{E}}(t) = 2Dmk\gamma v|t|\,. \tag{70}$$

Similarly, to calculate the wall kinetic energy we substitute the deviation $\delta X^M = \delta_z^M \Phi$ with the branon field Φ given by (54) into the integrand in (69) obtaining

$$T_1^{00} = \frac{\mu}{2}\left[\Big(-2\,\bar{h}_{00} + \bar{h}_{zz}\Big)\,\delta(z) - 2\,\Phi\,\delta'(z)\right]\,. \tag{71}$$

Since Φ is the function of the world-volume coordinates (t,r) only, the term $\Phi\,\delta'(z)$ vanishes upon integration over z, so the brane kinetic energy in the first order in $\varkappa_D$ does not depend on Φ. Substituting into (71) the particle metric deviation (44) one gets

$$T_1^{00} = \frac{\Gamma\left(\frac{D-3}{2}\right)}{2\pi^{\frac{D-1}{2}}}\Big((D-2)\gamma^2 v^2 + (2D-7)\Big)\,mk\chi\,\delta(z)\,,$$
$$\chi \equiv \frac{1}{[\gamma^2(z-vt)^2 + r^2]^{\frac{D-3}{2}}}\,, \tag{72}$$

which leads after integration to:

$$\mathcal{E} = \frac{\Gamma\left(\frac{D-3}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{D-2}{2}\right)}\Big((D-2)\gamma^2 v^2 + (2D-7)\Big)\,mkQ(a)\,,$$
$$a = \gamma v t\,, \tag{73}$$

where $Q(a)$ denotes the integral of χ over r including the volume factor

$$Q(a) = \int_0^\infty \frac{r^{D-3}\,dr}{(a^2+r^2)^{\frac{D-3}{2}}}\,. \tag{74}$$

This integral linearly diverges at the upper limit. This divergence is due to insufficiently fast decay of the particle gravity with the radial distance along the wall. Meanwhile, the resulting dependence of the kinetic energy on time can be trusted only for sufficiently small t satisfying the applicability condition (60) since the metric perturbation is legitimate only for small time intervals around the perforation moment $t=0$. So expanding $Q(t) = Q(0) + t\partial_t Q + \ldots$ we can trust only the linear term. Omitting the (infinite) constant, we are led to the following prescription for the regularized Q:

$$Q_{\text{reg}}(a) = a\frac{dQ}{da} = -(D-3)a^2 \int \frac{r^{D-3}\,dr}{(a^2+r^2)^{\frac{D-1}{2}}}\,. \tag{75}$$

The integral in the expression for Q_{reg} is finite and is evaluated by the substitution $1+(r/a)^2 = 1/y$ leading to the Euler beta-function:

$$Q_{\text{reg}} = -\frac{|a|(D-3)\sqrt{\pi}\,\Gamma\left(\frac{D-2}{2}\right)}{2\,\Gamma\left(\frac{D-1}{2}\right)}\,, \tag{76}$$

so the desired quantity reads:

$$\mathcal{E} = -\gamma v\,\Big((D-2)\gamma^2 v^2 + 2D - 7\Big)\,mk\,|t|\,. \tag{77}$$

The sum of this expression and the particle kinetic energy (70) is not zero. This is not surprising, since we still have to take into account the gravitational energy-momentum tensor $S^{MN}(h,\bar{h})$. Substituting into the quadratic form $S^{00}(h,\bar{h})$ the first order particle and wall metric deviations one finds the following non-zero terms:

1. The products of the first derivatives of both h_{MN} and $\bar{h}_{MN}$ whose resulting z-dependence is $\operatorname{sgn} z\,\partial_z\chi$. Since this function must be further integrated over z, we can replace $\operatorname{sgn} z\partial_z\chi \to -2\chi\delta(z)$, dropping the total derivative which gives a constant. The $\delta(z)$ appearing afterward indicates on the connection of this term with the wall.

2. Terms proportional to the second z-derivatives of $\bar{h}_{MN}$: with z-dependence $|z|\partial_z^2\chi$. These also become localized on the wall after integrating by parts twice.

3. The second derivatives of h_{MN}, which are proportional to $\delta(z)\chi$; these are directly attached to the wall.

4. Boxes of $\bar{h}_{MN}$: $-3\,h_{00}\,\Box\bar{h}_{00} + \frac{1}{2}\,h_{00}\,\Box\bar{h} + h_{PQ}\,\Box\bar{h}^{PQ}$. These are localized on the particle world-line after application of the linearized Einstein equations.

Thus, omitting the total derivatives, we can rewrite the stress term as

$$S^{00} = S_p\,\delta(z - vt) + S_w\,\delta(z)\,, \tag{78}$$

where

$$S_p = -2\left(\,(D+1)\,v^2 + \frac{4}{\gamma^2}\right) mk|z|\delta^{D-2}(\mathbf{r})\,,$$
$$S_w = -\frac{\Gamma\left(\frac{D-3}{2}\right)}{2\pi^{\frac{D-1}{2}}}\left[(D-2)\gamma^2 v^2\right.$$
$$\left. + 2D - 5 - \frac{2(D-3)}{\gamma^2}\right] mk\chi\,. \tag{79}$$

After integration over z we are left with the quantities which can be interpreted as effective potential energies of the wall V and the particle $\bar{V}$ in the gravitational field of the partner:

$$V = \int S_w\, d^{D-2}\mathbf{r}\,, \quad \bar{V} = \int S_p\, d^{D-2}\mathbf{r}\,. \tag{80}$$

The integral for V gives again the divergent quantity (74) which is regularized according to (75) as the corresponding quantity for kinetic energy. Performing the evaluation we get:

$$V = mk\left[(D-2)\gamma^2 v^2 + 2D - 5\right.$$
$$\left. - \frac{2(D-3)}{\gamma^2}\right]\gamma v|t|, \tag{81}$$
$$\bar{V} = -2mk\left(\,(D+1)\,v^2 + \frac{4}{\gamma^2}\right)\gamma v|t|\,. \tag{82}$$

The sum of the kinetic and potential terms therefore reads

$$\bar{\mathcal{E}} + \bar{V} = -(\mathcal{E} + V) = 2mk\gamma v|t|$$
$$\times\left(D - 4 - (D-3)\,v^2\right)\,. \tag{83}$$

This quantity has a meaning of the total energy transferred by the wall to the particle during the collision; it is opposite to the total energy transferred by the particle to the wall, so the energy balance is fulfilled indeed. The energy transfer linearly depends on time, this is the consequence of our linear approximation in the interaction constant. The sign depends on the relative velocity and the dimension. The somewhat unexpected feature here is the emergence of the potential terms due to effective localization of gravitational stresses. There is nothing mysterious here, however, since the gravitational interaction in this order looks like an action at a distance. The retardation effect is only in the branon wave, but as we have seen, this does not contribute to the energy transfer in the linear order. Though we did not consider here the transfer of the spatial momentum, it is worth noting that the corresponding balance equations are more complicated. First, the contribution of he branon waves is non-zero, and second, there is non-zero momentum flux through the lateral surface of the world tube [35].

6 Conclusion

We have discussed some unusual properties of gravitational interaction of the point particle with the Nambu-Goto domain wall. In the static case the interaction force is repulsive, so the particle impinging on the wall is decelerated. The wall is not simply repelled but it gets excited, which is interpreted as the branon. If the particle reaches the wall and perforate it, a second shock-like spherical branon wave is created which then propagates outward along the wall. Both these branons do not carry the energy within the lowest non-trivial approximation of the perturbation theory in terms of the gravitational coupling constant.

We then analyzed the energy conservation in the same order including the contribution of the mediating field. Generically, the field stresses are non-local, but we have discovered that in the lowest non-trivial order of the perturbation theory their contribution can be unambiguously split into two parts which are effectively localized on the particle world-line and the wall world-volume and can be therefore prescribed to the particle and the wall separately, leading to the notion of the relativistic potential energy in the collision problem.

A question arises whether similar picture can be valid in other collision problems. Actually, the nature of the mediating field is not essential. Two other features seem to be crucial for possibility to define relativistic potentials. One is the use of the perturbation theory in terms of the interaction coupling, and another is the locality of the colliding objects, which looks natural within the classical theory. In fact, iterations in terms of the coupling constant is the standard tool in the quantum field theory as well, but the use of non-localized particle states with definite momentum would make such a procedure obscure. However, in the case of localized wave packets this still may work, so further investigations along these lines are desirable.

Acknowledgments

This work was supported by the RFBR grant 14-02-01092.

The research of PS is partially implemented under the "ARISTEIA II" Action of the Operational Program "Education and Lifelong Learning"; it is also co-funded by the European Social Fund (ESF) and National Resources and is supported by the EU program "Thales" ESF/NSRF 2007-2013. Finally, PS is grateful to the non-commercial "Dynasty" foundation (Russian Federation) for financial support.

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Received 19.11.2014

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ГРАВИТАЦИОННОЕ СТОЛКНОВЕНИЕ ЧАСТИЦ С ДОМЕННЫМИ СТЕНКАМИ И РЕЛЯТИВИСТСКИЕ ПОТЕНЦИАЛЫ

Рассматривается столкновение точечной частицы с бесконечно тонкой плоской доменной стенкой в линейном и пост-линейном приближениях классической эйнштейновской гравитации в пространстве Минковского произвольной размерности. Оба сталкивающихся объекта трактуются динамически с учетом бранонного возбуждения доменной стенки. Баланс энергии в этом процессе нетривиален, так как сила взаимодействия не спадает с растоянием, и ни частица, ни стенка никогда не являются свободными. Показано, что гравитационные натяжения эффективно локализуются в мировых объемах частицы и стенки и дают релятивистские потенциалы каждого из объектов в гравитационном поле партнера. Вклад бранонов в энергию доменной стенки в низшем приближении по гравитационной постоянной равен нулю.

Ключевые слова: *гравитация, браны, доменные стенки, законы сохранения.*

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