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FAST SPINNING PARTICLE IN EXTERNAL ELECTROMAGNETIC AND GRAVITATIONAL FIELDS

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Three-dimensional acceleration in general relativity can be defined so that spinless particle in the process of evolution in an external gravitational field can not exceed the speed of light. We ask what happens if a spinless particle is replaced by test-particle with spin one-half. Inclusion of spin-gravitational interaction gives generalized Papapetrou equations with modified metric along the world-line. The modified metric should be used to calculate the acceleration of spinning particles. Inclusion of spin-electromagnetic interaction in flat space gives generalized Frenkel equations, where an effective metric along world-line arises for the particle with anomalous magnetic moment. This implies that intervals of time (and distance) probed by such particle in the presence of electromagnetic field slightly differ from those in empty space.

Keywords: *ultra-relativistic spinning particle, Frenkel equation, BMT equation, Papapetrou equation*

1 Introduction

Basic notions of Special and General Relativity have been formulated before the discovery of spin, so they describe space-time properties as they are seen by spinless test-particle. In this work we raise the question whether these notions remain the same if the spinless particle is replaced by more realistic spinning test-particle.

To account spin, we need a systematically constructed classical model for relativistic description of rotational degrees of freedom. In recent works [1–4], we have obtained equations of motion of spinning particle which generalize those of Frenkel-BMT (Papapetrou) to the case of an arbitrary electromagnetic (gravitational) fields. This allows us to start the detailed analysis of old suggestions [5–7] on possible modifications of metric properties of space-time as they are seen by spinning particle in the presence of electromagnetic (gravitational) fields.

Our starting observation is that speed of light does not represent special point of the complete equations of motion. As a consequence, if we preserve the usual special-relativity definitions of time and distance, the critical speed that the particle with anomalous magnetic moment can not overcome during its evolution in electromagnetic field slightly differs from the speed of light. To improve this point, we notice that interaction of spin with electromagnetic field induces the matrix (6), which can be considered as effective metric along the world-line. So, we can follow the general-relativity prescriptions to analyze the spinning particle in an external electromagnetic field.

Therefore we turn to the analysis of ultra-relativistic behavior of spinless particle moving along geodesic in gravitational field. We propose three-dimensional acceleration (which reduces to that of Landau-Lifshitz for the constant gravitational field), and show that this guarantees impossibility for spinless particle to exceed the speed of light. Then we apply the formalism to the effective metric which arose for spinning particle in Minkowski space in the presence of electromagnetic field. With our definition of acceleration, the critical speed coincides with the speed of light. The price to pay is that intervals of time (and distance) probed by such particle in the presence of electromagnetic field slightly differ from those in empty space. At last, we show that interaction of spin with gravitational background also implies slight deformation (24) of initial metric. The deformed metric need to be used for definition of the acceleration.

2 Spinless particle on electromagnetic background

Spinning particle in arbitrary electromagnetic and gravitational backgrounds obeys rather complicated equations of motion [2–4]. So it is instructive to begin our discussion with more simple case of spinless particle. Typical relativistic equations of motion (EM) have singularity at some value of a particle velocity. For instance, the standard Lagrangian of spinless particle in electromagnetic field

$$L=\frac{1}{2\lambda}\dot{x}^2-\frac{\lambda}{2}m^2c^2+\frac{e}{c}A\dot{x}, \qquad (1)$$

implies the manifestly relativistic and reparametrization invariant equations $DDx^\mu=\frac{e}{mc^2}F^\mu{}_\nu Dx^\nu$, where $D=\frac{1}{\sqrt{-\dot{x}^2}}\frac{d}{d\tau}$. They became singular as $\dot{x}^2\to 0$. Using reparametrization invariance of EM, we take physical time as the parameter, $\tau=t$, then $x^\mu=(ct,\mathbf{x}(t))$, $\dot{x}^\mu=(c,\mathbf{v}(t))$ and $\frac{1}{\sqrt{-\dot{x}^2}}=\frac{1}{\sqrt{c^2-\mathbf{v}^2}}$. For the acceleration $\mathbf{a}=\frac{d\mathbf{v}}{dt}$, the EM imply $\mathbf{va}=[c^2-\mathbf{v}^2]^{\frac{3}{2}}\frac{e(\mathbf{vE})}{mc^3}$, that is the longitudinal acceleration vanishes as $|\mathbf{v}| \to c$. Hence the singularity implies that the particles speed can not exceed the value c.

We will use the following terminology. The speed v_{cr} that a particle can not exceed during its evolution in an external field is called critical speed. The observer independent scale c of special relativity is called, as usual, the speed of light. According to the expression for $\mathbf{a}$ above, critical speed of a spinless particle coincides with the speed of light.

Let us point out two possible modifications which could yield non vanishing acceleration as $v \to c$.

First, this would be for the sufficiently singular external force. For example, for a particle in electromagnetic background, this could be $Dx^\nu\left[1+\kappa(DxF\partial_\alpha FDx)Dx^\alpha\right]$. The right dimension can be supplied by the constant κ proportional to $\frac{e^8}{m^5c^{10}}$ or $\frac{\hbar^4}{m^5c^6}$.

Second, in the presence of external fields, we can construct additional reparametrization invariants. For instance, we can use the derivative $D' \equiv \frac{1}{\sqrt{-\dot{x}g\dot{x}}}\frac{d}{d\tau}$, where

$$-\dot{x}^\mu g_{\mu\nu}\dot{x}^\nu = c^2 - \mathbf{v}^2 - k(\dot{x}FF\dot{x}), \qquad (2)$$

with k equal to $\frac{e^6}{m^4c^8}$ or $\frac{\hbar^3}{m^4c^5}$. The surface $\dot{x}g\dot{x} = 0$ is slightly different from the sphere $c^2 - \mathbf{v}^2 = 0$. For the particle with equations $D'D'x^\mu = \frac{e}{mc^2}F^\mu{}_\nu D'x^\nu$, acceleration vanishes at the critical velocity different from the speed of light. To see this, we compute

$$-\dot{x}FF\dot{x} = c^2E_i\left(\delta_{ij} - \frac{v_iv_j}{c^2}\right)E_j + \mathbf{v}^2B_iN_{ij}B_j.$$

Here $N \equiv 1 - \frac{\mathbf{v}\mathbf{v}}{\mathbf{v}^2}$ is projection operator on the plane orthogonal to the vector $\mathbf{v}$, so we can write $\mathbf{B}N\mathbf{B} = (N\mathbf{B})^2 = \mathbf{B}_\perp^2$. Then the factor (2) reads

$$-\dot{x}g\dot{x} = c^2 - \mathbf{v}^2 + k\left[c^2\mathbf{E}(1 - \frac{\mathbf{v}\mathbf{v}}{c^2})\mathbf{E} + \mathbf{v}^2\mathbf{B}_\perp^2\right].$$

Besides, $1 - \frac{\mathbf{v}\mathbf{v}}{c^2}$ turns into the projection operator N when $|\mathbf{v}| = c$. Hence $-\dot{x}g\dot{x} \overset{|\mathbf{v}|\to c}{\longrightarrow} kc^2[\mathbf{E}_\perp^2 + \mathbf{B}_\perp^2]$. If $\mathbf{E}$ and $\mathbf{B}$ are not mutually parallel in the laboratory system, this expression does not vanish for any orientation of $\mathbf{v}$. Thus, acceleration does not vanish at $|\mathbf{v}| = c$. The particle could exceed the speed of light and then approaches to the critical velocity.

Combining these observations, let us write relativistic equation of the form

$$[(\dot{x}g\dot{x})\eta^{\mu\nu} - \dot{x}^\mu(g\dot{x})^\nu]\,\ddot{x}^\nu = -(\dot{x}g\dot{x})^2f^\mu(F, D'x), \qquad (3)$$

with f^μ being polynomial on $(D'x)^{2n}$ up to $n = 2$. The system has the following properties. A) It is manifestly relativistic covariant, that is speed of light represents the invariant scale of the model. B) Due to reparametrization invariance, there are only only three independent equations in the system. This allows to interpret them as equations of motion of relativistic particle. C) The equations make sense for any velocity. D) If $f^\mu \sim (D'x)^2$ or less, acceleration of the particle may be non vanishing at $|\mathbf{v}| = c$, but vanishes at the critical velocity $\mathbf{v}_{cr}$. If $f^\mu \sim (D'x)^4$, Eq. (3) has no critical points. So the particle could exceed the speed of light and then continues accelerate. Below, we repeat this analysis for more realistic case of a particle with spin.

3 Spinning particle in electromagnetic background within the standard special-relativity notions

Consider spinning particle with mass m, electric charge e and magnetic moment μ interacting with an arbitrary electromagnetic field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The manifestly Poincare and reparametrization invariant Lagrangian on configuration space with coordinates $x^\mu(\tau)$, $\omega^\mu(\tau)$ and $\lambda(\tau)$ reads [4]

$$L = \frac{1}{4\lambda}\Big[\dot{x}N\dot{x} + \lambda D\omega ND\omega - \sqrt{[\dot{x}N\dot{x} + \lambda D\omega ND\omega]^2 - 4\lambda(\dot{x}ND\omega)^2}\Big] - \frac{\lambda}{2}m^2c^2 + \frac{\alpha}{2\omega^2} + \frac{e}{c}A\dot{x}. \qquad (4)$$

This depends on one free parameter $\alpha = \frac{3\hbar^2}{4}$, this particular value corresponds to spin one-half particle. Similarly to Eq. (1), the only auxiliary variable is λ, this provides the mass-shell condition. It has been denoted $N^{\mu\nu} \equiv \eta^{\mu\nu} - \frac{\omega^\mu\omega^\nu}{\omega^2}$, then $N^{\mu\nu}\omega_\nu = 0$. To introduce coupling of the position variable x with electromagnetic field, we have added the minimal interaction $\frac{e}{c}A_\mu\dot{x}^\mu$. As for basic variables of spin ω^μ, they couple with A^μ through the term $D\omega^\mu \equiv \dot{\omega}^\mu - \lambda\frac{e\mu}{c}F^{\mu\nu}\omega_\nu$. The Frenkel spin-tensor [8] is a composite quantity constructed from ω^μ and its conjugated momentum π^μ as follows[1]:

$$J^{\mu\nu} = 2(\omega^\mu\pi^\nu - \omega^\nu\pi^\mu) = (J^{i0} = D^i,\ J_{ij} = 2\epsilon_{ijk}S_k).$$

Here S_i is three-dimensional spin-vector and D_i is dipole electric moment.

The action implies EM which generalize those of Frenkel and Bargmann-Michel-Telegdi to the case of an arbitrary electromagnetic field. They have been studied in [2,4]. For the present discussion we need only symbolic form of the equation for position variable, this reads

$$\frac{d}{d\tau}\left[\frac{\dot{x}^\mu}{\sqrt{-\dot{x}g\dot{x}}}\right] = f^\mu\,, \qquad (5)$$

[1]The basic variables ω^μ do not represent observable quantities since they are not invariant under spin-plane local symmetry presented in the model [1].

where f^μ is the polynomial $f^\mu = \sqrt{-\dot{x}g\dot{x}}a_1 + a_2 + \frac{a_3}{\sqrt{-\dot{x}g\dot{x}}} + \frac{a_4}{\dot{x}g\dot{x}}$, with the coefficients a_i that are finite as $\dot{x}g\dot{x} \to 0$. The effective metric arises for the particle with anomalous magnetic moment $\mu \neq 1$

$$g_{\mu\nu} = [\eta + b(\mu - 1)(JF + FJ) \\ +b^2(\mu - 1)^2 FJJF]_{\mu\nu}, \tag{6}$$

where $b \equiv \frac{-2e}{4m^2c^3 - 3e\mu(JF)}$. The variational problem (4) yields also the value-of-spin and Frenkel conditions $J^{\mu\nu}J_{\mu\nu} = 6\hbar^2$, $J^{\mu\nu}\mathcal{P}_\nu = 0$, where the canonical momentum is $\mathcal{P}_\nu = \frac{\partial L}{\partial \dot{x}^\nu} - \frac{e}{c}A_\nu$. They provide consistent quantization which yields one-particle (positive energy) sector of the Dirac equation, see [1].

Inclusion of these constraints into a variational problem, as well as the search for an interaction consistent with them turn out to be rather non trivial tasks [5,9], and the expression (4) is probably the only solution of the problem. So, the appearance of effective metric (6) in Eq. (5) seems to be unavoidable in a systematically constructed model of spinning particle.

The speed of light does not represent special point of the manifestly relativistic equation (5). Singularity occurs at the critical velocity $\mathbf{v}_{cr}$ determined by the equation $\dot{x}g\dot{x} = 0$. It can be shown [4] that even in homogeneous fields there are configurations admitting $v_{cr} > c$. In general case we expect that $\mathbf{v}_{cr}$ is both field and spin-dependent quantity. Using spacial part of Eq. (5), we can estimate acceleration near v_{cr}. Explicit computation gives $\mathbf{va} \sim \sqrt{-\dot{x}g\dot{x}}$, that is the acceleration along the direction of velocity vanishes as $|\mathbf{v}| \to v_{cr} > c$.

In resume, if we insist to preserve the usual special-relativity definitions of time and distance, critical speed of spinning differs from the speed of light. To see, whether we can keep the condition $v_{cr} = c$, we use formal similarity of the matrix g appeared in (5) with space-time metric. Then we can follow the general-relativity prescription to define time and distance in the presence of electromagnetic field. So, let us stop for a moment to discuss the definitions of velocity and acceleration in general relativity.

4 Three-dimensional acceleration of spinless particle in general relativity

Consider pseudo Riemann space

$$\mathbf{M}^{(1,3)} = \{x^\mu,\ g_{\mu\nu}(x^\rho),\ g_{00} < 0\}. \tag{7}$$

Three-dimensional velocity and acceleration can be defined in such a way that speed of light represents coordinate independent quantity and, more over, a particle during its evolution in curved background can not exceed the speed of light.

To achieve this, we represent interval in 1+3 block-diagonal form [10]

$$-ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -c^2\left[\frac{\sqrt{-g_{00}}}{c}(dx^0 + \frac{g_{0i}}{g_{00}}dx^i)\right]^2 \\ + \left(g_{ij} - \frac{g_{0i}g_{0j}}{g_{00}}\right)dx^i dx^j.$$

This prompts to introduce infinitesimal time interval, distance and speed as follows:

$$dt = \frac{\sqrt{-g_{00}}}{c}(dx^0 + \frac{g_{0i}}{g_{00}}dx^i), \\ dl^2 = (g_{ij} - \frac{g_{0i}g_{0j}}{g_{00}})dx^i dx^j \equiv \gamma_{ij}dx^i dx^j, \quad v = \frac{dl}{dt}. \tag{8}$$

The conversion factor between the world time x^0 and the physical time t is

$$\frac{dt}{dx^0} = \frac{\sqrt{-g_{00}}}{c}(1 + \frac{g_{0i}}{g_{00}}\frac{dx^i}{dx^0}). \tag{9}$$

Introduce also the three-velocity vector

$$v^i = \left(\frac{dt}{dx^0}\right)^{-1}\frac{dx^i}{dx^0}, \tag{10}$$

or, symbolically $v^i = \frac{dx^i}{dt}$. This is consistent with the above definition of v: $v^2 = \left(\frac{dl}{dt}\right)^2 = \mathbf{v}^2 = v^i\gamma_{ij}v^j$. In the result, the interval acquires the form similar to special relativity (but now we have $\mathbf{v}^2 = \mathbf{v}\gamma\mathbf{v}$)

$$-ds^2 = -c^2dt^2 + dl^2 = -c^2dt^2\left(1 - \frac{\mathbf{v}^2}{c^2}\right). \tag{11}$$

This equality holds in any coordinate system x^μ. Hence a particle with $ds^2 = 0$ has the speed $\mathbf{v}^2 = c^2$.

The formalism remains manifestly covariant under subgroup of spacial transformations $x^0 = x'^0$, $x^i = x^i(x'^j)$, $a^i{}_j(x') \equiv \frac{\partial x^i}{\partial x'^j}$. Under these transformations g_{00} is a scalar function, g_{0i} is a vector while g_{ij} and γ_{ij} are tensors. Since $g^{ij}\gamma_{jk} = \delta^i{}_k$, the inverse metric of γ_{ij} turns out to be $(\gamma^{-1})^{ij} = g^{ij}$. The velocity (10) behaves as a vector $v^i(x^0) = a^i{}_j(x'^k(x^0))v'^j(x^0)$, so it is convenient to introduce the covariant derivatives D_k and D_0

$$D_k v^i = \partial_k v^i + \tilde{\Gamma}^i{}_{kj}(\gamma)v^j, \quad D_0 v^i = \frac{dx^k}{dx^0}D_k v^i. \tag{12}$$

The Christoffel symbols $\tilde{\Gamma}^i{}_{jk}(\gamma)$ are constructed with help of three-dimensional metric $\gamma_{ij}(x^0, x^k)$ written in Eq. (8), where x^0 is considered as a parameter

$$\tilde{\Gamma}^i{}_{jk} = \frac{1}{2}\gamma^{ia}(\partial_j\gamma_{ak} + \partial_k\gamma_{aj} - \partial_a\gamma_{jk}). \tag{13}$$

As a consequence, the metric γ is covariantly constant, $D_k\gamma_{ij} = 0$.

Hence we associated with $\mathbf{M}^{(1,3)}$ the one-parameter family of three-dimensional spaces $\mathbf{M}^3_{x^0} =$

$\{x^k, \ \gamma_{ij}, \ D_k\gamma_{ij} = 0\}$. Note that velocity has been defined above with help of the curve in $\mathbf{M}^3_{x^0}$ parameterized by this parameter, $x^i(x^0)$.

In the case of constant gravitational field $g_{\mu\nu}(x^k)$, we have usual three-dimensional Riemann geometry $\mathbf{M}^3 = \{x^i, \ \gamma_{ij}(x^k)\}$, and the standard notion of a parallel transport. So an analog of constant vector field of Euclidean geometry is the covariantly-constant field, $D_0\xi^i = 0$, and the acceleration with respect to physical time is defined by

$$a^i = \left(\frac{d\tau}{dx^0}\right)^{-1} D_0 v^i = \left(\frac{d\tau}{dx^0}\right)^{-1} \frac{dv^i}{dx^0} + \tilde{\Gamma}^i{}_{jk}v^j v^k. \quad (14)$$

To define an acceleration in general case, we need to adopt some notion of a constant vector field (parallel transport equation) along the trajectory $x^i(x^0)$ that cross the family $\mathbf{M}^3_{x^0}$. In Euclidean space the scalar product of two constant fields has the same value at any point. In particular, taking the scalar product along a line $x^i(x^0)$, we have $\frac{d}{dx^0}(\xi, \eta) = 0$. For the constant fields in our case it is natural to demand the same (necessary) condition: $\frac{d}{dx^0}[\xi^i(x^0)\gamma_{ij}(x^0, x^i(x^0))\eta^i(x^0)] = 0$. Taking into account that $D_k\gamma_{ij} = 0$, this condition can be written as follows

$$(D_0\xi + \frac{1}{2}(\xi\partial_0\gamma\gamma^{-1}), \eta) + (\xi, D_0\eta + \frac{1}{2}(\gamma^{-1}\partial_0\gamma\eta)) = 0.$$

So we take the parallel-transport equation to be

$$D_0\xi^i + \frac{1}{2}(\xi\partial_0\gamma\gamma^{-1})^i = 0. \quad (15)$$

Then we define the acceleration with respect to physical time as follows:

$$a^i = \left(\frac{dt}{dx^0}\right)^{-1} \left[D_0 v^i + \frac{1}{2}(\mathbf{v}\partial_0\gamma\gamma^{-1})^i\right]. \quad (16)$$

For the special case of constant gravitational field, $g_{\mu\nu}(x^i)$, the definition (16) reduces to that of Landau and Lifshitz, see page 251 in [10].

Let us estimate the acceleration as $v \to c$. Particle in general relativity propagates along geodesics of $\mathbf{M}^{(1,3)}$. If we take the proper time to be the parameter, geodesics obey the system

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0, \quad g_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = -1. \quad (17)$$

The system has no sense for the case we are interested in, $ds^2 \to 0$. So we rewrite it in arbitrary parametrization λ (here we denote $\dot{x}^\mu = \frac{dx^\mu}{d\lambda}$)

$$\frac{d}{d\lambda}\left(\frac{\dot{x}^\mu}{\sqrt{-\dot{x}g\dot{x}}}\right) + \Gamma^\mu{}_{\alpha\beta}(g)\dot{x}^\alpha\frac{\dot{x}^\beta}{\sqrt{-\dot{x}g\dot{x}}} = 0. \quad (18)$$

To see, which equation for a^i implied by (18), we write the latter through v^i. Using reparametrization invariance, we can take $\lambda = x^0$ in (18). Then spacial part of geodesic equation (18) reads

$$\left(\frac{d\tau}{dx^0}\right)^{-1}\frac{d}{dx^0}\frac{v^i}{\sqrt{1-\frac{\vec{v}\gamma\vec{v}}{c^2}}} = \frac{G^i}{\sqrt{1-\frac{\vec{v}\gamma\vec{v}}{c^2}}}, \quad (19)$$

where

$$G^i(g_{\mu\nu}, \mathbf{v}) = -\left(\frac{d\tau}{dx^0}\right)^{-2}\Gamma^i{}_{00} - \Gamma^i{}_{jk}v^j v^k -2\left(\frac{d\tau}{dx^0}\right)^{-1}\Gamma^i{}_{0k}v^k, \quad (20)$$

is non-singular function as $v \to c$. We compute derivative on l.h.s. of the equation (19)

$$\left(\frac{d\tau}{dx^0}\right)^{-1}\left[M^i{}_jD_0v^j + \frac{(\mathbf{v}\partial_0\gamma\mathbf{v})}{2(c^2-\mathbf{v}\gamma\mathbf{v})}v^i\right] = G^i + \tilde{\Gamma}^i{}_{jk}v^jv^k, \quad (21)$$

where $M^i{}_j = \delta^i{}_j + \frac{v^i(\mathbf{v}\gamma)_j}{c^2-\mathbf{v}\gamma\mathbf{v}}$. We apply the inverse matrix $\tilde{M}^i{}_j = \delta^i{}_j - \frac{v^i(\mathbf{v}\gamma)_j}{c^2}$, and then complete D_0v^i up to the acceleration (16). Then (21) reads

$$a^i = \frac{1}{2}\left(\frac{d\tau}{dx^0}\right)^{-1}\left[(\mathbf{v}\partial_0\gamma\gamma^{-1})^i - \frac{(\mathbf{v}\partial_0\gamma\mathbf{v})}{c^2}v^i\right] + \left(\delta^i{}_j - \frac{v^i(\mathbf{v}\gamma)_j}{c^2}\right)[\tilde{\Gamma}^j{}_{kl}(\gamma)v^kv^l + G^j]. \quad (22)$$

Then the acceleration along the velocity is

$$\mathbf{v}\gamma\mathbf{a} = \frac{1}{2}\left(\frac{d\tau}{dx^0}\right)^{-1}\left[(\mathbf{v}\partial_0\gamma\mathbf{v}) - (\mathbf{v}\partial_0\gamma\mathbf{v})\frac{(\mathbf{v}\gamma\mathbf{v})}{c^2}\right] + \left(1 - \frac{\mathbf{v}\gamma\mathbf{v}}{c^2}\right)(\mathbf{v}\gamma)_i[\tilde{\Gamma}^i{}_{kl}(\gamma)v^kv^l + G^i]. \quad (23)$$

This implies $\mathbf{v}\gamma\mathbf{a} \to 0$ as $\mathbf{v}\gamma\mathbf{v} \to c^2$. That is acceleration along the direction of velocity vanishes as the speed approximates the speed of light. With these definitions the spinless particle in an external gravitational field can not overcome the speed of light.

The last term in the definition (16) yields the important factor $(\mathbf{v}\partial_0\gamma\gamma^{-1})^i$ in Eq. (23). As EM (22) and (23) do not contain $\sqrt{c^2-\mathbf{v}\gamma\mathbf{v}}$, they have sense even for $v > c$. Without this factor, the particle in gravitational field could exceed c and then continues accelerate.

5 World-line geometry probed by spinning particle

The construction can be applied, without modifications, to the effective metric (6) appeared in EM of spinning particle (5). The metric depends on x^i via the field strength $F(x^0, x^i)$, and on x^0 via the field strength as well as via the spin-tensor $J(x^0)$. The geodesic equation is replaced now by Eq. (5). Adopting that velocity and acceleration of spinning particle in electromagnetic field are given by Eqs. (10) and (16), we obtain the theory with critical speed equal to the speed of light. The intervals of time (and distance) probed by such particle are given now by Eq. (8). They slightly differ from those in empty space.

Due to $\mu - 1$-factor in Eq. (6), the deformation of world-line geometry in electromagnetic field will be seen only by a particle with anomalous magnetic moment. In a gravitational field the deformed geometry could be probed by any spinning particle. To see this, let us consider the Frenkel electron in a curved background with the metric $\tilde{g}_{\mu\nu}$. We use the model constructed in [3]. The Lagrangian can be obtained from (4) (with $A^\mu = 0$) replacing $\eta_{\mu\nu}$ by $\tilde{g}_{\mu\nu}$ and usual derivative of ω^μ by the covariant derivative, $\dot{\omega}^\mu \rightarrow D\omega^\mu = \dot{\omega}^\mu + \Gamma^\mu_{\alpha\beta}(\tilde{g})\dot{x}^\alpha\omega^\beta$. This leads to EM consistent with those of Papapetrou [11], the latter are widely assumed as the reasonable equations of spinning particle in gravitational fields. In the EM (5) now appears the effective metric [3]

$$g_{\mu\nu} = \left[\tilde{g} + \beta(J\theta + \theta J) + \beta^2\theta JJ\theta\right]_{\mu\nu}, \qquad (24)$$

where $\beta \equiv \frac{1}{8m^2c^2 - J\cdot\theta}$, $\theta_{\mu\nu} \equiv R_{\mu\nu\alpha\beta}J^{\alpha\beta}$ and $R_{\mu\nu\alpha\beta}(\tilde{g})$ is the curvature tensor. Hence, to guarantee the observer-independence of c and the equality $v_{cr} = c$, we need to define velocity (10) and acceleration (16) using the deformed metric g instead of $\tilde{g}$.

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УЛЬТРАРЕЛЯТИВИСТСКАЯ СПИНОВАЯ ЧАСТИЦА ВО ВНЕШНЕМ ГРАВИТАЦИОННОМ И ЭЛЕКТРОМАГНИТНОМ ПОЛЯХ

Трехмерное ускорение в общей теории относительности можно определить так, что бесспиновая частица в процессе эволюции во внешнем гравитационном поле не сможет превысить скорость света. Мы выясняем, что происходит если бесспиновую частицу заменить на частицу со спином $\frac{1}{2}$. Учет спин-гравитационного взаимодействия приводит к обобщенным уравнениям Папапетроу с модифицированной вдоль мировой линии метрикой. Именно модифицированная метрика должна использоваться для вычисления трехмерного ускорения спиновой частицы. Взаимодействие спина с электромагнитным полем в плоском пространстве приводит к обобщенным уравнениям Френкеля, в которых возникает эффективная метрика вдоль мировой линии частицы с аномальным магнитным моментом. Для такой частицы интервалы времени (расстояния) в электромагнитном поле и в пустоте отличаются друг от друга.

Ключевые слова: *ультра-релятивистская спиновая частица, уравнения Френкеля, уравнения Папапетроу.*

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